E & M - Basic Physical Concepts

Electric force and electric field

Electric force between 2 point charges:

$|F| = k\,\frac{|q_1|\,|q_2|}{r^2}$

$k = 8.987551787 \times 10^9\ \mathrm{N\,m^2/C^2}$

$\epsilon_0 = \frac{1}{4\,\pi\,k} = 8.854187817 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$

$q_p = -q_e = 1.60217733\,(49) \times 10^{-19}\ \mathrm{C}$

$m_p = 1.672623\,(10) \times 10^{-27}\ \mathrm{kg}$

$m_e = 9.1093897\,(54) \times 10^{-31}\ \mathrm{kg}$

Electric field: $\vec{E} = \frac{\vec{F}}{q}$

Point charge: $|E| = k\,\frac{|Q|}{r^2}$, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$

Field patterns: point charge, dipole, $\|$ plates, rod, spheres, cylinders,...

Charge distributions:

Linear charge density: $\lambda = \frac{\Delta Q}{\Delta x}$

Area charge density: $\sigma_A = \frac{\Delta Q}{\Delta A}$

Surface charge density: $\sigma_{surf} = \frac{\Delta Q_{surf}}{\Delta A}$

Volume charge density: $\rho = \frac{\Delta Q}{\Delta V}$

Electric flux and Gauss' law

Flux: $\Delta\Phi = E\,\Delta A_\perp = \vec{E}\cdot\hat{n}\Delta A$

Gauss law: Outgoing Flux from S, $\Phi_S = \frac{Q_{enclosed}}{\epsilon_0}$

Steps: to obtain electric field

–Inspect $\vec{E}$ pattern and construct S

–Find $\Phi_s = \oint_{surface} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$, solve for $\vec{E}$

Spherical: $\Phi_s = 4\,\pi\,r^2\,E$

Cylindrical: $\Phi_s = 2\,\pi\,r\,\ell\,E$

Pill box: $\Phi_s = E\,\Delta A$, 1 side; $= 2\,E\,\Delta A$, 2 sides

Conductor: $\vec{E}_{in} = 0$, $E^{\|}_{surf} = 0$, $E^{\perp}_{surf} = \frac{\sigma_{surf}}{\epsilon_0}$

Potential

Potential energy: $\Delta U = q\,\Delta V$ $1\ \mathrm{eV} \approx 1.6 \times 10^{-19}\ \mathrm{J}$

Positive charge moves from high V to low V

Point charge: $V = \frac{k\,Q}{r}$ $V = V_1 + V_2 = \ldots$

Energy of a charge-pair: $U = \frac{k\,q_1\,q_2}{r_{12}}$

Potential difference: $|\Delta V| = |E\,\Delta s_{\|}|$,

$\Delta V = -\vec{E}\cdot\Delta\vec{s}$, $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$

$E = -\frac{d\,V}{dr}$, $E_x = -\frac{\Delta V}{\Delta x}\Big|_{fix\ y,z} = -\frac{\partial V}{\partial x}$, *etc.*

Capacitances $Q = C\,V$

Series: $V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots$, $Q = Q_i$

Parallel: $Q = C_{eq}\,V = C_1\,V + C_2\,V + \cdots$, $V = V_i$

Parallel plate-capacitor: $C = \frac{Q}{V} = \frac{Q}{E\,d} = \frac{\epsilon_0\,A}{d}$

Energy: $U = \int_0^Q V\,dq = \frac{1}{2}\,\frac{Q^2}{C}$, $u = \frac{1}{2}\,\epsilon_0\,E^2$

Dielectrics: $C = \kappa C_0$, $U_\kappa = \frac{1}{2\,\kappa}\frac{Q^2}{C_0}$, $u_\kappa = \frac{1}{2}\,\epsilon_0\,\kappa\,E_\kappa^2$

Spherical capacitor: $V = \frac{Q}{4\,\pi\,\epsilon_0\,r_1} - \frac{Q}{4\,\pi\,\epsilon_0\,r_2}$

Potential energy: $U = -\vec{p}\cdot\vec{E}$

Current and resistance

Current: $I = \frac{d\,Q}{dt} = n\,q\,v_d\,A$

Ohm's law: $V = I\,R$, $E = \rho J$

$E = \frac{V}{\ell}$, $J = \frac{I}{A}$, $R = \frac{\rho\ell}{A}$

Power: $P = I\,V = \frac{V^2}{R} = I^2\,R$

Thermal coefficient of ρ: $\alpha = \frac{\Delta\rho}{\rho_0\Delta T}$

Motion of free electrons in an ideal conductor:

$a\,\tau = v_d \to \frac{q\,E}{m}\,\tau = \frac{J}{n\,q} \to \rho = \frac{m}{n\,q^2\,\tau}$

Direct current circuits $V = I\,R$

Series: $V = I\,R_{eq} = I\,R_1 + I\,R_2 + I\,R_3 + \cdots$, $I = I_i$

Parallel: $I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots$, $V = V_i$

Steps: in application of Kirchhoff's Rules

–*Label currents:* $i_1, i_2, i_3, \ldots$

–*Node equations:* $\sum i_{in} = \sum i_{out}$

–*Loop equations:* "$\sum(\pm\mathcal{E}) + \sum(\mp iR) = 0$"

–*Natural:* "+" for loop-arrow entering − terminal

"−" for loop-arrow-parallel to current flow

RC circuit: if $\frac{d\,y}{dt} + \frac{1}{R\,C}\,y = 0$, $y = y_0\exp(-\frac{t}{R\,C})$

Charging: $\mathcal{E} - V_c - R\,i = 0$, $\frac{1}{c}\,\frac{d\,q}{dt} + R\,\frac{d\,i}{dt} = \frac{i}{c} + R\,\frac{d\,i}{dt} = 0$

Discharge: $0 = V_c - R\,i = \frac{q}{c} + R\,\frac{d\,q}{dt}$, $\frac{i}{c} + R\,\frac{d\,i}{dt} = 0$

Magnetic field and magnetic force

$\mu_0 = 4\,\pi \times 10^{-7}\ \mathrm{T\,m/A}$

Wire: $B = \frac{\mu_0\,i}{2\,\pi\,r}$ **Axis of loop:** $B = \frac{\mu_0\,a^2\,i}{2\,(a^2+x^2)^{3/2}}$

Magnetic force: $\vec{F}_M = i\,\vec{\ell}\times\vec{B} \to q\,\vec{v}\times\vec{B}$

Loop-magnet ID: $\vec{\tau} = i\,\vec{A}\times\vec{B}$, $\vec{\mu} = i\,A\,\hat{n}$

Circular motion: $F = \frac{m\,v^2}{r} = q\,v\,B$, $T = \frac{1}{f} = \frac{2\,\pi\,r}{v}$

Lorentz force: $\vec{F} = q\,\vec{E} + q\,\vec{v}\times\vec{B}$

Hall effect: $V_H = \frac{F_M\,d}{q}$, $U = -\vec{\mu}\cdot\vec{B}$

Sources of $\vec{B}$ and magnetism of matter

Biot-Savart Law: $\Delta\vec{B} = \frac{\mu_0}{4\,\pi}\,\frac{i\,\Delta\vec{\ell}\times\hat{r}}{r^2}$, $B = \frac{\mu_0}{4\,\pi}\,\frac{q\,\vec{v}\times\hat{r}}{r^2}$

$\Delta B = \frac{\mu_0}{4\,\pi}\,\frac{i\,\Delta y}{r^2}\sin\theta$, $\sin\theta = \frac{a}{r}$, $\Delta y = \frac{r^2\,\Delta\theta}{a}$

Ampere's law: $\mathcal{M} = \oint_L \vec{B}\cdot d\vec{s} = \mu_0\,I_{encircled}$

Steps: to obtain magnetic field

–Inspect $\vec{B}$ pattern and construct loop L

–Find $\mathcal{M}$ and I_{encl}, and solve for $\vec{B}$.

Displ. current: $I_d = \epsilon_0\,\frac{d\,\Phi_E}{dt} = \epsilon_0\,\frac{d\,(E\,A)}{dt} = \frac{d\,Q_A}{dt}$

Magnetism in atom:

Orbital motion: $\mu = i\,A = \frac{e}{2\,m}\,L$

$L = m\,v\,r = n\,\hbar$, $\hbar = \frac{h}{2\,\pi} = 1.06 \times 10^{-34}\ \mathrm{J\,s}$

$\mu_{orbit} = n\,\mu_B$, $\mu_B = \frac{e\,\hbar}{2\,m} = 9.27 \times 10^{-24}\ \mathrm{J/T}$

Spin: $S = \frac{\hbar}{2}$, $\mu_{spin} = \mu_B$

Magnetism in matter:

$B = B_0 + B_M = (1+\chi)\,B_0 = (1+\chi)\,\mu_0\,\frac{B_0}{\mu_0} = \kappa_m\,H$

Ferromagnetic: $\chi \gg 1$ **Diamagnetic:** $-1 \ll \chi < 0$

Paramagnetic: $0 < \chi \ll 1$, $M = \frac{C}{T}B$

Faraday's law

$$\mathcal{E} = -N\,\frac{d\phi_B}{dt}, \qquad \phi_B = \int \vec{B}\cdot d\vec{A},$$
$$\mathcal{E} = \int \vec{E}\cdot d\vec{s}, \qquad \vec{E} = \frac{\vec{F}_M}{q}$$

Lenz law: Induced $\vec{B}$ opposes change of Φ_B

$$\frac{d\phi_B}{dt} = \frac{d(B\,A_\perp)}{dt} = \frac{dB}{dt}\,A_\perp + B\,\frac{dA_\perp}{dt}$$

Moving rods: $\frac{dA}{dt} = \ell\, v, \quad \frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}\,R\cdot R\,\theta\right)$

Rotating loop: $\frac{dA_\perp}{dt} = \frac{d}{dt}\,(A\,\cos\omega t)$

Cutting B lines $\rightarrow$ change $\phi_B \rightarrow E_{ind} \rightarrow \mathcal{E}_{ind}$

Maxwell equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}, \qquad \oint \vec{B}\cdot d\vec{A} = 0\,,$$
$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi_B}{dt}, \quad \oint \vec{B}\cdot d\vec{s} = \mu_0\left[I + \epsilon_0\,\frac{d\phi_E}{dt}\right]$$

Inductance

Mutual: $\mathcal{E}_2 = -M_{21}\,\frac{di_1}{dt}, \quad M_{21} = M_{12} = \frac{N_2\,\phi_{21}}{i_1}$

Self: $\mathcal{E} = -L\,\frac{di}{dt}, \quad L = \frac{N\,\phi}{i}, \quad V_L = L\,\frac{di}{dt}$

Long solenoid: $L = \frac{N\,B\,A}{i}, \quad B = \mu_0\,n\,i$

Energies: $U_L = \frac{1}{2}\,L\,i^2, \quad u_B = \frac{1}{2\,\mu_0}\,B^2$

$$U_C = \frac{1}{2\,C}\,q^2, \quad u_E = \frac{1}{2}\,\epsilon_0\,E^2$$

L C: $V_L + V_C = 0 \Rightarrow L\,\frac{di}{dt} = -\frac{q}{C}\ \ q = q_0\cos(\omega\,t + \delta),$

$$\omega = \sqrt{\frac{1}{L\,C}}, \quad U_C + U_L = U_{C\,\max} = U_{L\,\max} = U_0$$

Decay Equations: $\frac{dy}{dt} = -a\,y, \quad y = y_0\,\exp(-a\,t)$

L R: $\mathcal{E} = V_L + R\,i, \quad \frac{dV_L}{dt} + \frac{R\,V_L}{L} = 0,$

$$V_L = \mathcal{E}\,\exp\left(-\frac{Rt}{L}\right), \quad i = \frac{\mathcal{E}}{R}\left[1 - \exp\left(\frac{-R\,t}{L}\right)\right]$$

L R C:

$$Q \approx Q_0\,e^{-\frac{R}{2L}\,t}\cos\omega_d\,t, \quad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2\,L}\right)^2}$$

Underdamped, critically damped & overdamped

A C Circuits

Impedance: $[\text{Ohm} \equiv \Omega] \quad Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$

Inductive $X_L = \omega\,L$, Capactive $X_C = \frac{1}{\omega\,C}$

Mean value: $\bar{f}(t) = \frac{1}{T}\int_0^T f(t)\,dt$

$$[\sin\omega\,t]_{rms} = \overline{[\sin^2\omega\,t]}^{\frac{1}{2}} = \overline{[\tfrac{1}{2}\,(1 - \cos 2\,\omega\,t)]}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Electromagnetic waves

Properties of em waves:

$$E = E_m\cos(k\,z - \omega\,t), \quad B = \frac{E}{c}$$
$$v = \frac{dz}{dt} = \frac{\omega}{k} = \lambda\,f = \frac{\lambda}{T}, \quad n = \frac{c}{v}$$

speed of light: $c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} = 2.99792458 \times 10^8$ m/s

$\vec{B} \perp \vec{E}$**, propagating along:** $\vec{E}\times\vec{B}$

$$u = u_E + u_B, \quad u_E = u_B$$

Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}, \quad \bar{S} = \bar{I} = \frac{E_{rms}B_{rms}}{\mu_0}$

Intensity: $I = \frac{P}{A} = \frac{\Delta U}{A\,\Delta z}\,\frac{dz}{dt} = u\,c$

Energy conservation: $\int \vec{S}\cdot d\vec{A} = \frac{dU}{dt} + P_R$

Complete absorption: Momentum $p = \frac{U}{c}$

Pressure: $P = \frac{F}{A} = \frac{\Delta p}{\Delta t}\,\frac{1}{A} = \frac{\Delta U}{c\,\Delta t}\,\frac{1}{A} = u = \frac{S}{c}$

Complete reflection: $P = \frac{2\,U}{c}, \quad P = \frac{2\,S}{c}$

Reflection and Refraction

Index of refraction: $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

Snell's law: $n_1\sin\theta_1 = n_2\sin\theta_2$

Critical angle: $n_2 > n_1, \quad n_2\sin\theta_c = n_1\sin 90^\circ$

Total reflection: $\theta > \theta_c$

Mirrors and lenses

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Ray tracing rules:

Mirror: At symm pt S, reflected symmetrically through center of sphere, undeflected. Parallel to axis, converges toward F (or diverges away from F), $f = \frac{R}{2}$.

Lens: Through center of lens, undeflected. Parallel to axis, converges toward F (or diverges away from F)

Image: $q > 0$ (real), $q < 0$ (virtual)

Focal point F**:** at $p = \infty, \quad q = f$

$f = \pm|f|$, "+" convergent, "−" divergent

Magnification: $M = \frac{h'}{h} = -\frac{q}{p}$

Refraction at spherical surface: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

R is coordinate of center with origin at S, with S the symmetry point of surface on the axis

Lens maker: $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Two media: $M = \frac{h'}{h} = -\frac{q}{p}\,\frac{n_1}{n_2}$

Huygen's principles:

Points in wave front are sources of next wavelets

Forward tangent surface is next wave front

Interference

Maxima $\phi = 0, 2\,\pi, 4\,\pi, \cdots$; Minima $\phi = \pi, 3\,\pi, 5\,\pi, \cdots$

Double slits: $I_{average} = I_0\cos^2\left(\frac{\phi}{2}\right), \quad \phi = k\,\Delta\,.$

$\sin\theta = \frac{\Delta}{d},\ \tan\theta = \frac{y}{L}, \qquad$ for small $\theta,\ \theta \approx \sin\theta \approx \tan\theta$

Phasor diagram: $\vec{A} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \cdots$

$$A_x = A_{1x} + A_{2x} + A_{3x} + \cdots, \quad A_y = A_{1y} + A_{2y} + \cdots$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

First minimum for N **slits:** $\phi = \frac{2\,\pi}{N}$

Thin film: $\phi = k\,\Delta + |\phi_{1_{reflected}} - \phi_{2_{reflected}}|, \quad \Delta = 2\,t$

$\phi_{reflected} = \pi$ (denser medium); $=0$ (lighter medium)

Diffraction

Single slit: $I = I_0\left[\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right]^2, \quad \beta = k\Delta, \quad \Delta = a\,\sin\theta$

Resolution criterion: $\theta_{criterion} = 1.22\,\frac{\lambda}{D}$

Grating: Principle maxima $\Delta = m\,\lambda$

Polarization

Brewster $(n_1 < n_2)$**:** $n_1\sin\theta_{br} = n_2\sin(\frac{\pi}{2} - \theta_{br})$

Polarizer: $E_{transmit} = E_0\cos\theta, \quad I = I_0\cos^2\theta$

Unpolarized light: $\frac{\Delta I}{\Delta\theta} = \frac{I_0}{2\,\pi}$

Transmitted Intensity: $\Delta I' = \Delta I\,\cos^2\theta$

$$I' = \frac{I_0}{2\,\pi}\int_0^{2\,\pi}\cos^2\theta\,d\theta = \frac{I_0}{2}$$

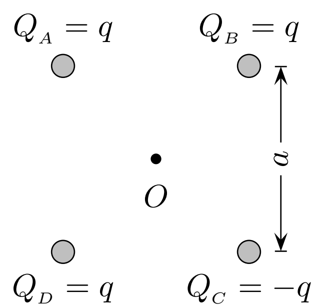
This print-out should have 36 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. V1:1, V2:1, V3:1, V4:1, V5:2.

Four Charges in Square JMS

23:03, trigonometry, multiple choice, < 1 min, fixed.

001 (part 1 of 1) 10 points

Consider charges in a square again, but this time with a different assignment of charges (shown in the figure below).



Find E_O at O.

1. $E_O = 4\,\frac{k\,q}{a^2}$ **correct**

2. $E_O = \sqrt{2}\,\frac{k\,q}{a^2}$

3. $E_O = 2\,\sqrt{2}\,\frac{k\,q}{a^2}$

4. $E_O = \frac{k\,q}{a^2}$

5. $E_O = \frac{1}{\sqrt{2}}\,\frac{k\,q}{a^2}$

6. $E_O = \frac{1}{5\,\sqrt{2}}\,\frac{k\,q}{a^2}$

7. $E_O = \frac{1}{4\,\sqrt{2}}\,\frac{k\,q}{a^2}$

8. $E_O = 3\,\frac{k\,q}{a^2}$

9. $E_O = 3\,\sqrt{2}\,\frac{k\,q}{a^2}$

10. $E_O = \frac{1}{3\,\sqrt{2}}\,\frac{k\,q}{a^2}$

Explanation:

The magnitudes of all four E-components at O are equal to $E_A = 2\,k\,\frac{q}{a^2}$. Draw a diagram, similar to the one in the explanation to part 1, to show the directions of the field vectors at O.

You should find that the contributions from B and D cancel, whereas the contributions from A and C add. This means the magnitude of the total field is

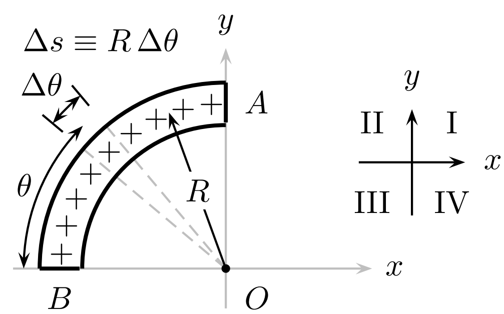
$$E = (2)\,(2)\,k\,\frac{q}{a^2} = \boxed{4\,k\,\frac{q}{a^2}}.$$

Charged Arc JMS

, , , < 1 min, .

002 (part 1 of 1) 10 points

A uniformly charged circular arc AB of radius R is shown in the figure. It covers a quarter of a circle and it is located in the second quadrant. The total charge on the arc is $Q > 0$.



The direction of the electric field vector $\vec{E}$ at the origin, due to the charge distribution, is

1. in quadrant IV. **correct**

2. along the positive x-axis.

3. along the positive y-axis.

4. along the negative y-axis.

5. along the negative x-axis.

6. in quadrant I.

7. in quadrant III.

8. in quadrant II.

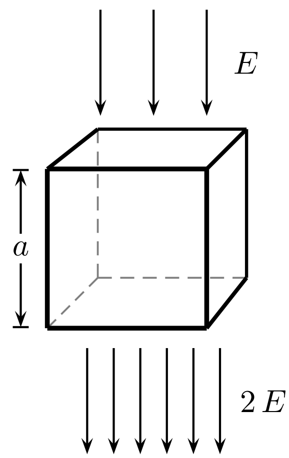
Explanation:

The electric field for a positive charge is directed away from it. In this case, the electric field generated by each Δq will be directed into quadrant IV, so the total electric field will be in the same quadrant.

Charge Inside a Box 02
24:02, calculus, multiple choice, < 1 min, fixed.

003 (part 1 of 1) 10 points

A cubic box of side a, oriented as shown, contains an unknown charge. The vertically directed electric field has a uniform magnitude E at the top surface and $2E$ at the bottom surface.



How much charge Q is inside the box?

1. $Q_{encl} = 0$

2. $Q_{encl} = 2\,\epsilon_0\,E\,a^2$

3. $Q_{encl} = \epsilon_0\,E\,a^2$ **correct**

4. $Q_{encl} = \dfrac{1}{2}\,\epsilon_0\,E\,a^2$

5. $Q_{encl} = 3\,\epsilon_0\,E\,a^2$

6. $Q_{encl} = 2\,\dfrac{E}{\epsilon_0\,a^2}$

7. $Q_{encl} = \dfrac{E}{\epsilon_0\,a^2}$

8. $Q_{encl} = 3\,\dfrac{E}{\epsilon_0\,a^2}$

9. $Q_{encl} = 6\,\epsilon_0\,E\,a^2$

10. insufficient information

Explanation:

Electric flux through a surface S is, by convention, positive for electric field lines going *out of* the surface S and negative for lines going in.

Here the surface is a cube and no flux goes through the vertical sides. The top receives

$$\Phi_{\text{top}} = -E\,a^2$$

(inward is negative) and the bottom

$$\Phi_{\text{bottom}} = 2\,E\,a^2\,.$$

The total electric flux is

$$\Phi_E = -E\,a^2 + 2\,E\,a^2 = E\,a^2\,.$$

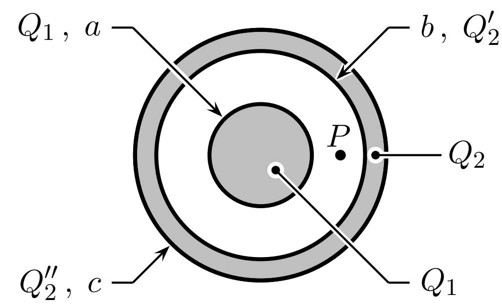
Using Gauss's Law, the charge inside the box is

$$Q_{\text{encl}} = \epsilon_0\,\Phi_E = \boxed{\epsilon_0\,E\,a^2}\,.$$

Concentric Conductors JMS
24:04, calculus, multiple choice, > 1 min, fixed.

004 (part 1 of 3) 10 points

Consider a solid conducting sphere with a radius a and charge Q_1 on it. There is a conducting spherical shell concentric to the sphere. The shell has an inner radius b (with $b > a$) and outer radius c and a net charge Q_2 on the shell. Denote the charge on the inner surface of the shell by Q_2' and that on the outer surface of the shell by Q_2''.



Find the charge Q_2''.

1. $Q_2'' = Q_1 + Q_2$ **correct**

2. $Q_2'' = Q_1 - Q_2$

3. $Q_2'' = Q_2 - Q_1$

4. $Q_2'' = 2\,(Q_1 + Q_2)$

5. $Q_2'' = 2\,(Q_1 - Q_2)$

6. $Q_2'' = 2\,(Q_2 - Q_1)$

7. $Q_2'' = \dfrac{Q_1 + Q_2}{2}$

8. $Q_2'' = \dfrac{Q_2 - Q_1}{2}$

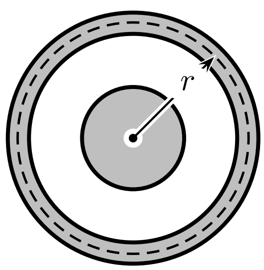
9. $Q_2'' = \dfrac{Q_1 - Q_2}{2}$

10. $Q_2'' = \dfrac{(Q_1 + Q_2)^2}{Q_1 - Q_2}$

Explanation:

Basic Concepts: Gauss' Law

Sketch a concentric Gaussian surface S (dashed line) within the shell.



Since the electrostatic field in a conducting medium is zero, according to Gauss's Law,

$$\begin{aligned}\Phi_S &= \frac{Q_1 + Q_2'}{\epsilon_0} \\ &= 0 \\ Q_2' &= -Q_1\end{aligned}$$

But the net charge on the shell is

$$Q_2 = Q_2' + Q_2'',$$

so the charge on the outer surface of the shell is

$$\begin{aligned}Q_2'' &= Q_2 - Q_2' \\ &= \boxed{Q_2 + Q_1}.\end{aligned}$$

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005 (part 2 of 3) 10 points

Find the magnitude of the electric field at point P $\left(\|\vec{E}_P\| \equiv E_P\right)$, where the distance from P to the center is $r = \dfrac{a+b}{2}$.

1. $E_P = \dfrac{4\,k_e\,Q_1}{(a+b)^2}$ **correct**

2. $E_P = 0$

3. $E_P = \dfrac{4\,k_e\,Q_2}{(a+b)^2}$

4. $E_P = \dfrac{4\,k_e\,(Q_1 - Q_2)}{(a+b)^2}$

5. $E_P = \dfrac{2\,k_e\,Q_1}{(a+b)^2}$

6. $E_P = \dfrac{2\,k_e\,Q_2}{(a+b)^2}$

7. $E_P = \dfrac{2\,k_e\,(Q_1 - Q_2)}{(a+b)^2}$

8. $E_P = \dfrac{4\,k_e\,(Q_1 + Q_2)}{(a+b)^2}$

9. $E_P = \dfrac{2\,k_e\,(Q_1 + Q_2)}{(a+b)^2}$

10. $E_P = \dfrac{2\,k_e\,Q_1\,a}{(a+b)^3}$

Explanation:

Choose the spherical surface S centered at O, which passes through P. Here,

$$\begin{aligned}4\,\pi\,r^2\,E_P &= \frac{Q_1}{\epsilon_0} \\ E_P &= \frac{Q_1}{4\,\pi\,\epsilon_0\,r^2} \\ &= \frac{k_e\,Q_1}{r^2} \\ &= \boxed{\frac{4\,k_e\,Q_1}{(a+b)^2}}.\end{aligned}$$

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006 (part 3 of 3) 10 points

Assume: The potential at $r = \infty$ is zero.

Find the potential V_P at point P.

1. $V_P = \dfrac{2\,k_e\,Q_1}{a+b} - \dfrac{k_e\,Q_1}{b} + \dfrac{k_e(Q_1 + Q_2)}{c}$ **correct**

2. $V_P = \dfrac{2\,k_e\,Q_1}{a+b}$

3. $V_P = \dfrac{2\,k_e\,(Q_1 - Q_2)}{a+b}$

4. $V_P = 0$

5. $V_P = \dfrac{2\,k_e\,Q_1}{a+b} + \dfrac{k_e\,Q_2}{c}$

6. $V_P = \dfrac{k_e\,Q_1}{a+b} - \dfrac{k_e\,Q_2}{b}$

7. $V_P = \dfrac{2\,k_e\,Q_1}{a+b} - \dfrac{2\,k_e\,Q_2}{b}$

8. $V_P = \dfrac{2\,k_e\,Q_1}{a+b} - \dfrac{k_e\,Q_2}{c}$

9. $V_P = \dfrac{2\,k_e\,Q_1}{a+b} + \dfrac{k_e\,Q_1}{b} - \dfrac{k_e(Q_1 - Q_2)}{c}$

10. $V_P = \dfrac{2\,k_e\,Q_1}{a}$

Explanation:

Using the superposition principle, adding the 3 concentric charge distributions; *i.e.*, Q_1 at a, $-Q$ at b and $Q_1 + Q_2$ at c, gives

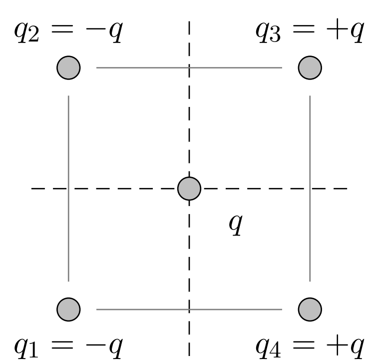
$$V = \boxed{\frac{2\,k_e\,Q_1}{a+b} - \frac{k_e\,Q_1}{b} + \frac{k_e\,(Q_1+Q_2)}{c}}\,.$$

Add a Charge to Four JMS

25:01, highSchool, multiple choice, < 1 min, fixed.

007 (part 1 of 1) 10 points

Four charges are placed at the corners of a square of side a, with $q_1 = q_2 = -q$, $q_3 = q_4 = +q$, where q is positive. Initially there is no charge at the center of the square.



Find the work required to bring the charge q from infinity and place it at the center of the square.

1. $W = 0$ **correct**

2. $W = \dfrac{4\,k\,q^2}{a^2}$

3. $W = \dfrac{2\,k\,q^2}{a^2}$

4. $W = \dfrac{-2\,k\,q^2}{a^2}$

5. $W = \dfrac{-4\,k\,q^2}{a^2}$

6. $W = \dfrac{4\,k\,q^2}{a}$

7. $W = \dfrac{2\,k\,q^2}{a}$

8. $W = \dfrac{-2\,k\,q^2}{a}$

9. $W = \dfrac{-4\,k\,q^2}{a}$

10. $W = \dfrac{8\,k\,q^2}{a^2}$

Explanation:

Based on the superposition principle, the potential at the center due to the charges at the corners is

$$\begin{aligned} V &= V_1 + V_2 + V_3 + V_4 \\ &= \frac{k\,q}{r}(-1 - 1 + 1 + 1) = 0\,. \end{aligned}$$

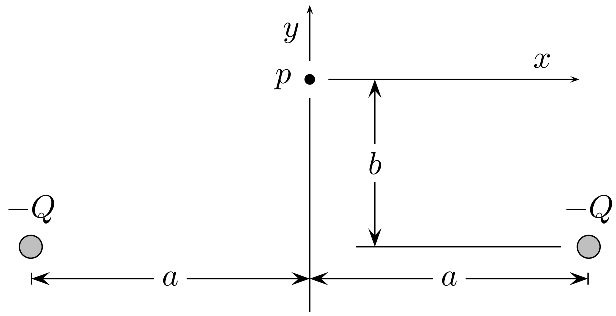
Here r is the common distance from the center to the corners. The work required to bring the charge q from infinity to the center is then $W = q\,V = 0$.

Electric Potential or FieldJMS

25:03, trigonometry, multiple choice, > 1 min, wording-variable.

008 (part 1 of 2) 10 points

Two charges are located in the (x, y) plane as shown in the figure below. The fields produced by these charges are observed at the origin, $p = (0, 0)$.



Use Coulomb's law to find the x-component of the electric field at p.

1. $E_x = 0$ **correct**

2. $E_x = -\dfrac{4\,k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

3. $E_x = \dfrac{4\,k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

4. $E_x = \dfrac{2\,k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

5. $E_x = -\dfrac{2\,k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

6. $E_x = \dfrac{k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

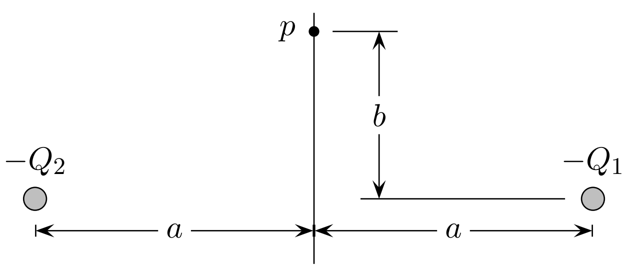
7. $E_x = -\dfrac{k_e\,Q\,a}{(a^2+b^2)^{3/2}}$

8. $E_x = \dfrac{2\,k_e\,Q}{a^2+b^2}$

9. $E_x = -\dfrac{2\,k_e\,Q}{a^2+b^2}$

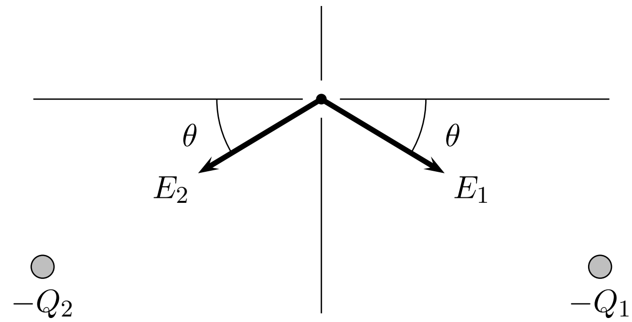
Explanation:

Let: $k_e = 8.98755 \times 10^9\ \mathrm{N\,m^2/C^2}$.



$$\begin{aligned} r_1 &= \sqrt{x_1^2 + y_1^2} \\ &= \sqrt{a^2+b^2}\,. \end{aligned}$$

$$\begin{aligned} r_2 &= \sqrt{x_2^2 + y_2^2} \\ &= \sqrt{(-a)^2 + b^2} \\ &= \sqrt{a^2+b^2}\,, \quad \text{so} \\ r_2 &= r_1 = r\,. \end{aligned}$$



where

$$\begin{aligned} |\sin\theta| &= \frac{b}{r} = \frac{b}{\sqrt{a^2+b^2}} \\ |\cos\theta| &= \frac{a}{r} = \frac{a}{\sqrt{a^2+b^2}}\,. \end{aligned}$$

In the x-direction, the contributions from the two charges are

$$\begin{aligned} E_{x_1} &= -k_e\,\frac{(-Q)}{r_1^2}\,|\cos(\theta)| \qquad (1) \\ &= -k_e\,\frac{(-Q)}{(a^2+b^2)}\,\frac{a}{\sqrt{a^2+b^2}} \\ &= +k_e\,\frac{Q\,a}{(a^2+b^2)^{3/2}} \end{aligned}$$

$$\begin{aligned} E_{x_2} &= -k_e\,\frac{(+Q)}{r_2^2}\,|\cos(\theta)| \qquad (2) \\ &= -k_e\,\frac{(+Q)}{(a^2+b^2)}\,\frac{a}{\sqrt{a^2+b^2}} \\ &= -k_e\,\frac{Q\,a}{(a^2+b^2)^{3/2}} \end{aligned}$$

$$\begin{aligned} E_x &= E_{x_1} + E_{x_2} \\ &= 0\,. \end{aligned}$$

---

009 (part 2 of 2) 10 points

Let: $V = 0$ at infinity.

Find the electric potential at p.

1. $V_y = -\dfrac{2\,k_e\,Q}{\sqrt{a^2+b^2}}$ **correct**

2. $V_y = +\dfrac{2\,k_e\,Q}{\sqrt{a^2+b^2}}$

3. $V_y = -\dfrac{4\,k_e\,Q}{\sqrt{a^2+b^2}}$

4. $V_y = \dfrac{4\,k_e\,Q}{\sqrt{a^2+b^2}}$

5. $V_y = -\dfrac{2\,k_e\,Q\,a}{\sqrt{a^2+b^2}}$

6. $V_y = \dfrac{2\,k_e\,Q\,a}{\sqrt{a^2+b^2}}$

7. $V_y = -\dfrac{4\,k_e\,Q\,a}{\sqrt{a^2+b^2}}$

8. $V_y = \dfrac{4\,k_e\,Q\,a}{\sqrt{a^2+b^2}}$

9. $V_y = 0$

Explanation:

The potential for a point charge $-Q$ is

$$V = k_e\,\frac{-Q}{r}\,.$$

For the two charges in this problem, we have

$$V_1 = k_e\,\frac{-Q}{\sqrt{a^2+b^2}}\,.$$

$$V_2 = k_e\,\frac{-Q}{\sqrt{a^2+b^2}}\,.$$

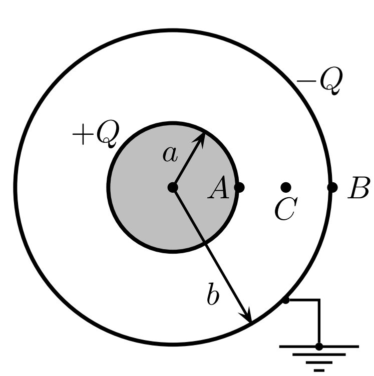
$$\begin{aligned} V_p &= V_1 + V_2 \\ &= \frac{k_e}{\sqrt{a^2+b^2}}\,[-Q+(-Q)] \\ &= -\frac{2\,k_e\,Q}{\sqrt{a^2+b^2}}\,. \end{aligned}$$

Spherical Capacitor JMS

26:02, calculus, multiple choice, > 1 min, fixed.

010 (part 1 of 1) 10 points

Given a spherical capacitor with radius of the inner conducting sphere a and the outer shell b. The outer shell is grounded. The charges are $+Q$ and $-Q$. A point C is located at $r = \dfrac{R}{2}$, where $R = a + b$.



The capacitance of this spherical capacitor is

1. $C = \dfrac{k_e}{b}\,.$

2. $C = \dfrac{a}{k_e}\,.$

3. $C = \dfrac{b}{k_e}\,.$

4. $C = \dfrac{a+b}{k_e}\,.$

5. $C = \dfrac{1}{k_e\,(a+b)}\,.$

6. $C = \dfrac{1}{k_e\,(a-b)}\,.$

7. $C = \dfrac{k_e}{a}\,.$

8. $C = \dfrac{1}{k_e\left(\dfrac{1}{a}-\dfrac{1}{b}\right)}\,.$ **correct**

9. $C = \dfrac{b\,-\,a}{2\,k_e\,ln\left(\dfrac{b}{a}\right)},.$

10. $C = \dfrac{b^2}{4\,k_e\,(\,b\,-\,a)},.$

Explanation:

$$\begin{aligned} \Delta V &= V_a - V_b \\ &= k_e\,Q\left(\frac{1}{a}-\frac{1}{b}\right) - 0 \end{aligned}$$

since V_b is grounded. The charge on the inside of the shell doesn't affect the grounded potential.

The capacitance of this spherical capacitor is

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e\, Q\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{k_e\left(\frac{1}{a} - \frac{1}{b}\right)}\,.$$

Introduce a Dielectric JMS

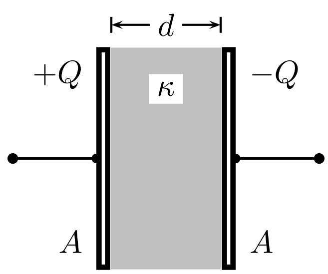
26:05, calculus, multiple choice, < 1 min, fixed.

011 (part 1 of 2) 10 points

Consider an air-filled parallel plate capacitor with plate area A and gap width d. The plate charge is Q.

Subsequent to full charging of the capacitor, the battery is disconnected.

Now, the gap is filled with of dielectric of dielectric constant κ.



The voltage within the gap in the presence of the dielectric is given by

1. $V' = \frac{Q^2}{\kappa\,\epsilon_0\,A}\,d\,.$

2. $V' = \frac{Q\,\kappa}{\epsilon_0\,A}\,d\,.$

3. $V' = \frac{Q\,A}{\kappa\,\epsilon_0}\,d\,.$

4. $V' = \frac{Q}{\kappa\,\epsilon_0\,d}\,A\,.$

5. $V' = \frac{Q^2}{\kappa\,\epsilon_0}\,A\,.$

6. $V' = \frac{Q^2\,\kappa}{\epsilon_0\,A}\,d\,.$

7. $V' = \frac{Q}{\kappa\,\epsilon_0\,A}\,d\,.$ **correct**

8. $V' = \frac{Q^2}{\kappa\,\epsilon_0\,d}\,A\,.$

Explanation:

$$V' = \frac{V}{\kappa} = \frac{E\,d}{\kappa} = \frac{Q}{\kappa\,\epsilon_0\,A}\,d$$

012 (part 2 of 2) 10 points

The energy within the gap in the presence of the dielectric is given by

1. $U' = \frac{Q^2}{2\,\kappa\,\epsilon_0\,d}\,A\,.$

2. $U' = \frac{Q^2}{2\,\epsilon_0\,A}\,d\,.$

3. $U' = \frac{Q^2}{2\,\kappa\,\epsilon_0\,A}\,d\,.$ **correct**

4. $U' = \frac{Q^2\,\kappa}{2\,\epsilon_0\,A}\,d\,.$

5. $U' = \frac{Q}{\kappa\,\epsilon_0\,A}\,d\,.$

6. $U' = \frac{Q}{\epsilon_0\,A}\,d\,.$

7. $U' = \frac{Q}{\kappa\,\epsilon_0\,d}\,A\,.$

8. $U' = \frac{Q\,\kappa}{\epsilon_0\,A}\,d\,.$

Explanation:

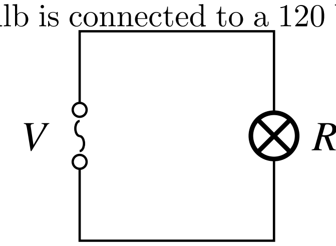
$$U' = \frac{Q^2}{2\,C'} = \frac{Q^2}{2\,\left(\frac{\kappa\,\epsilon_0\,A}{d}\right)} = \frac{Q^2}{2\,\kappa\,\epsilon_0\,A}\,d\,.$$

Light Bulb in a Circuit JMS

, , , < 1 min, .

013 (part 1 of 2) 10 points

A 75 W bulb is connected to a 120 V source.



What is the current through the bulb?

1. 0.466667 A

2. 0.506306 A

3. 0.561789 A

4. 0.608182 A

5. 0.625 A **correct**

6. 0.645669 A

7. 0.653043 A

8. 0.670588 A

9. 0.696581 A

10. 0.705385 A

Explanation:

$$\text{Given}: \quad P = 75 \text{ W}, \quad \text{and} \quad V = 120 \text{ V}.$$

The current is

$$I = \frac{P}{V} = \frac{75 \text{ W}}{120 \text{ V}} = \boxed{0.625 \text{ A}}.$$

Dimensional analysis for I:

$$\frac{\text{W}}{\text{V}} = \frac{\text{J/s}}{\text{J/C}} = \frac{\text{J}}{\text{s}} \cdot \frac{\text{C}}{\text{J}} = \frac{\text{C}}{\text{s}} = \text{A}$$

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014 (part 2 of 2) 10 points

A lamp dimmer puts a resistance in series with the bulb.

What resistance would be needed to reduce the current to 0.3 A?

1. 32.7125 Ω

2. 45.0553 Ω

3. 57.0368 Ω

4. 58.2651 Ω

5. 92.1429 Ω

6. 120.044 Ω

7. 122.723 Ω

8. 132.777 Ω

9. 208 Ω **correct**

10. 212.982 Ω

Explanation:

$$R_{total} = R + R_1, \quad \text{and}$$
$$V = I_1 R_{total} = I_1 R + I_1 R_1$$

so that

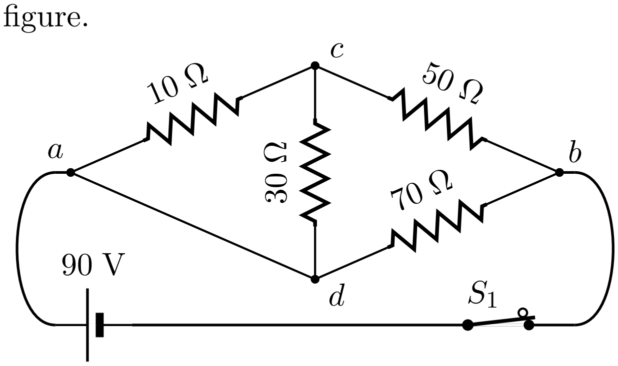
$$R_1 = \frac{V - I_1 R}{I_1} = \frac{V}{I_1} - R = \frac{120 \text{ V}}{0.3 \text{ A}} - 192\ \Omega = \boxed{208\ \Omega}.$$

Four Resistors JMS

28:02, highSchool, multiple choice, > 1 min, normal.

015 (part 1 of 1) 10 points

Four resistors are connected as shown in the figure.



Find the resistance between points a and b.

1. 31.5686 Ω **correct**

2. 33.3855 Ω

3. 34.4127 Ω

4. 36.0099 Ω

5. 37.6052 Ω

6. 38.1779 Ω

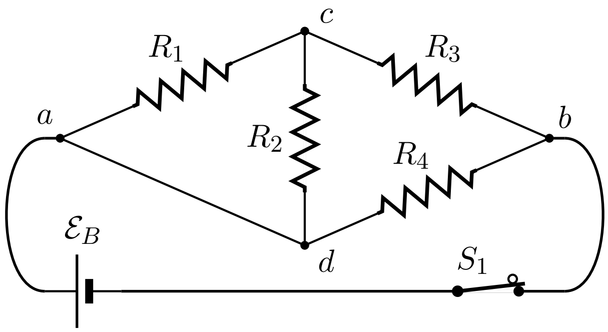
7. 38.9958 Ω

8. 39.4313 Ω

9. 40.046 Ω

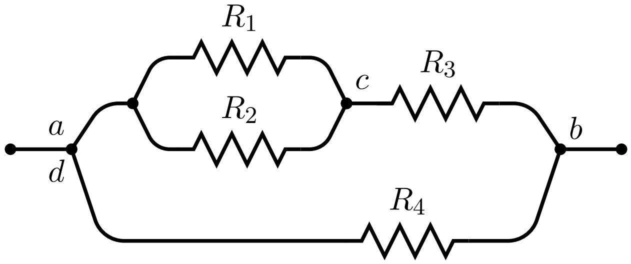
10. 42.0635 Ω
Explanation:

$$\begin{aligned} \textit{Given}: \quad & R_1 = 10\ \Omega\,, \\ & R_2 = 30\ \Omega\,, \\ & R_3 = 50\ \Omega\,, \\ & R_4 = 70\ \Omega\,, \quad \text{and} \\ & \mathcal{E}_B = 90\ \text{V}\,. \end{aligned}$$



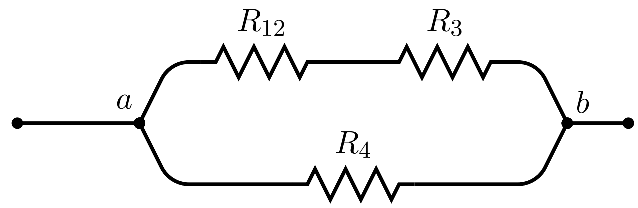
Ohm's law is $V = I\,R$.

A good rule of thumb is to eliminate junctions connected by zero resistance.



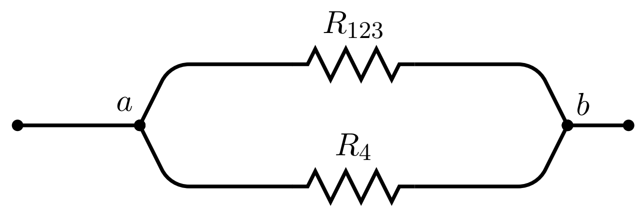
The parallel connection of R_1 and R_2 gives the equivalent resistance

$$\begin{aligned} \frac{1}{R_{12}} &= \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1\,R_2} \\ R_{12} &= \frac{R_1\,R_2}{R_1 + R_2} \\ &= \frac{(10\ \Omega)\,(30\ \Omega)}{10\ \Omega + 30\ \Omega} \\ &= 7.5\ \Omega\,. \end{aligned}$$



The series connection of R_{12} and R_3 gives the equivalent resistance

$$\begin{aligned} R_{123} &= R_{12} + R_3 \\ &= 7.5\ \Omega + 50\ \Omega \\ &= 57.5\ \Omega\,. \end{aligned}$$

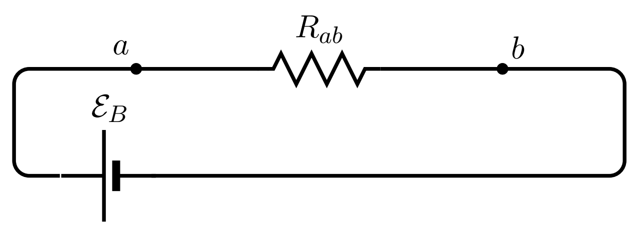


The parallel connection of R_{123} and R_4 gives the equivalent resistance

$$\begin{aligned} \frac{1}{R_{ab}} &= \frac{1}{R_{123}} + \frac{1}{R_4} = \frac{R_4 + R_{123}}{R_{123}\,R_4} \\ R_{ab} &= \frac{R_{123}\,R_4}{R_{123} + R_4} \\ &= \frac{(57.5\ \Omega)\,(70\ \Omega)}{57.5\ \Omega + 70\ \Omega} \\ &= 31.5686\ \Omega\,. \end{aligned}$$

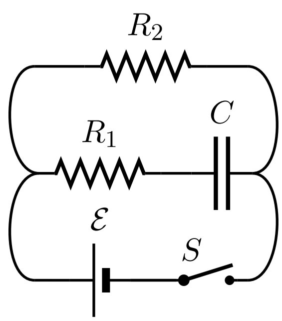
or combining the above steps, the equivalent resistance is

$$\begin{aligned} R_{ab} &= \frac{\left(\dfrac{R_1\,R_2}{R_1 + R_2} + R_3\right) R_4}{\dfrac{R_1\,R_2}{R_1 + R_2} + R_3 + R_4} \\ &= \frac{\left[\dfrac{(10\ \Omega)\,(30\ \Omega)}{10\ \Omega + 30\ \Omega} + 50\ \Omega\right](70\ \Omega)}{\dfrac{(10\ \Omega)\,(30\ \Omega)}{10\ \Omega + 30\ \Omega} + 50\ \Omega + 70\ \Omega} \\ &= 31.5686\ \Omega\,. \end{aligned}$$



RC Circuit 02
28:04, calculus, multiple choice, < 1 min, fixed.

016 (part 1 of 2) 10 points
Consider the circut below, which consists of two conducting loops.



After the switch S is closed, the current through resistor R_2 is,

1. oscillating with constant amplitude.

2. from right to left through R_2.

3. zero at all times.

4. oscillating with decreasing amplitude.

5. from left to right through R_2. **correct**

6. Exponentially increasing

7. Exponentially damping

8. not well defined

9. impossible to calculate

10. Increasing linearly

Explanation:
Since the potential drop across resistor R_2 is fixed to be $\mathcal{E}$ after the switch is closed, the current is also a fixed value and the direction is from left to right on R_2.

017 (part 2 of 2) 10 points
After the switch S has been closed for a very long time, the currents in the two circuits are

1. zero through both resistors

2. $i_1 = \dfrac{\mathcal{E}}{R_1}$ through R_1 and zero through R_2.

3. $i_1 = \dfrac{\mathcal{E}}{R_1}$ through R_1 and $i_2 = \dfrac{\mathcal{E}}{R_2}$ in circuit 2.

4. oscillating with constant amplitude in both circuits.

5. zero through R_1 and $i_2 = \dfrac{\mathcal{E}}{R_2}$ through R_2. **correct**

6. impossible to calculate

7. not well defined

8. $i_1 = i_2 = \dfrac{\mathcal{E}\,(R_1 + R_2)}{R_1\,R_2}$

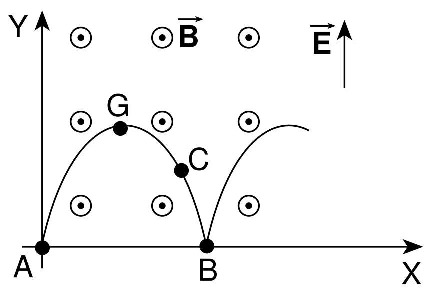
9. $i_1 = i_2 = \dfrac{\mathcal{E}}{R_1 + R_2}$

10. infinite

Explanation:
As mentioned above, the current in R_2 remains unchanged to be $\frac{\mathcal{E}}{R_2}$, while for R_1, after a long time, the current in the circuit tends to an equilibrium state, namely the capacitor doesn't get charged or release charge any more. There is no current through the capacitor as well as resistor R_1 after a long time.

Charged Particle in a FieldJMS
29:02, trigonometry, multiple choice, > 1 min, fixed.

018 (part 1 of 2) 10 points
A particle of mass m and charge q starts from rest at the origin (point A in the figure below).



There is a uniform electric field $\vec{E}$ in the positive y-direction and a uniform magnetic field $\vec{B}$ directed towards the reader. It can be shown that the path is a cycloid whose radius of curvature at the top point is twice the y-coordinate at that level.

What is the relation between kinetic energy of the charge at points A and B?

1. The kinetic energy of the particle at point B is the same as it was at point A. **correct**

2. The kinetic energy of the particle at point B is larger than the energy at point A.

3. The kinetic energy of the particle at point B is smaller than the energy at point A.

4. The relationship between the kinetic energy of the particle at point A and at point B cannot be determined by the information given.

5. This setup is inherently unphysical, and hence, any discussion regarding energy is meaningless.

Explanation:

When the particle has reached point B, its displacement in the direction of $\vec{E}$ is zero. Therefore the net work done by the conservative electric force is zero. The magnetic force never does any work. Therefore the work-energy theorem, ($W = \Delta K$) says that the kinetic energy of the particle at point B must be the same as it was at point A. Thus at B the particle is again at rest.

---

019 (part 2 of 2) 10 points

How much is the work done by the external forces as the particle moves from A to C, where point C is any point on the path, with coordinates $(x\,,\,y)$.

1. $W = q\,E\,x$

2. $W = q\,E\sqrt{x^2+y^2}$

3. $W = q\,B\,x$

4. $W = q\,B\,y$

5. $W = q\,B\sqrt{x^2+y^2}$

6. $W = q\,E\,y$ **correct**

7. $W = q\,E\,y + q\,B\,x$

8. $W = q\,B\,y + q\,E\,x$

9. $W = q\,(E+B)\sqrt{x^2+y^2}$

10. $W = 0$

Explanation:

Because the magnetic force does not do any work on the particle, the net work is done by the conservative electric force; *i.e.*,

$$W = F_e\,y = q\,E\,y\,,$$

where y is the displacement of the particle in the direction of $\vec{E}$ as the particle reaches the point C.

Current on a Cube JMS

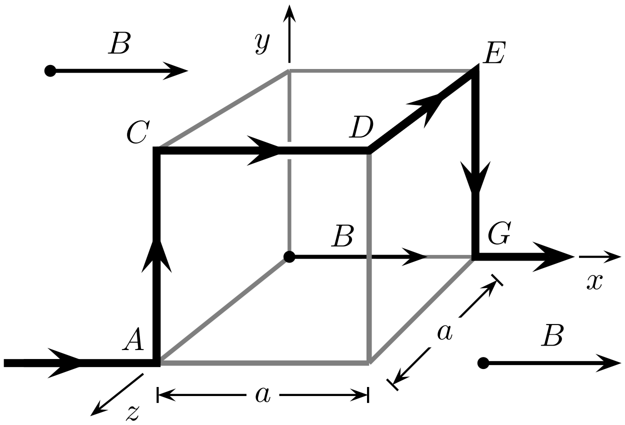
, , , < 1 min, .

020 (part 1 of 1) 10 points

Note: The conventional Cartesian notation of $\hat{\imath}$ (a unit vector along the positive x axis), $\hat{\jmath}$ (a unit vector along the positive y axis), and $\hat{k}$ (a unit vector along the positive z axis), is used.

Given a current segment which flows along the edges of a cube as shown in the figure. The cube has sides of length a. The current flows along the path $A \to C \to D \to E \to G$.

There is a uniform magnetic field $\vec{B} = B\,\hat{\imath}$.



Find the direction $\widehat{F} \equiv \dfrac{\vec{F}}{\|\vec{F}\|}$ of the resultant magnetic force on the current segment $ACDEG$.

1. $\widehat{F} = -\hat{\jmath}$ **correct**

2. $\widehat{F} = -\hat{k}$

3. $\widehat{F} = \hat{\imath}$

4. $\widehat{F} = -\hat{\imath}$

5. $\widehat{F} = \hat{\jmath}$

6. $\widehat{F} = \hat{k}$

7. Undetermined, since the magnitude of the force is zero.

8. $\widehat{F} = \dfrac{1}{\sqrt{2}} \left(\hat{\jmath} - \hat{k}\right)$

9. $\widehat{F} = \dfrac{1}{\sqrt{2}} \left(\hat{k} - \hat{\jmath}\right)$

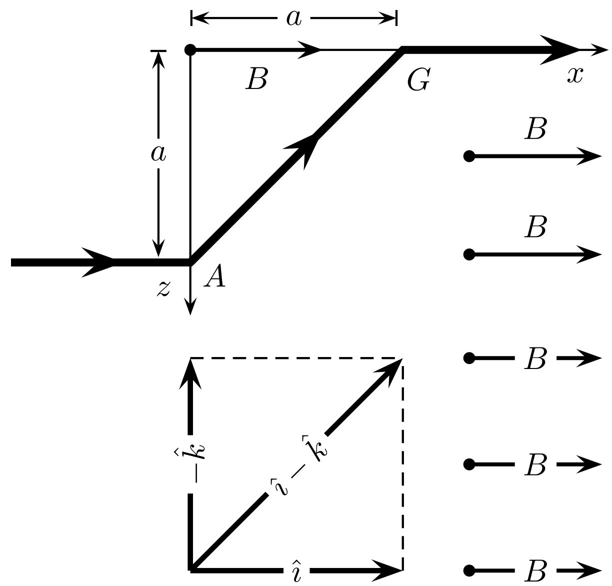
10. $\widehat{F} = \dfrac{1}{\sqrt{2}} \left(\hat{\jmath} + \hat{k}\right)$

Explanation:

Note: The current in wire segment CD flows in the $\hat{\imath}$ direction and the current in wire segment DE flows in the $-\hat{k}$.

Refer to the following sketch when reading the explanation

Top View



The magnetic force on a wire is given by

$$\vec{F}_{\text{mag}} = I\,\vec{\ell} \times \vec{B}\,.$$

The vector $\vec{\ell}$ is given by the sum of the current segments

$$\vec{\ell} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EG}\,,$$

and this is the vector $\overrightarrow{AG}$, (see figure above). The magnitude is given by

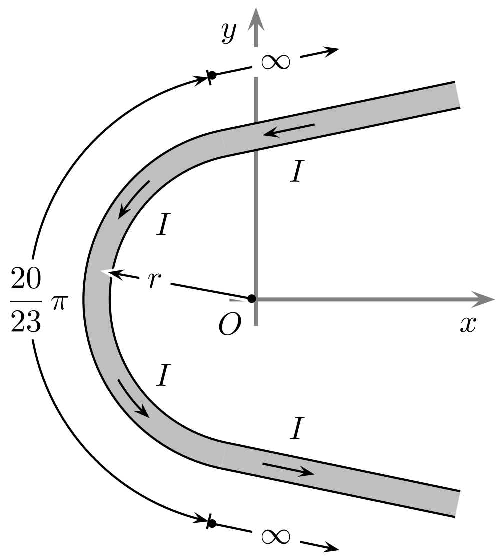
$$\begin{aligned} \vec{F} &\simeq \vec{\ell} \times \vec{B} \\ &\simeq (\hat{\imath} - \hat{k}) \times (\hat{\imath}) \\ &= (\hat{\imath} \times \hat{\imath}) - (\hat{k} \times \hat{\imath}) \\ &= 0 - \hat{\jmath} \\ \widehat{F} &= \boxed{-\hat{\jmath}}\,. \end{aligned}$$

Magnetic Field from an Arc JMS

30:01, calculus, multiple choice, > 1 min, wording-variable.

021 (part 1 of 1) 10 points

Consider two radial legs (extending to infinity) and a connecting $\dfrac{20}{23}\pi$ circular arc carrying a current I as shown below.



What is the magnitude of the magnetic field B_o (at the origin O) due to the current through this path?

1. $B_o = \frac{5}{23}\,\frac{\mu_0 I}{r} + \frac{\mu_0 I}{2\pi r}$ **correct**

2. $B_o = \frac{5}{23}\,\frac{\mu_0 I}{r} + \frac{\mu_0 I}{4\pi r}$

3. $B_o = \frac{5}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{2\pi r}$

4. $B_o = \frac{5}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{4\pi r}$

5. $B_o = \frac{5}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{2 r}$

6. $B_o = \frac{2}{23}\,\frac{\mu_0 I}{r} + \frac{\mu_0 I}{2\pi r}$

7. $B_o = \frac{2}{23}\,\frac{\mu_0 I}{r} + \frac{\mu_0 I}{4\pi r}$

8. $B_o = \frac{2}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{2\pi r}$

9. $B_o = \frac{2}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{4\pi r}$

10. $B_o = \frac{2}{23}\,\frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{2 r}$

Explanation:

Note: The magnetic field at B_o for the entire path points in the same direction.

The two straight wire segments produce the same magnetic field at B_o as a single long straight wire. Using Ampére's law, for the magnetic field a distance r from a straight wire, we have

$$\begin{aligned}\oint \vec{B}\cdot d\vec{s} &= \mu_0 I\\ \oint B\,ds &= \mu_0 I\\ B\oint ds &= \mu_0 I\\ B\,2\pi r &= \mu_0 I\,, \quad \text{so}\\ B_o &= \frac{\mu_0 I}{2\pi r}\,. \qquad (1)\end{aligned}$$

However, around the arc we will use the Biot-Savart law, where $|d\vec{s}\times\hat{r}| = ds = r\,d\theta$.

The magnetic field at at the center of an arc with a current I is

$$\begin{aligned}B_o &= \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}\\ &= \frac{\mu_0 I}{4\pi r^2}\int ds\\ &= \frac{\mu_0 I}{4\pi r^2}\int r\,d\theta\\ &= \frac{\mu_0 I}{4\pi r}\int_0^{\frac{20}{23}\pi} d\theta\\ &= \frac{\mu_0 I}{4\pi r}\,\theta\Big|_0^{\frac{20}{23}\pi}\\ &= \frac{\mu_0 I}{4\pi r}\left(\frac{20}{23}\pi - 0\right)\\ &= \frac{5}{23}\,\frac{\mu_0 I}{r}\,. \qquad (2)\end{aligned}$$

The magnetic field at B_o for the entire path is the sum of Eqs. 2 and 1.

$$B_o = \frac{5}{23}\,\frac{\mu_0 I}{r} + \frac{\mu_0 I}{2\pi r}$$

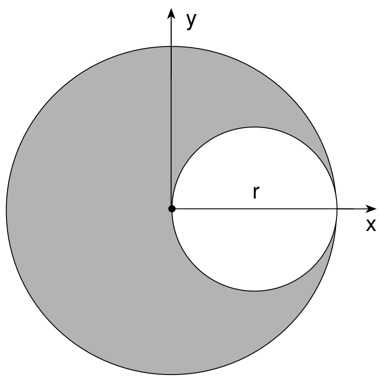
into the page or out of the page

Off Centered Hole

30:03, calculus, numeric, > 1 min, wording-variable.

022 (part 1 of 1) 10 points

A total current of 50 mA flows through an infinitely long cylinder of radius $r = 4$ cm which has an infinitely long cylindrical hole through it of diameter r centered at $\frac{r}{2}$ along the x-axis (as in figure 1).



What is the magnitude of the magnetic field at a distance of 12 cm along the positive x-axis? Assume that the magnitude of the current density is the same in the cylinder and in the hole and that the currents in the cylinder and the hole flow in opposite directions with respect to each other.

1. 1.40851×10^{-8} T

2. 2.33987×10^{-8} T

3. 4.25256×10^{-8} T

4. 5.32468×10^{-8} T

5. 5.88477×10^{-8} T

6. 7.08751×10^{-8} T

7. 7.77778×10^{-8} T **correct**

8. 8.64532×10^{-8} T

9. 1.14872×10^{-7} T

10. 1.19632×10^{-7} T

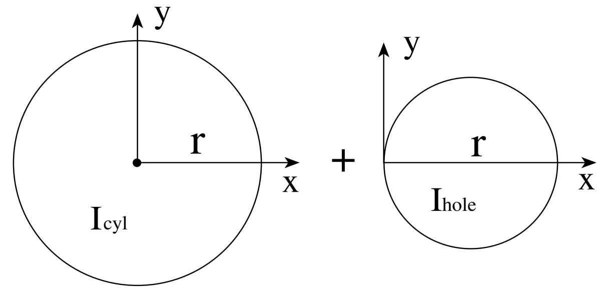
Explanation:

Basic Concepts: Magnetic Field due to a Long Cylinder

$$B = \frac{\mu_0 I}{2\pi r}.$$

Principle of Superposition.

Solution: Our goal is to model the given situation, which is complex and lacks symmetry, by adding together the fields from combinations of simpler current configurations which together match the given current distribution. The combination of the currents in Fig. 2 will do so if we choose I_{cyl} and I_{hole} correctly.



Since the current is uniform, the current density $J = \dfrac{I}{A}$ is constant. Then

$$J = I_{cyl}\, A_{cyl} = -I_{hole}\, A_{hole}\,.$$

Clearly, $A_{cyl} = \pi\, r^2$, and $A_{hole} = \dfrac{\pi\, r^2}{4}$. Thus $I_{hole} = -\dfrac{I_{cyl}}{4}$.

Note: The minus sign means I_{hole} is flowing in the direction opposite I_{cyl} and I, as it must if it is going to cancel with I_{cyl} to model the hole.

We also require $I = I_{cyl} + I_{hole}$. We then have $I_{cyl} = \dfrac{4}{3}\, I$, and $I_{hole} = -\dfrac{1}{3}\, I$. With these currents, the combination of the two cylinders in figure 2 gives the same net current and current distribution as the conductor in our problem.

The magnetic fields are

$$B_{cyl} = \frac{\mu_0 \left(\dfrac{4}{3}\, I\right)}{2\,\pi\, x}$$
$$B_{hole} = \frac{\mu_0 \left(-\dfrac{1}{3}\, I\right)}{2\,\pi\,(x - r/2)}\,.$$

Thus the total magnetic field is

$$\begin{aligned} B_{total} &= B_{cyl} + B_{hole} \\ &= \frac{\mu_0\, I}{6\,\pi}\left(\frac{4}{x} - \frac{1}{x - \dfrac{r}{2}}\right) \\ &= \frac{\mu_0\, I}{6\,\pi}\left[\frac{3\,x - 2\,r}{x\left(x - \dfrac{r}{2}\right)}\right] \\ &= \frac{(4\,\pi \times 10^{-7}\ \mathrm{T\,m/A})\,(50\ \mathrm{mA})}{6\,\pi} \end{aligned}$$

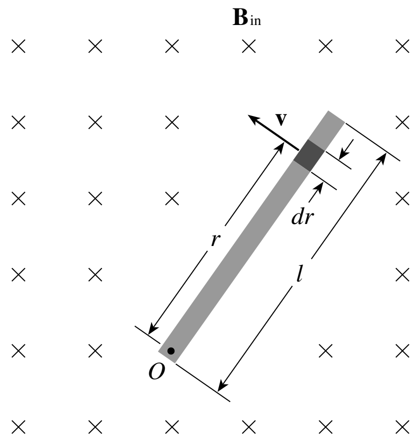
$$\times \left[\frac{3\,(12\text{ cm}) - 2\,(4\text{ cm})}{(12\text{ cm})\left((12\text{ cm}) - \dfrac{(4\text{ cm})}{2}\right)} \right]$$
$$= 7.77778 \times 10^{-8}\text{ T}.$$

Rotating Metal Bar 02

31:02, calculus, numeric, > 1 min, normal.

023 (part 1 of 1) 10 points

A metal bar spins at a constant rate in the magnetic field of the Earth as in Figure. The rotation occurs in a region where the component of the Earth's magnetic field perpendicular to the plane of rotation is 3.3×10^{-5} T. The bar is 1 m in length and its angular speed is $5\,\pi$.



What potential difference is developed between its ends?

1. 2.86804×10^{-5} V

2. 7.05979×10^{-5} V

3. 8.13233×10^{-5} V

4. 0.000141863 V

5. 0.000162982 V

6. 0.00022808 V

7. 0.000252191 V

8. 0.000259181 V **correct**

9. 0.000461814 V

10. 0.000600358 V

Explanation:

Basic Concept:

Motional emf

$\mathcal{E} = B{\cdot}l{\cdot}v$

For a point on the bar, the velocity with which the point moves changes linearly with the distance from the point to the rotation center. So, the effective velocity for the whole bar equals:

$$v_{eff} = \frac{\omega \cdot l}{2} = \frac{2\pi f \cdot l}{2} = 7.85398\text{ m/s},$$

and the induced emf in the bar is

$$\mathcal{E} = B \cdot l \cdot v_{eff} = 0.000259181\text{ V}.$$

Therefore, the potential difference between the ends of the bar is:

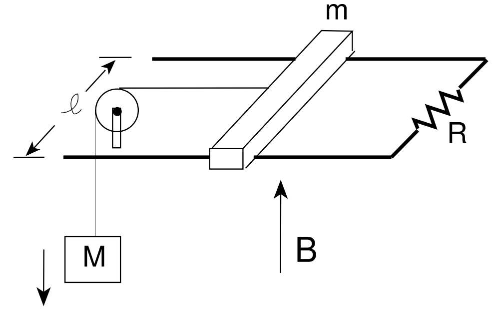
$$\Delta V = \mathcal{E} = 0.000259181\text{ V}.$$

Bar Pulled Through Field JMS

31:03, calculus, multiple choice, > 1 min, fixed.

024 (part 1 of 1) 10 points

A bar of negligible resistance and mass m in the figure below is pulled horizontally across frictionless parallel rails, also of negligible resistance, by a massless string that passes over an ideal pulley and is attached to a suspended mass M. The uniform magnetic field has a magnitude B, and the distance between the rails is ℓ. The rails are connected at one end by a load resistor R. Use g.



What is the magnitude of the terminal velocity (*i.e.*, the eventual steady-state speed v_∞) reached by the bar?

1. $v_\infty = \dfrac{M\,g\,R}{\ell\,B}$

2. $v_\infty = \dfrac{M\,g\,R}{\ell^2\,B^2}$ **correct**

3. $v_\infty = \dfrac{M\,g\,R}{\ell\,B^2}$

4. $v_\infty = \dfrac{M\,g\,R}{\ell^2\,B}$

5. $v_\infty = \dfrac{M\,g\,R^2}{\ell^2\,B^2}$

6. $v_\infty = \dfrac{M\,g\,R^2}{\ell\,B^2}$

7. $v_\infty = \dfrac{M\,g\,R^2}{\ell^2\,B}$

8. $v_\infty = \dfrac{M\,g\,R^2}{\ell\,B}$

9. $v_\infty = \dfrac{M^2\,g^2\,R^2}{\ell^2\,B^2}$

10. $v_\infty = \dfrac{M^2\,g^2\,R}{\ell\,B}$

Explanation:

Basic Concepts:

$$\vec{F}_g = M\,\vec{g}$$

$$\vec{F}_m = I\,\vec{\ell} \times \vec{B}$$

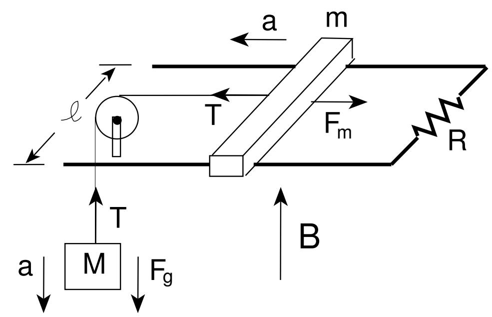
$$\vec{F}_{net} = (M+m)\,\vec{a} = \vec{F}_g - \vec{F}_m$$

$$\mathcal{E} = I\,R = -\frac{d\,\Phi_B}{dt}$$

$$\Phi_B = \vec{B} \cdot \vec{A}$$

$$\mathcal{E} = B\,\ell\,v$$

Solution: It follows from Lenz's law that the magnetic force opposes the motion of the bar. When the wire acquires steady-state speed, the gravitational force F_g is counter-balanced by the magnetic force F_m (see figure below):



$$F_g = M\,g = F_m = \ell\,I\,B \quad (1)$$

$$I = \frac{M\,g}{\ell\,B} \quad (2)$$

To find the induced current, we use Ohm's law and substitute in the induced emf, $\mathcal{E} = -\dfrac{d\,\Phi}{dt}$

$$I = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\,\frac{d\,\Phi}{dt} \quad (3)$$

Note, we have ignored the minus sign from the induced emf $\mathcal{E}$ because we will eventually evaluate the magnitude of the terminal velocity. The flux is $\Phi = B A$. So

$$\frac{d\,\Phi}{dt} = B\,\frac{dA}{dt} = B\,\ell\,v \quad (4)$$

$$I = \frac{B\,\ell\,v}{R} \quad (5)$$

Using (2) and (5) and noting that v is the terminal velocity v_∞

$$\frac{M\,g}{\ell\,B} = \frac{B\,\ell\,v_\infty}{R}\,. \quad (6)$$

Solving for the magnitude of the terminal velocity v_∞

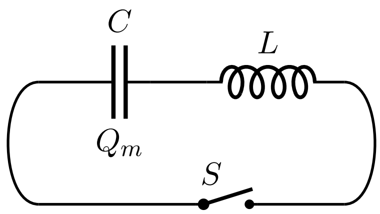
$$v_\infty = \frac{M\,g\,R}{\ell^2\,B^2}\,. \quad (7)$$

Energy in an LC Circuit JMS

32:05, calculus, multiple choice, < 1 min, fixed.

025 (part 1 of 2) 10 points

Consider the LC circuit shown below. Switch S is initially open, and the capacitor has a charge Q_m on its plates. At t=0 the switch is closed.



What will be the energy U_C stored in the capacitor as a function of time?

1. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\cos^2\left(\frac{t}{\sqrt{LC}}\right)$ **correct**

2. $U_C = \left(\frac{Q_m^2}{C}\right)\sin^2\left(\frac{t}{\sqrt{LC}}\right)$

3. $U_C = \frac{Q_m^2}{2\,C}$

4. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\exp\left(-\frac{t}{\sqrt{LC}}\right)$

5. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\left(1 - \exp\left[-\frac{t}{\sqrt{LC}}\right]\right)$

6. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\cos\left(t\sqrt{LC}\right)$

7. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\sin^2\left(\frac{t}{\sqrt{LC}}\right)$

8. $U_C = \left(\frac{Q_m^2}{C}\right)\cos^2\left(\frac{t}{\sqrt{LC}}\right)$

9. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\sin^2\left(t\sqrt{\frac{L}{C}}\right)$

10. $U_C = \left(\frac{Q_m^2}{2\,C}\right)\cos\left(\frac{t}{\sqrt{LC}}\right)$

Explanation:

Solution: The charge on the capacitor in the $L\,C$ circuit satisfies

$$\frac{d^2\,Q}{dt^2} = -\frac{1}{L\,C}\,Q$$

The solution is

$$Q = Q_m\,\cos\left(\frac{t}{\sqrt{LC}}\right)$$

where Q_m is the initial charge on the capacitor. Thus the energy is given by

$$U_c = \frac{Q^2}{2\,C} = \frac{Q_m^2}{2\,C}\cos^2\left(\frac{t}{\sqrt{L\,C}}\right)$$

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026 (part 2 of 2) 10 points

What will be the total energy U as a function of time?

1. $U = \frac{Q_m^2}{2\,C}$ **correct**

2. $U = \left(\frac{Q_m^2}{2\,C}\right)\cos\left(\frac{t}{\sqrt{LC}}\right)$

3. $U = \left(\frac{Q_m^2}{2\,C}\right)\cos^2\left(t\sqrt{LC}\right)$

4. $U = \left(\frac{Q_m^2}{2\,C}\right)\exp\left(-\frac{t}{\sqrt{LC}}\right)$

5. $U = \left(\frac{Q_m^2}{2\,C}\right)\left(1 - \exp\left[-\frac{t}{\sqrt{LC}}\right]\right)$

6. $U = \frac{Q_m^2}{C}$

7. $U = \frac{1}{\sqrt{LC}}$

8. $U = \frac{Q_m^2}{4\,C}$

9. $U = \frac{2\,Q_m^2}{C}$

10. $U = \sqrt{LC}$

Explanation:

This is just the sum of Part 1 and Part 2:

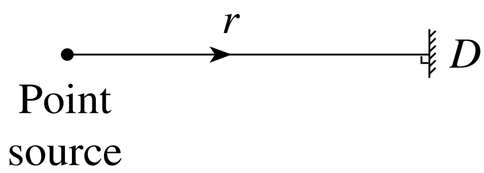
$$\begin{aligned} U &= U_L + U_c \\ &= \frac{Q_m^2}{2\,C}\left[\cos^2\left(\frac{t}{\sqrt{L\,C}}\right) + \sin^2\left(\frac{t}{\sqrt{L\,C}}\right)\right] \\ &= \frac{Q_m^2}{2\,C}. \end{aligned}$$

Point Light Source JMS

34:03, trigonometry, multiple choice, > 1 min, fixed.

027 (part 1 of 2) 10 points

A point light source delivers a time-averaged power P. It radiates light isotropically. A piece of small flat surface is placed at D, which is a distance r away. This piece has a cross section A_{surf}. The surface reflects $\frac{1}{4}$ of the light and absorbs $\frac{3}{4}$ of the light. Assume the light hitting the various parts of the surface is perpendicular to them.



The time-averaged energy density hitting the surface is given by:

1. $\overline{u} = 4\,\pi\,r^2\,\frac{P}{c}$

2. $\overline{u} = \pi\,r^2\,\frac{P}{c}$

3. $\overline{u} = A_{\text{surf}}\,\frac{P}{c}$

4. $\overline{u} = \frac{P}{4\,\pi\,c\,r^2}$ **correct**

5. $\overline{u} = \frac{P}{c\,A_{\text{surf}}}$

6. $\overline{u} = 4\,\pi\,r^2\,P$

7. $\overline{u} = \pi\,r^2\,P$

8. $\overline{u} = A_{\text{surf}}\,P$

9. $\overline{u} = \frac{P}{A_{\text{surf}}}$

10. $\overline{u} = \frac{P}{4\,\pi\,r^2}$

Explanation:

Basic Concepts EM Wave

The time-averaged energy density at D is given by

$$\overline{u} = \frac{I}{c} = \frac{P}{4\,\pi\,r^2\,c}.$$

---

028 (part 2 of 2) 10 points

Find the total time-averaged force on the surface in terms of the intensity I of the light at D.

1. $\overline{F} = \frac{A_{\text{surf}}\,I}{c}$

2. $\overline{F} = \frac{7}{4}\,\frac{4\,\pi\,I}{c}$

3. $\overline{F} = \frac{3\,A_{\text{surf}}\,I}{2\,c}$

4. $\overline{F} = \frac{7\,A_{\text{surf}}\,I}{4\,c}$

5. $\overline{F} = \frac{2\,A_{\text{surf}}\,I}{c}$

6. $\overline{F} = \frac{4\,\pi\,I}{c}$

7. $\overline{F} = \frac{5}{4}\,\frac{4\,\pi\,I}{c}$

8. $\overline{F} = \frac{3}{2}\,\frac{4\,\pi\,I}{c}$

9. $\overline{F} = \frac{5\,A_{\text{surf}}\,I}{4\,c}$ **correct**

10. $\overline{F} = 2\,\frac{4\,\pi\,I}{c}$

Explanation:

The time-average force is

$$\begin{aligned} F &= \text{Pressure}\,A_{\text{surf}} \\ &= F_{\text{abs}} + F_{\text{refl}} \\ &= \left(\frac{3}{4}\,\overline{u} + \frac{1}{4}\,2\,\overline{u}\right)\,A_{\text{surf}} \\ &= \frac{5\,A_{\text{surf}}\,I}{4\,c} \end{aligned}$$

Diamond Critical Angle

35:07, calculus, numeric, > 1 min, normal.

029 (part 1 of 1) 10 points

Assume: Refraction index for diamond $n_{diamond} = 2.419$.

The smallness of the critical angle θ_c for diamond means that light is easily "trapped" within a diamond and eventually emerges from the many cut faces. This makes a diamond more brilliant than stones with smaller n and larger θ_c. Traveling inside a diamond, a light ray is incident on the interface between diamond and air.

What is the critical angle for total internal reflection?

1. 20.9248 °

2. 21.1623 °

3. 21.9091 °

4. 22.9934 $^\circ$

5. 23.4786 $^\circ$

6. 24.4182 $^\circ$ **correct**

7. 24.7343 $^\circ$

8. 25.7715 $^\circ$

9. 26.5148 $^\circ$

10. 28.1446 $^\circ$

Explanation:

Basic Concept: Critical angle θ_c for total internal reflection

$$\sin\theta_c = \frac{n_2}{n_1}\,.$$

Solution: For diamond, the critical angle

$$\sin\theta_c = \frac{1}{2.419}\,.$$

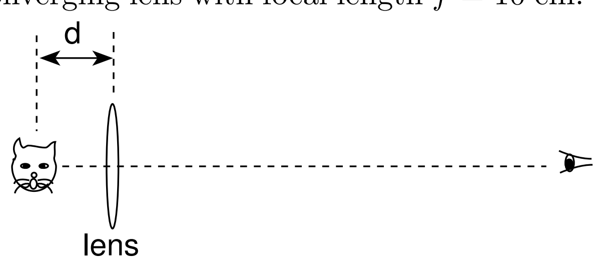
$$\theta_c = 24.4182^\circ\,.$$

Image of a Cat JMS

36:02, trigonometry, multiple choice, > 1 min, normal.

030 (part 1 of 1) 10 points

A cat is a distance $d = 15$ cm from a thin converging lens with focal length $f = 10$ cm.



How far from the lens is the image of the cat due only to this lens?

1. $\left(\frac{1}{f} + \frac{1}{d}\right)^{-1}$

2. $\frac{1}{f - d}$

3. $\frac{f}{\sqrt{\left(\frac{1}{f}\right)^2 + \left(\frac{1}{d}\right)^2}}$

4. $\left(\frac{1}{f} - \frac{1}{d}\right)^{-1}$ **correct**

5. $\left(\frac{1}{d} - \frac{1}{f}\right)^{-1}$

6. $\frac{d}{\sqrt{\left(\frac{1}{f}\right)^2 + \left(\frac{1}{d}\right)^2}}$

7. $\frac{1}{d - f}$

8. $\left(\frac{2}{f} - \frac{2}{d}\right)^{-1}$

9. $d + f$

10. $\left(\frac{2}{d} + \frac{2}{f}\right)^{-1}$

Explanation:

Basic Concepts:

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$		$m = \frac{h'}{h} = -\frac{q}{p}$
Converging Lens		$f > 0$
$\infty > p > f$	$f < q < \infty$	$0 > m > -\infty$
$f > p > 0$	$-\infty < q < 0$	$\infty > m > 1$
Diverging Lens		$0 > f$
$\infty > p > 0$	$f < q < 0$	$0 < m < 1$

Solution: Using the thin lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}\,,$$

we can compute the position of the image which would be:

$$\begin{aligned} x &= \left(\frac{1}{f} - \frac{1}{d}\right)^{-1} \\ &= \left(\frac{1}{10\text{ cm}} - \frac{1}{15\text{ cm}}\right)^{-1} \\ &= 30\text{ cm} \end{aligned}$$

MultiSlits JMS

37:04, trigonometry, multiple choice, < 1 min, wording-variable.

031 (part 1 of 1) 10 points

Given: The setup of a six slit diffraction experiment shown in the figure.

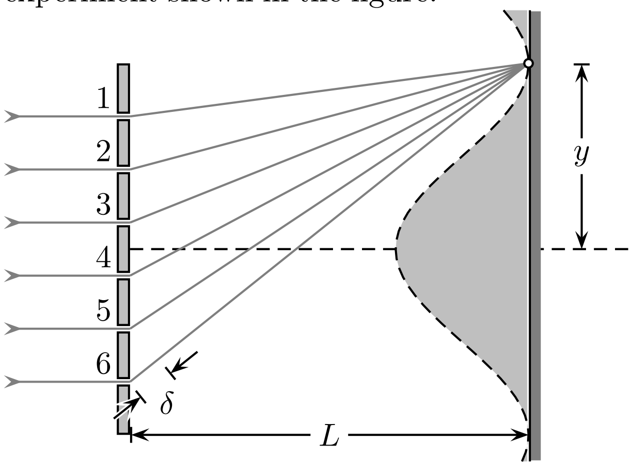


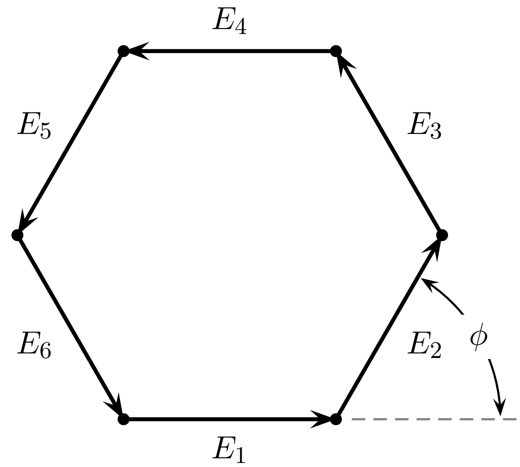
Figure: Not drawn to scale.

Find the path difference difference between two rays from adjacent slits which gives rise to the first minimum.

1. $\delta = \frac{1}{6}\lambda$ **correct**
2. $\delta = \frac{1}{4}\lambda$
3. $\delta = \frac{1}{5}\lambda$
4. $\delta = \frac{2}{5}\lambda$
5. $\delta = \frac{3}{4}\lambda$
6. $\delta = \frac{3}{5}\lambda$
7. $\delta = \frac{2}{3}\lambda$
8. $\delta = \frac{1}{2}\lambda$
9. $\delta = 2\lambda$
10. $\delta = \lambda$

Explanation:

Basic Concept: Light Interference



The first minimum occurs when the six phasor vectors of the six rays in the phasor diagram form a closed hexagon. Thus, the relative phas angle ϕ between the adjacent phasor vectors is given by

$$\phi = \frac{360^\circ}{6} = 60^\circ = \frac{1}{3}\pi\,,$$

and the path difference δ is

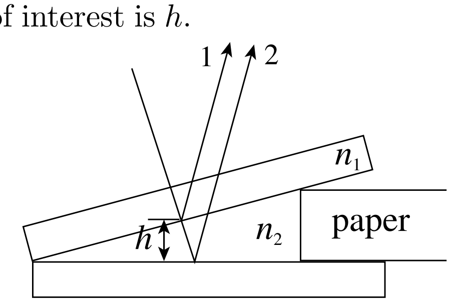
$$\delta = \frac{\lambda}{2\pi}\,\phi = \frac{\lambda}{2\pi}\,\frac{1}{3}\pi = \frac{1}{6}\,\lambda\,.$$

Thin Wedge of Air 03

37:06, calculus, multiple choice, > 1 min, fixed.

032 (part 1 of 2) 10 points

Let us do the air wedge problem without making the approximation that the index of refraction of air is unity. Let the wavelength of the incident light waves in the vacuum be λ_{vac}. As shown in the figure, denote the index of refraction of the glass as n_1 and that of air as n_2. The height of the thin wedge at the point of interest is h.



The phase angle difference between reflected rays # 1 and # 2 due to their path difference is given by

1. $\phi_{\text{path}} = \dfrac{4\,\pi}{n_1\,\lambda_{\text{vac}}}\,h\,.$

2. $\phi_{\text{path}} = \dfrac{4\,\pi\,n_1}{\lambda_{\text{vac}}}\,h\,.$

3. $\phi_{\text{path}} = \dfrac{4\,\pi\,n_1}{n_2\,\lambda_{\text{vac}}}\,h\,.$

4. $\phi_{\text{path}} = \dfrac{4\,\pi\,n_2}{\lambda_{\text{vac}}}\,h\,.$ **correct**

5. $\phi_{\text{path}} = \dfrac{4\,\pi}{n_2\,\lambda_{\text{vac}}}\,h\,.$

6. $\phi_{\text{path}} = \dfrac{2\,\pi\,n_2}{\lambda_{\text{vac}}}\,h\,.$

7. $\phi_{\text{path}} = \dfrac{2\,\pi\,n_1}{\lambda_{\text{vac}}}\,h\,.$

8. $\phi_{\text{path}} = \dfrac{2\,\pi\,n_1}{n_2\,\lambda_{\text{vac}}}\,h\,.$

9. $\phi_{\text{path}} = \dfrac{2\,\pi}{n_1\,\lambda_{\text{vac}}}\,h\,.$

10. $\phi_{\text{path}} = \dfrac{2\,\pi}{n_2\,\lambda_{\text{vac}}}\,h\,.$

Explanation:

The wavelength in air is related to the wavelength in the vacuum by

$$\lambda_{\text{air}} = \frac{\lambda_{\text{vac}}}{n_2}\,.$$

The ϕ_{path} is related to the path difference $\Delta = 2\,h$ by

$$\begin{aligned}\phi_{\text{path}} &= 2\,\pi\,\frac{\Delta}{\lambda_{\text{air}}}\\ &= 2\,\pi\,\frac{2\,h\,n_2}{\lambda_{\text{vac}}}\\ &= \frac{4\,\pi\,n_2}{\lambda_{\text{vac}}}\,h\,.\end{aligned}$$

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033 (part 2 of 2) 10 points

If the maximum phase difference due to the path difference is 40 radians, what is the total number of dark fringes, including the dark fringe at zero separation along the point of contact?

1. $N_{\text{total}} = 13$

2. $N_{\text{total}} = 5$

3. $N_{\text{total}} = 6$

4. $N_{\text{total}} = 8$

5. $N_{\text{total}} = 9$

6. $N_{\text{total}} = 10$

7. $N_{\text{total}} = 11$

8. $N_{\text{total}} = 12$

9. $N_{\text{total}} = 7$ **correct**

10. $N_{\text{total}} = 14$

Explanation:

Since there's a phase change π at the air glass interface, the total phase difference is

$$\phi = \phi_{\text{path}} + \pi\,.$$

Generally, destructive interference occurs when

$$(2n-1)\pi = \phi_{\text{path}} + \pi\,, \qquad n = 1, 2, 3\,\cdots\,.$$

Note: When $\phi_{\text{path}} = 0$, the equation is satisfied by $n = 1$.

So the above expression includes the minimum at zero separation. Now, the maximum number of dark fringes, N, for $\phi_{\text{path}} = 40$ rad can be found by considering

$$(2N-1)\pi \le \phi_{\text{max}} = 40 + \pi\,.$$

Solving for N yields

$$N \le \frac{40}{2\pi} + 1\,.$$

Since N must be an integer, we arrive at

$$N = \text{int}\left(\frac{40}{2\pi} + 1\right) = 7\,.$$

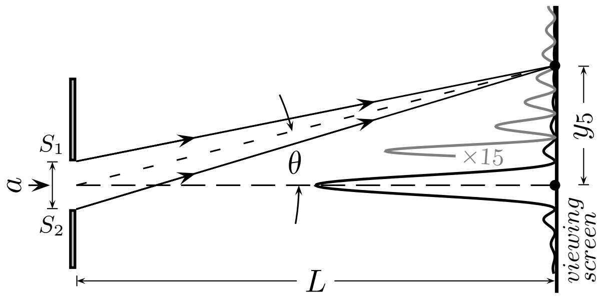
Dark Fringe Position

38:02, trigonometry, multiple choice, > 1 min, wording-variable.

034 (part 1 of 1) 10 points

Hint: Use a small angle approximation; *e.g.*, $\sin\theta = \tan\theta$.

Consider the setup of a single slit experiment.



Determine the height y_5, where the fifth minimum occurs.

1. $y_5 = 5\,\dfrac{\lambda\,L}{a}$ **correct**

2. $y_5 = \dfrac{11}{2}\,\dfrac{\lambda\,L}{a}$

3. $y_5 = 6\,\dfrac{\lambda\,L}{a}$

4. $y_5 = \dfrac{13}{2}\,\dfrac{\lambda\,L}{a}$

5. $y_5 = 7\,\dfrac{\lambda\,L}{a}$

6. $y_5 = \dfrac{15}{2}\,\dfrac{\lambda\,L}{a}$

7. $y_5 = \dfrac{9}{2}\,\dfrac{\lambda\,L}{a}$

8. $y_5 = 4\,\dfrac{\lambda\,L}{a}$

9. $y_5 = \dfrac{7}{2}\,\dfrac{\lambda\,L}{a}$

10. $y_5 = 3\,\dfrac{\lambda\,L}{a}$

Explanation:

Basic Concepts: Light Diffraction

$$\frac{I}{I_0} = \left(\frac{\sin\dfrac{\beta}{2}}{\dfrac{\beta}{2}}\right)^2,$$

where the minima are at

$$\frac{\beta}{2} = \pi\,,\ 2\,\pi\,,\ 3\,\pi\,,\ 4\,\pi\,,\ 5\,\pi\,,\ 6\,\pi\,,\ \cdots\,,\quad \text{or}$$
$$\beta = 2\,\pi\,,\ 4\,\pi\,,\ 6\,\pi\,,\ 8\,\pi\,,\ 10\,\pi\,,\ 12\,\pi\,,\ \cdots\,,$$
$$= 2\,m\,\pi\,,$$

where m is the first, second, third, fourth, $\cdots$, minimum in the diffraction pattern.

Solution: The first minimum is at $\beta = 2\,\pi$, where $\beta = 2\,\phi = 2\,\pi$, where $\phi = \pi$ is the phase difference of the two rays for destructive interference.

The fifth minimum occurs at $\beta = 10\,\pi$, which corresponds to a path difference δ between two end rays

$$\begin{aligned}
\delta &= \frac{\beta}{k} \\
&= \frac{10\,\pi}{\left(\dfrac{2\,\pi}{\lambda}\right)} \\
&= 5\,\lambda \\
\theta &= \frac{\delta}{a} \\
&= \frac{y_5}{L} \\
y_5 &= \frac{\delta}{a}\,L \\
&= 5\,\frac{\lambda\,L}{a}\,,
\end{aligned}$$

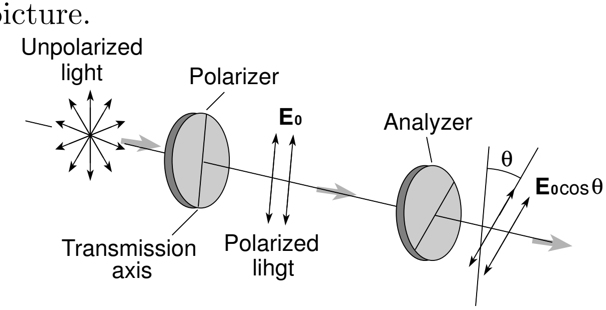
where $k \equiv \dfrac{2\,\pi}{\lambda}$.

Beam Intensity

38:06, calculus, multiple choice, < 1 min, fixed.

035 (part 1 of 1) 10 points

An unpolarized light beam with intensity of I_0 passes through 2 polarizers shown in the picture.



If $\theta = 30^\circ$,what is the beam intensity after the second polarizer?

1. $\dfrac{1}{16} I_0$

2. $\frac{3}{8} I_0$ **correct**

3. $\frac{1}{8} I_0$

4. $\frac{3}{16} I_0$

5. $\frac{1}{4} I_0$

6. $\frac{5}{16} I_0$

7. $\frac{7}{16} I_0$

8. $\frac{1}{2} I_0$

9. $\frac{9}{16} I_0$

10. $\frac{5}{8} I_0$

Explanation:

The beam intensity after the first polarizer is

$$I_1 = \frac{I_0}{2}$$

We use the formula for the intensity of the transmitted (polarized) light. Thus the beam intensity after the second polarizer is

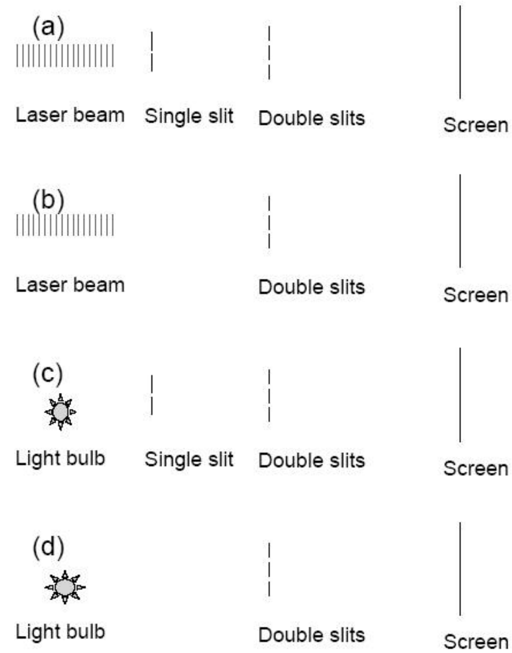
$$\begin{aligned} I &= I_1 \cos^2 \theta \\ &= \frac{I_0}{2} \cos^2(30^\circ) \\ &= \frac{3\, I_0}{8} \end{aligned}$$

Coherence and Slits

38:99, trigonometry, multiple choice, < 1 min, fixed.

036 (part 1 of 1) 10 points

For this problem, consider a screen illuminated by various combinations of slits and light sources, as described by the following diagram:



Knowing that laser light, in contrast to ordinary light sources, is generated with very well-defined phase (the laser light is coherent), which of the above setups will produce an interference pattern on the screen?

Note: the light bulb emits monochromatic (one-colored) light.

1. (a) (b) and (c) **correct**

2. (a) and (b)

3. (c) and (d)

4. (b) and (d)

5. (a) and (c)

6. (a) (b) and (d)

7. (a) (c) and (d)

8. (b) (c) and (d)

9. all of them

10. none of them

Explanation:

Laser light is coherent. Consequently, applying simple double and single slits to it will not destroy its coherence. Consequently, both (a) and (b) will produce intereference patterns.

Similarly, by filtering the light through a single slit apparatus, one constrains the path of the light from the light bulb to the screen. This makes the light leaving the single slit coherent. Consequently, when this newly coherent light passes through the double slit, an intereference pattern will result. If one only looks at light of a given wavelength, the pattern will be very similar to that generated by passing laser light through a double slit.

When the single slit is not availible to filter the light, however, the phases of the light bulb light hitting the double slit are essentially random. Consequently, any effect due to path differences is washed out by this randomness, and no pattern is observed.

Therefore, the correct answer is (a) (b) and (c).