Chapter 23 Solutions

23.1 (a) $N = \left(\frac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(47.0\ \frac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$

(b) $\#\text{ electrons added} = \frac{Q}{e} = \frac{1.00\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = 6.25\times10^{15}$

or $\boxed{2.38 \text{ electrons for every } 10^{9} \text{ already present}}$

23.2 (a) $F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{(3.80\times10^{-10}\text{ m})^2} = \boxed{1.59\times10^{-9}\text{ N}}$ (repulsion)

(b) $F_g = \frac{G m_1 m_2}{r^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(1.67\times10^{-27}\text{ kg})^2}{(3.80\times10^{-10}\text{ m})^2} = \boxed{1.29\times10^{-45}\text{ N}}$

The electric force is $\boxed{\text{larger by } 1.24\times10^{36} \text{ times}}$

(c) If $k_e\frac{q_1q_2}{r^2} = G\frac{m_1m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11}\text{ C/kg}}$$

23.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \approx \left(\frac{70{,}000 \text{ grams}}{18 \text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right) \approx 2.3\times10^{28}\text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = (0.01)(1.6\times10^{-19}\text{ C})(2.3\times10^{28}) = 3.7\times10^{7}\text{ C}$$

So $F = k_e\frac{q_1q_2}{r^2} = (9\times10^{9})\frac{(3.7\times10^{7})^2}{0.6^2}\text{ N} = 4\times10^{25}\text{ N}$ $\boxed{\sim 10^{26}\text{ N}}$

This force is almost enough to lift a "weight" equal to that of the Earth:

$$Mg = (6\times10^{24}\text{ kg})(9.8\text{ m/s}^2) = 6\times10^{25}\text{ N} \sim 10^{26}\text{ N}$$

23.4 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \quad \text{so} \quad q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times10^4\text{ N}}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times10^{-3}\text{ C}$$

The number of electron transferred is then

$$N_{xfer} = \left(1.05\times10^{-3}\text{ C}\right)\Big/\left(1.60\times10^{-19}\text{ C / e}^-\right) = 6.59\times10^{15}\text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{tot} = \left(\frac{10.0\text{ g}}{107.87\text{ g / mol}}\right)\left(6.02\times10^{23}\text{ atoms / mol}\right)\left(47\text{ e}^-\text{ / atom}\right) = 2.62\times10^{24}\text{ e}^-$$

The fraction transferred is then

$$f = \frac{N_{xfer}}{N_{tot}} = \left(\frac{6.59\times10^{15}}{2.62\times10^{24}}\right) = \boxed{2.51\times10^{-9}} = 2.51\text{ charges in every billion}$$

23.5

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(6.02\times10^{23}\right)^2}{\left[2(6.37\times10^6\text{ m})\right]^2} = \boxed{514\text{ kN}}$$

***23.6** (a) The force is one of attraction. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(12.0\times10^{-9}\text{ C}\right)\left(18.0\times10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{2.16\times10^{-5}\text{ N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of repulsion, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(3.00\times10^{-9}\text{ C}\right)\left(3.00\times10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{8.99\times10^{-7}\text{ N}}$$

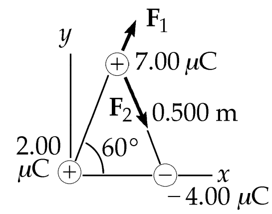
23.7 $$F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\ \text{C}^2)(7.00\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 0.503\ \text{N}$$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2\ /\ \text{C}^2)(7.00\times10^{-6}\ \text{C})(4.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 1.01\ \text{N}$$

$$F_x = (0.503 + 1.01)\cos 60.0° = 0.755\ \text{N}$$

$$F_y = (0.503 - 1.01)\sin 60.0° = -0.436\ \text{N}$$

$$\mathbf{F} = (0.755\ \text{N})\mathbf{i} - (0.436\ \text{N})\mathbf{j} = \boxed{0.872\ \text{N at an angle of } 330°}$$



Goal Solution

Three point charges are located at the corners of an equilateral triangle as shown in Figure P23.7. Calculate the net electric force on the 7.00–μC charge.

G: **G**ather Information: The 7.00–μC charge experiences a repulsive force $\mathbf{F}_1$ due to the 2.00–μC charge, and an attractive force $\mathbf{F}_2$ due to the –4.00–μC charge, where $F_2 = 2F_1$. If we sketch these force vectors, we find that the resultant appears to be about the same magnitude as F_2 and is directed to the right about 30.0° below the horizontal.

O: **O**rganize: We can find the net electric force by adding the two separate forces acting on the 7.00–μC charge. These individual forces can be found by applying Coulomb's law to each pair of charges.

A: **A**nalyze: The force on the 7.00–μC charge by the 2.00–μC charge is $\mathbf{F}_1 = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_1 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\ \text{C}^2\right)\left(7.00\times10^{-6}\ \text{C}\right)\left(2.00\times10^{-6}\ \text{C}\right)}{\left(0.500\ \text{m}\right)^2}\left(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}\right) = \mathbf{F}_1 = \left(0.252\mathbf{i} + 0.436\mathbf{j}\right)\ \text{N}$$

Similarly, the force on the 7.00–μC by the –4.00–μC charge is $\mathbf{F}_2 = k_e \frac{q_1 q_3}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_2 = -\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(7.00\times10^{-6}\ \text{C}\right)\left(-4.00\times10^{-6}\ \text{C}\right)}{\left(0.500\ \text{m}\right)^2}\left(\cos 60°\,\mathbf{i} - \sin 60°\,\mathbf{j}\right) = \left(0.503\mathbf{i} - 0.872\mathbf{j}\right)\ \text{N}$$

Thus, the total force on the 7.00–μC, expressed as a set of components, is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(0.755\ \mathbf{i} - 0.436\ \mathbf{j}\right)\ \text{N} = 0.872\ \text{N at } 30.0° \text{ below the } +x \text{ axis}$$

L: **L**earn: Our calculated answer agrees with our initial estimate. An equivalent approach to this problem would be to find the net electric field due to the two lower charges and apply **F**=q**E** to find the force on the upper charge in this electric field.

***23.8** Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\mathbf{F} = \frac{k_e(3q)Q}{x^2}\mathbf{i} + \frac{k_e(q)Q}{(d-x)^2}(-\mathbf{i})$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

***23.9** (a) $F = \frac{k_e e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{(0.529\times10^{-10}\ \text{m})^2} = \boxed{8.22\times10^{-8}\ \text{N}}$

(b) We have $F = \frac{mv^2}{r}$ from which $v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(8.22\times10^{-8}\ \text{N})(0.529\times10^{-10}\ \text{m})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{2.19\times10^6\ \text{m/s}}$

23.10 The top charge exerts a force on the negative charge $\frac{k_eqQ}{(d/2)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}(d/2x)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_eqQ}{(d^2/4 + x^2)}\right)\left(\frac{(-x)\mathbf{i}}{(d^2/4 + x^2)^{1/2}}\right) = m\mathbf{a} \qquad \text{or for } x << d/2, \qquad \mathbf{a} \approx \frac{-2k_eqQ}{md^3/8}\mathbf{x}$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\mathbf{a} = -\omega^2\mathbf{x}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_eqQ}{md^3}$.

$T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_eqQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $v_{\max} = \omega A = \boxed{4a\sqrt{\frac{k_eqQ}{md^3}}}$

23.11 For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$, or $q\mathbf{E} = -mg(-\mathbf{j})$. Thus, $\mathbf{E} = \dfrac{mg}{q}\mathbf{j}$.

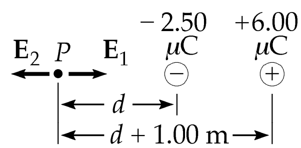
(a) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{(9.11\times10^{-31}\text{ kg})(9.80\text{ m/s}^2)}{\left(-1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\mathbf{j}}$$

(b) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\mathbf{j}}$$

23.12 $\sum F_y = 0$: $\quad QE\mathbf{j} + mg(-\mathbf{j}) = 0$

$$\therefore \qquad m = \frac{QE}{g} = \frac{(24.0\times10^{-6}\text{ C})(610\text{ N / C})}{9.80\text{ m / s}^2} = \boxed{1.49\text{ grams}}$$

***23.13** The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are



$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\text{ C})}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-6}\text{ C})}{(d+1.00\text{ m})^2} \qquad (2)$$

Equate the right sides of (1) and (2) to get $\quad (d + 1.00\text{ m})^2 = 2.40d^2$

or $\quad d + 1.00\text{ m} = \pm 1.55d$

which yields $\quad d = 1.82\text{ m}$ or $d = -0.392\text{ m}$

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction. Thus, $\boxed{d = 1.82\text{ m to the left of the -2.50 }\mu\text{C charge.}}$

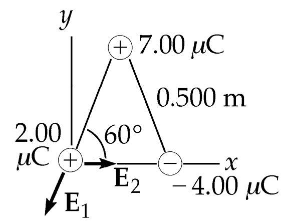
23.14 If we treat the concentrations as point charges,

$$\mathbf{E}_+ = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}(-\mathbf{j})\text{ (downward)}$$

$$\mathbf{E}_- = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}(-\mathbf{j})\text{ (downward)}$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \boxed{7.20\times10^5\text{ N / C downward}}$$

***23.15** (a) $E_1 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)}{(0.500)^2} = 2.52\times10^5\ \text{N/C}$



$E_2 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.500)^2} = 1.44\times10^5\ \text{N/C}$

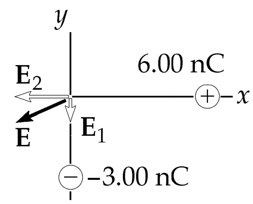
$E_x = E_2 - E_1\cos 60° = 1.44\times10^5 - 2.52\times10^5 \cos 60.0° = 18.0\times10^3\ \text{N/C}$

$E_y = -E_1 \sin 60.0° = -2.52\times10^5 \sin 60.0° = -218\times10^3\ \text{N/C}$

$\mathbf{E} = [18.0\,\mathbf{i} - 218\,\mathbf{j}]\times10^3\ \text{N/C} = \boxed{[18.0\,\mathbf{i} - 218\,\mathbf{j}]\ \text{kN/C}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(2.00\times10^{-6}\ \text{C}\right)\left(18.0\,\mathbf{i} - 218\,\mathbf{j}\right)\times10^3\ \text{N/C} = \left(36.0\,\mathbf{i} - 436\,\mathbf{j}\right)\times10^{-3}\ \text{N} = \boxed{\left(36.0\,\mathbf{i} - 436\,\mathbf{j}\right)\text{mN}}$

***23.16** (a) $\mathbf{E}_1 = \dfrac{k_e|q_1|}{r_1^2}(-\mathbf{j}) = \dfrac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}(-\mathbf{j}) = -\left(2.70\times10^3\ \text{N/C}\right)\mathbf{j}$



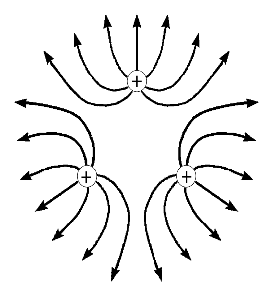
$\mathbf{E}_2 = \dfrac{k_e|q_2|}{r_2^2}(-\mathbf{i}) = \dfrac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}(-\mathbf{i}) = -\left(5.99\times10^2\ \text{N/C}\right)\mathbf{i}$

$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-\left(5.99\times10^2\ \text{N/C}\right)\mathbf{i} - \left(2.70\times10^3\ \text{N/C}\right)\mathbf{j}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(5.00\times10^{-9}\ \text{C}\right)\left(-599\,\mathbf{i} - 2700\,\mathbf{j}\right)\text{N/C}$

$\mathbf{F} = \left(-3.00\times10^{-6}\,\mathbf{i} - 13.5\times10^{-6}\,\mathbf{j}\right)\text{N} = \boxed{\left(-3.00\,\mathbf{i} - 13.5\,\mathbf{j}\right)\ \mu\text{N}}$

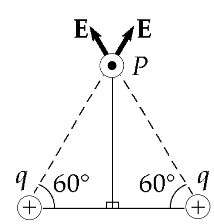
23.17 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.



(b) You may need to review vector addition in Chapter Three.

$\mathbf{E} = k_e \sum_i \dfrac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$

The magnitude of the field at point P due to each of the charges along the base of the triangle is $E = k_e\, q/a^2$. The direction of the field in each case is along the line joining the charge in question to point P as shown in the diagram at the right. The x components add to zero, leaving



$\mathbf{E} = \dfrac{k_e q}{a^2}(\sin 60.0°)\,\mathbf{j} + \dfrac{k_e q}{a^2}(\sin 60.0°)\,\mathbf{j} = \boxed{\sqrt{3}\,\dfrac{k_e q}{a^2}\,\mathbf{j}}$

Goal Solution

Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (**Hint:** Sketch the field lines in the plane of the charges.) (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

G: The electric field has the general appearance shown by the black arrows in the figure to the right. This drawing indicates that $\mathbf{E} = 0$ at the center of the triangle, since a small positive charge placed at the center of this triangle will be pushed away from each corner equally strongly. This fact could be verified by vector addition as in part (b) below.

The electric field at point P should be directed upwards and about twice the magnitude of the electric field due to just one of the lower charges as shown in Figure P23.17. For part (b), we must ignore the effect of the charge at point P, because a charge cannot exert a force on itself.

O: The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

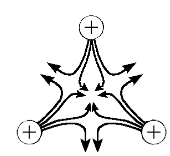
A: (b) The electric field from a point charge is $\mathbf{E} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$

As shown in the solution figure above, $\mathbf{E}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

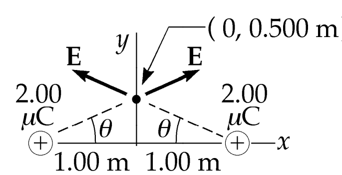
$\mathbf{E}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2}\left[(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) + (-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j})\right] = k_e \frac{q}{a^2}\left[2(\sin 60°\,\mathbf{j})\right] = 1.73 k_e \frac{q}{a^2}\,\mathbf{j}$$

L: The net electric field at point P is indeed nearly twice the magnitude due to a single charge and is entirely vertical as expected from the symmetry of the configuration. In addition to the center of the triangle, the electric field lines in the figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.



23.18 (a) $E = \dfrac{k_e q}{r^2} = \dfrac{(8.99\times 10^9)(2.00\times 10^{-6})}{(1.12)^2} = 14{,}400\ \text{N/C}$



$E_x = 0$ and $E_y = 2(14{,}400)\sin 26.6° = 1.29\times 10^4\ \text{N/C}$

so $\boxed{\mathbf{E} = 1.29\times 10^4\,\mathbf{j}\ \text{N/C}}$

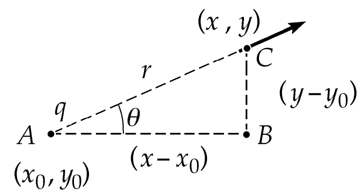
(b) $\mathbf{F} = \mathbf{E}q = (1.29\times 10^4\,\mathbf{j})(-3.00\times 10^{-6}) = \boxed{-3.86\times 10^{-2}\,\mathbf{j}\ \text{N}}$

23.19 (a) $\mathbf{E} = \dfrac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \dfrac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \dfrac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \dfrac{k_e(2q)}{a^2}\mathbf{i} + \dfrac{k_e(3q)}{2a^2}(\mathbf{i}\cos 45.0° + \mathbf{j}\sin 45.0°) + \dfrac{k_e(4q)}{a^2}\mathbf{j}$

$$\mathbf{E} = 3.06\frac{k_e q}{a^2}\mathbf{i} + 5.06\frac{k_e q}{a^2}\mathbf{j} = \boxed{5.91\frac{k_e q}{a^2} \text{ at } 58.8°}$$

(b) $\mathbf{F} = q\mathbf{E} = \boxed{5.91\dfrac{k_e q^2}{a^2} \text{ at } 58.8°}$

23.20 The magnitude of the field at (x,y) due to charge q at (x_0,y_0) is given by $E = k_e q/r^2$ where r is the distance from (x_0,y_0) to (x,y). Observe the geometry in the diagram at the right. From triangle ABC, $r^2 = (x-x_0)^2 + (y-y_0)^2$, or



$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \quad \sin\theta = \frac{(y-y_0)}{r}, \quad \text{and} \quad \cos\theta = \frac{(x-x_0)}{r}$$

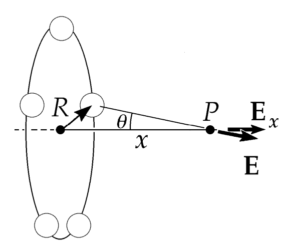
Thus, $E_x = E\cos\theta = \dfrac{k_e q}{r^2}\dfrac{(x-x_0)}{r} = \boxed{\dfrac{k_e q(x-x_0)}{[(x-x_0)^2+(y-y_0)^2]^{3/2}}}$

and $E_y = E\sin\theta = \dfrac{k_e q}{r^2}\dfrac{(y-y_0)}{r} = \boxed{\dfrac{k_e q(y-y_0)}{[(x-x_0)^2+(y-y_0)^2]^{3/2}}}$

23.21 The electric field at any point x is $E = \dfrac{k_e q}{(x-a)^2} - \dfrac{k_e q}{(x-(-a))^2} = \dfrac{k_e q(4ax)}{(x^2-a^2)^2}$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{(4a)(k_e q)}{x^3}}$

23.22 (a) One of the charges creates at P a field $\mathbf{E} = \dfrac{k_e Q/n}{R^2+x^2}$ at an angle θ to the x-axis as shown.



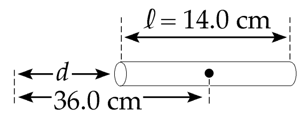
When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is $\dfrac{nk_e(Q/n)\mathbf{i}}{R^2+x^2}\cos\theta = \boxed{\dfrac{k_e Q x\mathbf{i}}{(R^2+x^2)^{3/2}}}$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field.}}$.

23.23 $$\mathbf{E} = \sum \frac{k_e q}{r^2} \sim= \frac{k_e q}{a^2}(-\,\mathbf{i}) + \frac{k_e q}{(2a)^2}(-\,\mathbf{i}) + \frac{k_e q}{(3a)^2}(-\,\mathbf{i}) + \;\ldots\; = \frac{-\,k_e q\,\mathbf{i}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^3} + \;\ldots\right) = \boxed{-\,\frac{\pi^2 k_e q}{6a^2}\,\mathbf{i}}$$

23.24 $$E = \frac{k_e \lambda \ell}{d(\ell + d)} = \frac{k_e(Q/\ell)\ell}{d(\ell + d)} = \frac{k_e Q}{d(\ell + d)} = \frac{(8.99 \times 10^9)(22.0 \times 10^{-6})}{(0.290)(0.140 + 0.290)}$$



$\mathbf{E} = \boxed{1.59 \times 10^6 \text{ N/C}}$, $\boxed{\text{directed toward the rod}}$.

23.25 $$E = \int \frac{k_e\, dq}{x^2} \quad \text{where } dq = \lambda_0\, dx$$

$$E = k_e\, \lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e \left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e \lambda_0}{x_0}}$$ $\boxed{\text{The direction is } -\mathbf{i} \text{ or left for } \lambda_0 > 0}$

23.26 $$\mathbf{E} = \int d\mathbf{E} = \int_{x_0}^{\infty}\left[\frac{k_e \lambda_0 x_0\, dx(-\mathbf{i})}{x^3}\right] = -\,k_e \lambda_0 x_0\,\mathbf{i}\int_{x_0}^{\infty} x^{-3}\, dx = -\,k_e \lambda_0 x_0\,\mathbf{i}\left(-\frac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right) = \boxed{\frac{k_e \lambda_0}{2x_0}(-\mathbf{i})}$$

23.27 $$E = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} = \frac{(8.99 \times 10^9)(75.0 \times 10^{-6})x}{(x^2 + 0.100^2)^{3/2}} = \frac{6.74 \times 10^5\, x}{(x^2 + 0.0100)^{3/2}}$$

(a) At $x = 0.0100$ m, $\mathbf{E} = 6.64 \times 10^6\, \mathbf{i}$ N/C = $\boxed{6.64\,\mathbf{i} \text{ MN/C}}$

(b) At $x = 0.0500$ m, $\mathbf{E} = 2.41 \times 10^7\, \mathbf{i}$ N/C = $\boxed{24.1\,\mathbf{i} \text{ MN/C}}$

(c) At $x = 0.300$ m, $\mathbf{E} = 6.40 \times 10^6\, \mathbf{i}$ N/C = $\boxed{6.40\,\mathbf{i} \text{ MN/C}}$

(d) At $x = 1.00$ m, $\mathbf{E} = 6.64 \times 10^5\, \mathbf{i}$ N/C = $\boxed{0.664\,\mathbf{i} \text{ MN/C}}$

23.28 $E = \dfrac{k_e Q x}{(x^2 + a^2)^{3/2}}$

For a maximum, $\dfrac{dE}{dx} = Qk_e\left[\dfrac{1}{(x^2+a^2)^{3/2}} - \dfrac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$

$x^2 + a^2 - 3x^2 = 0$ or $x = \dfrac{a}{\sqrt{2}}$

Substituting into the expression for E gives

$$E = \frac{k_e Qa}{\sqrt{2}(\frac{3}{2}a^2)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{2k_e Q}{3\sqrt{3}\,a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi \epsilon_0 a^2}}$$

23.29 $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$$E = 2\pi(8.99\times10^9)(7.90\times10^{-3})\left(1 - \frac{x}{\sqrt{x^2+(0.350)^2}}\right) = 4.46\times10^8\left(1 - \frac{x}{\sqrt{x^2+0.123}}\right)$$

(a) At x = 0.0500 m, $E = 3.83\times10^8$ N/C = $\boxed{383\text{ MN/C}}$

(b) At x = 0.100 m, $E = 3.24\times10^8$ N/C = $\boxed{324\text{ MN/C}}$

(c) At x = 0.500 m, $E = 8.07\times10^7$ N/C = $\boxed{80.7\text{ MN/C}}$

(d) At x = 2.00 m, $E = 6.68\times10^8$ N/C = $\boxed{6.68\text{ MN/C}}$

23.30 (a) From Example 23.9: $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$\sigma = \dfrac{Q}{\pi R^2} = 1.84\times10^{-3}\text{ C/m}^2$

$E = (1.04\times10^8\text{ N/C})(0.900) = 9.36\times10^7\text{ N/C} = \boxed{93.6\text{ MN/C}}$

appx: $E = 2\pi k_e \sigma = \boxed{104\text{ MN/C (about 11\% high)}}$

(b) $E = (1.04\times10^8\text{ N / C})\left(1 - \dfrac{30.0\text{ cm}}{\sqrt{30.0^2+3.00^2}\text{ cm}}\right) = (1.04\times10^8\text{ N/C})(0.00496) = \boxed{0.516\text{ MN/C}}$

appx: $E = k_e\dfrac{Q}{r^2} = (8.99\times10^9)\dfrac{5.20\times10^{-6}}{(0.30)^2} = \boxed{0.519\text{ MN/C (about 0.6\% high)}}$

23.31 The electric field at a distance x is $E_x = 2\pi k_e \sigma\left[1 - \dfrac{x}{\sqrt{x^2 + R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e \sigma\left[1 - \dfrac{1}{\sqrt{1 + R^2/x^2}}\right]$

For large x, $R^2/x^2 << 1$ and $\sqrt{1 + \dfrac{R^2}{x^2}} \approx 1 + \dfrac{R^2}{2x^2}$

so $E_x = 2\pi k_e \sigma\left(1 - \dfrac{1}{\left[1 + R^2/(2x^2)\right]}\right) = 2\pi k_e \sigma \dfrac{\left(1 + R^2/(2x^2) - 1\right)}{\left[1 + R^2/(2x^2)\right]}$

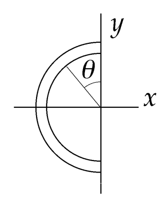
Substitute $\sigma = Q/\pi R^2$, $E_x = \dfrac{k_e Q\left(1/x^2\right)}{\left[1 + R^2/(2x^2)\right]} = k_e Q\left(x^2 + \dfrac{R^2}{2}\right)$

But for $x >> R$, $\dfrac{1}{x^2 + R^2/2} \approx \dfrac{1}{x^2}$, so $\boxed{E_x \approx \dfrac{k_e Q}{x^2} \text{ for a disk at large distances}}$

23.32 The sheet must have negative charge to repel the negative charge on the Styrofoam. The magnitude of the upward electric force must equal the magnitude of the downward gravitational force for the Styrofoam to "float" (i.e., $F_e = F_g$).

Thus, $-qE = mg$, or $-q\left(\dfrac{\sigma}{2\text{e}_0}\right) = mg$ which gives $\sigma = \boxed{-\dfrac{2\text{e}_0 mg}{q}}$

23.33 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE \sin\theta = k_e \int \dfrac{dq \sin\theta}{r^2}$



where $dq = \lambda ds = \lambda r\, d\theta$, so that, $E_x = \dfrac{k_e\lambda}{r}\int_0^{\pi} \sin\theta\, d\theta = \dfrac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi} = \dfrac{2k_e\lambda}{r}$

where $\lambda = \dfrac{q}{L}$ and $r = \dfrac{L}{\pi}$. Thus, $E_x = \dfrac{2k_e q\pi}{L^2} = \dfrac{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(7.50 \times 10^{-6}\text{ C})\pi}{(0.140 \text{ m})^2}$

Solving, $E = E_x = 2.16 \times 10^7$ N/C

Since the rod has a negative charge, $\mathbf{E} = (-2.16 \times 10^7\, \mathbf{i})$ N/C $= \boxed{-21.6\,\mathbf{i}\text{ MN/C}}$

23.34 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge Qdx/h and produces, at the chosen point, a field

$$d\mathbf{E} = \frac{k_e x}{(x^2 + R^2)^{3/2}} \frac{Q\,dx}{h}\,\mathbf{i}$$

The total field is

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Q x\,dx}{h(x^2 + R^2)^{3/2}}\,\mathbf{i} = \frac{k_e Q\mathbf{i}}{2h} \int_{x=d}^{d+h} (x^2 + R^2)^{-3/2}\, 2x\,dx$$

$$\mathbf{E} = \frac{k_e Q\mathbf{i}}{2h} \frac{(x^2 + R^2)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\mathbf{i}}{h}\left[\frac{1}{(d^2 + R^2)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $Q\,dx/h$, and charge-per-area $\sigma = Q\,dx/\pi R^2 h$. One disk produces a field

$$d\mathbf{E} = \frac{2\pi k_e Q\,dx}{\pi R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

So, $$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_{x=d}^{d+h} \frac{2k_e Q\,dx}{R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

$$\mathbf{E} = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[\int_d^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h} (x^2 + R^2)^{-1/2}\, 2x\,dx\right] = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{(x^2 + R^2)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\mathbf{E} = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[d + h - d - \left((d+h)^2 + R^2\right)^{1/2} + (d^2 + R^2)^{1/2}\right]$$

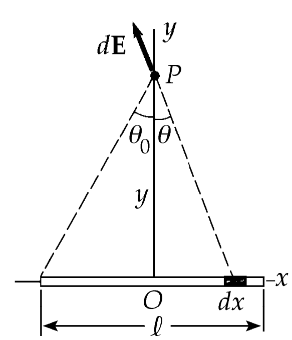
$$\mathbf{E} = \boxed{\frac{2k_e Q\mathbf{i}}{R^2 h}\left[h + (d^2 + R^2)^{1/2} - \left((d+h)^2 + R^2\right)^{1/2}\right]}$$

23.35 (a) The electric field at point P due to each element of length dx, is $dE = \dfrac{k_e dq}{(x^2 + y^2)}$ and is directed along the line joining the element of length to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda\,dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta \quad \text{where} \quad \cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Therefore, $$E = 2k_e \lambda y \int_0^{\ell/2} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$$



(b) For a bar of infinite length, $\theta \to 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{y}}$

***23.36** (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)2\pi(0.0250\ \text{m})[0.0250\ \text{m} + 0.0600\ \text{m}] = \boxed{2.00\times10^{-10}\ \text{C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)2\pi(0.0250\ \text{m})(0.0600\ \text{m}) = \boxed{1.41\times10^{-10}\ \text{C}}$$

(c) $Q = \rho V = \rho\pi r^2 L = \left(500\times10^{-9}\ \text{C/m}^3\right)\pi(0.0250\ \text{m})^2(0.0600\ \text{m}) = \boxed{5.89\times10^{-11}\ \text{C}}$

***23.37** (a) Every object has the same volume, $V = 8(0.0300\ \text{m})^3 = 2.16\times10^{-4}\ \text{m}^3$.

For each, $Q = \rho V = \left(400\times10^{-9}\ \text{C/m}^3\right)\left(2.16\times10^{-4}\ \text{m}^3\right) = \boxed{8.64\times10^{-11}\ \text{C}}$

(b) We must count the $9.00\ \text{cm}^2$ squares painted with charge:

(i) $6\times4 = 24$ squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)24.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{3.24\times10^{-10}\ \text{C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)34.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.59\times10^{-10}\ \text{C}}$$

(iii) 34 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)34.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.59\times10^{-10}\ \text{C}}$$

(iv) 32 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)32.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.32\times10^{-10}\ \text{C}}$$

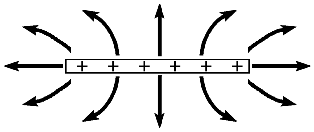
(c) (i) total edge length: $\ell = 24\times(0.0300\ \text{m})$

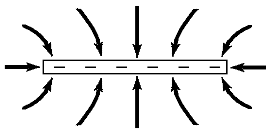
$$Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)24\times(0.0300\ \text{m}) = \boxed{5.76\times10^{-11}\ \text{C}}$$

(ii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)44\times(0.0300\ \text{m}) = \boxed{1.06\times10^{-10}\ \text{C}}$

(iii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)64\times(0.0300\ \text{m}) = \boxed{1.54\times10^{-10}\ \text{C}}$

(iv) $Q = \lambda \ell = \left(80.0 \times 10^{-12}\ \text{C/m}\right) 40 \times (0.0300\ \text{m}) = \boxed{0.960 \times 10^{-10}\ \text{C}}$

22.38 

22.39 

23.40 (a) $\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

23.41 $F = qE = ma \quad a = \frac{qE}{m}$

$v = v_i + at \qquad v = \frac{qEt}{m}$

electron: $v_e = \frac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{9.11\times10^{-31}} = \boxed{4.39\times10^{6} \text{ m/s}}$

in a direction opposite to the field

proton: $v_p = \frac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{1.67\times10^{-27}} = \boxed{2.39\times10^{3} \text{ m/s}}$

in the same direction as the field

23.42 (a) $|a| = \frac{qE}{m} = \frac{(1.602\times10^{-19})(6.00\times10^{5})}{(1.67\times10^{-27})} = 5.76\times10^{13} \text{ m/s}$ so $\quad \mathbf{a} = \boxed{-5.76\times10^{13}\,\mathbf{i} \text{ m/s}^2}$

(b) $v = v_i + 2a(x - x_i)$

$0 = v_i^{\,2} + 2(-5.76\times10^{13})(0.0700) \qquad \boxed{\mathbf{v}_i = 2.84\times10^{6}\,\mathbf{i} \text{ m/s}}$

(c) $v = v_i + at$

$0 = 2.84\times10^{6} + (-5.76\times10^{13})t \qquad t = \boxed{4.93\times10^{-8} \text{ s}}$

23.43 (a) $a = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\right)(640)}{\left(1.67 \times 10^{-27}\right)} = \boxed{6.14 \times 10^{10} \text{ m/s}^2}$

(b) $v = v_i + at$

$1.20 \times 10^6 = (6.14 \times 10^{10})t$

$\boxed{t = 1.95 \times 10^{-5} \text{ s}}$

(c) $x - x_i = \frac{1}{2}\left(v_i + v\right)t$

$x = \frac{1}{2}\left(1.20 \times 10^6\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7 \text{ m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(1.20 \times 10^6 \text{ m / s})^2 = \boxed{1.20 \times 10^{-15} \text{ J}}$

23.44 The required electric field will be $\boxed{\text{in the direction of motion}}$. We know that Work = ΔK

So, $-Fd = -\frac{1}{2} m v_i^2$ (since the final velocity = 0)

This becomes $Eed = \frac{1}{2}mv_i^2$ or $E = \frac{\frac{1}{2} m v_i^2}{e d}$

$E = \frac{1.60 \times 10^{-17} \text{ J}}{(1.60 \times 10^{-19} \text{ C})(0.100 \text{ m})} = \boxed{1.00 \times 10^3 \text{ N/C}}$ (in direction of electron's motion)

23.45 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done = ΔK so, $-Fd = -\frac{1}{2} m v_i^2$ (since the final velocity = 0)

which becomes $eEd = K$ and $\boxed{E = \frac{K}{e d}}$

Goal Solution

The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that stops these electrons in a distance of d?

G: We should expect that a larger electric field would be required to stop electrons with greater kinetic energy. Likewise, **E** must be greater for a shorter stopping distance, d. The electric field should be in the same direction as the motion of the negatively charged electrons in order to exert an opposing force that will slow them down.

O: The electrons will experience an electrostatic force $\mathbf{F} = q\mathbf{E}$. Therefore, the work done by the electric field can be equated with the initial kinetic energy since energy should be conserved.

A: The work done on the charge is $W = \mathbf{F} \cdot \mathbf{d} = q\mathbf{E} \cdot \mathbf{d}$

and $K_i + W = K_f = 0$

Assuming $\mathbf{v}$ is in the $+x$ direction, $K + (-e)\mathbf{E} \cdot d\mathbf{i} = 0$

$$e\mathbf{E} \cdot (d\mathbf{i}) = K$$

E is therefore in the direction of the electron beam: $\mathbf{E} = \dfrac{K}{ed}\mathbf{i}$

L: As expected, the electric field is proportional to K, and inversely proportional to d. The direction of the electric field is important; if it were otherwise the electron would speed up instead of slowing down! If the particles were protons instead of electrons, the electric field would need to be directed opposite to **v** in order for the particles to slow down.

23.46 The acceleration is given by $v^2 = v_i^2 + 2a(x - x_i)$ or $v^2 = 0 + 2a(-h)$

Solving, $a = -\dfrac{v^2}{2h}$

Now $\sum \mathbf{F} = m\mathbf{a}$: $-mg\mathbf{j} + q\mathbf{E} = -\dfrac{mv^2\,\mathbf{j}}{2h}$

Therefore $q\mathbf{E} = \left(-\dfrac{mv^2}{2h} + mg\right)\mathbf{j}$

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2(9.80\text{ m/s}^2)(5.00\text{ m})} = 9.90\text{ m/s}$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $$q = \frac{m}{E}\left(\frac{v^2}{2h} - g\right) = \frac{1.00 \times 10^{-3}\text{ kg}}{1.00 \times 10^4\text{ N/C}}\left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right)\left(\frac{(21.0\text{ m/s})^2}{2(5.00\text{ m})} - 9.80\text{ m/s}^2\right) = \boxed{3.43\ \mu\text{C}}$$

23.47 (a) $t = \frac{x}{v} = \frac{0.0500}{4.50 \times 10^5} = 1.11 \times 10^{-7}$ s $= \boxed{111 \text{ ns}}$

(b) $a_y = \frac{qE}{m} = \frac{(1.602 \times 10^{-19})(9.60 \times 10^3)}{(1.67 \times 10^{-27})} = 9.21 \times 10^{11} \text{ m / s}^2$

$y - y_i = v_{yi} t + \frac{1}{2} a_y t^2$

$y = \frac{1}{2}(9.21 \times 10^{11})(1.11 \times 10^{-7})^2 = 5.67 \times 10^{-3}$ m $= \boxed{5.67 \text{ mm}}$

(c) $v_x = \boxed{4.50 \times 10^5 \text{ m/s}}$

$v_y = v_{yi} + a_y = (9.21 \times 10^{11})(1.11 \times 10^{-7}) = \boxed{1.02 \times 10^5 \text{ m/s}}$

23.48 $a_y = \frac{qE}{m} = \frac{(1.602 \times 10^{-19})(390)}{(9.11 \times 10^{-31})} = 6.86 \times 10^{13} \text{ m / s}^2$

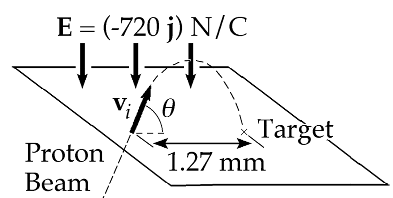
(a) $t = \frac{2 v_i \sin\theta}{a_y}$ from projectile motion equations

$t = \frac{2(8.20 \times 10^5)\sin 30.0°}{6.86 \times 10^{13}} = 1.20 \times 10^{-8}$ s) $= \boxed{12.0 \text{ ns}}$

(b) $h = \frac{v_i^2 \sin^2\theta}{2a_y} = \frac{(8.20 \times 10^5)^2 \sin^2 30.0°}{2(6.86 \times 10^{13})} = \boxed{1.23 \text{ mm}}$

(c) $R = \frac{v_i^2 \sin 2\theta}{2a_y} = \frac{(8.20 \times 10^5)^2 \sin 60.0°}{2(6.86 \times 10^{13})} = \boxed{4.24 \text{ mm}}$

23.49 $v_i = 9.55 \times 10^3$ m/s



(a) $a_y = \frac{eE}{m} = \frac{(1.60 \times 10^{-19})(720)}{(1.67 \times 10^{-27})} = 6.90 \times 10^{10} \text{ m/s}^2$

$R = \frac{v_i^2 \sin 2\theta}{a_y} = 1.27 \times 10^{-3}$ m so that $\frac{(9.55 \times 10^3)^2 \sin 2\theta}{6.90 \times 10^{10}} = 1.27 \times 10^{-3}$

$\sin 2\theta = 0.961$ $\theta = \boxed{36.9°}$ $90.0° - \theta = \boxed{53.1°}$

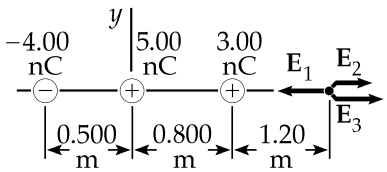
(b) $t = \frac{R}{v_{ix}} = \frac{R}{v_i \cos\theta}$ If $\theta = 36.9°$, $t = \boxed{167 \text{ ns}}$ If $\theta = 53.1°$, $t = \boxed{221 \text{ ns}}$

***23.50** (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})}{(2.50\ \text{m})^2}\mathbf{i} = -5.75\mathbf{i}\ \text{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge are

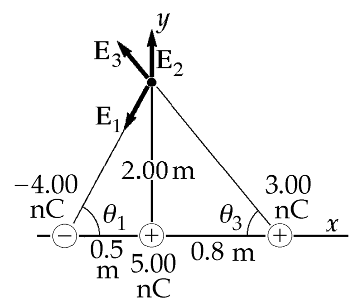
$$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\ \text{C})}{(2.00\ \text{m})^2}\mathbf{i} = 11.2\ \text{N/C}$$



$$\mathbf{E}_3 = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(1.20\ \text{m})^2}\mathbf{i} = 18.7\ \text{N/C}$$

$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \boxed{24.2\ \text{N/C}}$ in $+x$ direction.

(b) $\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \text{N/C})(0.243\mathbf{i} + 0.970\mathbf{j})$



$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \text{N/C})(+\mathbf{j})$

$\mathbf{E}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \text{N/C})(-0.371\mathbf{i} + 0.928\mathbf{j})$

$E_x = E_{1x} + E_{3x} = -4.21\mathbf{i}$ N/C $\quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\mathbf{j}$ N/C

$E_R = \boxed{9.42\ \text{N/C}} \quad \theta = \boxed{63.4^\circ\ \text{above}\ -x\ \text{axis}}$

23.51 The proton moves with acceleration $|a_p| = \frac{qE}{m} = \frac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 6.13\times10^{10}\ \text{m/s}^2$

while the e$^-$ has acceleration $|a_e| = \frac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{9.11\times10^{-31}\ \text{kg}} = 1.12\times10^{14}\ \text{m/s}^2 = 1836\,a_p$

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \text{cm} = \tfrac{1}{2}a_p t^2 + \tfrac{1}{2}a_e t^2 = 1837\left(\tfrac{1}{2}a_p t^2\right)$$

Thus, $d = \frac{1}{2}a_p t^2 = \frac{4.00\ \text{cm}}{1837} = \boxed{21.8\ \mu\text{m}}$

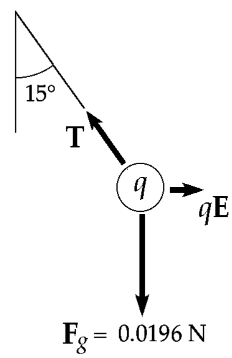
(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$$4.00\text{ cm} = \tfrac{1}{2}a_{\text{Na}}t^2 + \tfrac{1}{2}a_{\text{Cl}}t^2:\; 4.00\text{ cm} = \frac{1}{2}\left(\frac{eE}{22.99\text{ u}}\right)t^2 + \frac{1}{2}\left(\frac{eE}{35.45\text{ u}}\right)t^2$$

This may be written as $4.00\text{ cm} = \frac{1}{2}a_{\text{Na}}t^2 + \frac{1}{2}(0.649a_{\text{Na}})t^2 = 1.65\left(\frac{1}{2}a_{\text{Na}}t^2\right)$

so $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2 = \dfrac{4.00\text{ cm}}{1.65} = \boxed{2.43\text{ cm}}$

23.52 From the free-body diagram shown, $\sum F_y = 0$



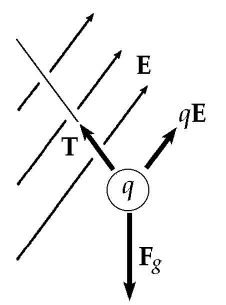
and $T\cos 15.0° = 1.96\times10^{-2}\text{ N}$

So $T = 2.03\times10^{-2}\text{ N}$

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$

or $q = \dfrac{T\sin 15.0°}{E} = \dfrac{(2.03\times10^{-2}\text{ N})\sin 15.0°}{1.00\times10^{3}\text{ N/C}} = 5.25\times10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}$

23.53 (a) Let us sum force components to find



Free Body Diagram for Goal Solution

$\sum F_x = qE_x - T\sin\theta = 0$, and $\sum F_y = qE_y + T\cos\theta - mg = 0$

Combining these two equations, we get

$$q = \frac{mg}{(E_x\cot\theta + E_y)} = \frac{(1.00\times10^{-3})(9.80)}{(3.00\cot 37.0° + 5.00)\times10^{5}} = 1.09\times10^{-8}\text{ C} = \boxed{10.9\text{ nC}}$$

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qE_x}{\sin 37.0°} = 5.44\times10^{-3}\text{ N} = \boxed{5.44\text{ mN}}$$

Goal Solution

A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as shown in Fig. P23.53. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N / C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

G: (a) Since the electric force must be in the same direction as **E**, the ball must be positively charged. If we examine the free body diagram that shows the three forces acting on the ball, the sum of which must be zero, we can see that the tension is about half the magnitude of the weight.

O: The tension can be found from applying Newton's second law to this statics problem (electrostatics, in this case!). Since the force vectors are in two dimensions, we must apply $\Sigma\mathbf{F} = m\mathbf{a}$ to both the x and y directions.

A: Applying Newton's Second Law in the x and y directions, and noting that $\Sigma\mathbf{F} = \mathbf{T} + q\mathbf{E} + \mathbf{F}_g = 0$,

$$\Sigma F_x = qE_x - T\sin 37.0° = 0 \qquad (1)$$

$$\Sigma F_y = qE_y + T\cos 37.0° - mg = 0 \qquad (2)$$

We are given $E_x = 3.00 \times 10^5$ N / C and $E_y = 5.00 \times 10^5$ N / C; substituting T from (1) into (2):

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37.0°}\right)} = \frac{(1.00 \times 10^{-3}\text{ kg})(9.80\text{ m / s}^2)}{\left(5.00 + \dfrac{3.00}{\tan 37.0°}\right) \times 10^5\text{ N / C}} = 1.09 \times 10^{-8}\text{ C}$$

(b) Using this result for q in Equation (1), we find that the tension is $T = \dfrac{qE_x}{\sin 37.0°} = 5.44 \times 10^{-3}$ N

L: The tension is slightly more than half the weight of the ball ($F_g = 9.80 \times 10^{-3}$ N) so our result seems reasonable based on our initial prediction.

23.54 (a) Applying the first condition of equilibrium to the ball gives:

$$\Sigma F_x = qE_x - T\sin\theta = 0 \quad \text{or} \quad T = \frac{qE_x}{\sin\theta} = \frac{qA}{\sin\theta}$$

and $$\Sigma F_y = qE_y + T\cos\theta - mg = 0 \quad \text{or} \quad qB + T\cos\theta = mg$$

Substituting from the first equation into the second gives:

$$q(A\cot\theta + B) = mg, \quad \text{or} \quad q = \boxed{\frac{mg}{(A\cot\theta + B)}}$$

(b) Substituting the charge into the equation obtained from ΣF_x yields

$$T = \frac{mg}{(A\cot\theta + B)}\left(\frac{A}{\sin\theta}\right) = \boxed{\frac{mgA}{A\cos\theta + B\sin\theta}}$$

Goal Solution

A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (A\mathbf{i} + B\mathbf{j})$ N / C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find (a) the charge on the ball and (b) the tension in the string.

G: This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

O: The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 53.

A: Again, Newton's second law: $-T\sin\theta + qA = 0$ (1)

and $+T\cos\theta + qB - mg = 0$ (2)

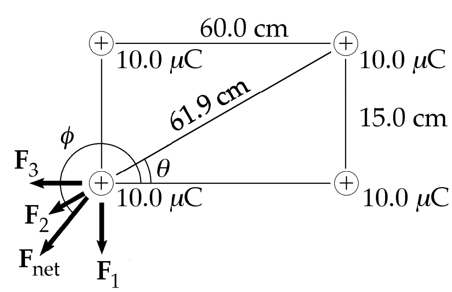
(a) Substituting $T = \dfrac{qA}{\sin\theta}$, into Eq. (2), $\dfrac{qA\cos\theta}{\sin\theta} + qB = mg$

Isolating q on the left, $q = \dfrac{mg}{(A\cot\theta + B)}$

(b) Substituting this value into Eq. (1), $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$

L: If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 53. If you find this problem more difficult than problem 53, the little list at the Gather step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the Analysis step, and for recognizing when we have an answer.

23.55 $F = \dfrac{k_e q_1 q_2}{r^2}$ $\tan\theta = \dfrac{15.0}{60.0}$ $\theta = 14.0°$



$$F_1 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.150)^2} = 40.0 \text{ N}$$

$$F_3 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.600)^2} = 2.50 \text{ N}$$

$$F_2 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.619)^2} = 2.35 \text{ N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78 \text{ N}$$

$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.6 \text{ N}$$

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78)^2 + (-40.6)^2} = \boxed{40.9 \text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78} \qquad \phi = \boxed{263°}$$

23.56 From Fig. A: $d\cos 30.0° = 15.0 \text{ cm}$, or $d = \dfrac{15.0 \text{ cm}}{\cos 30.0°}$

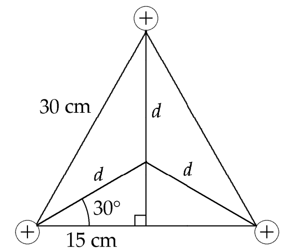


Figure A

From Fig. B: $\theta = \sin^{-1}\left(\dfrac{d}{50.0 \text{ cm}}\right) = \sin^{-1}\left(\dfrac{15.0 \text{ cm}}{50.0 \text{ cm}(\cos 30.0°)}\right) = 20.3°$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

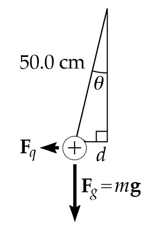


Figure B

From Fig. C: $F_q = 2F\cos 30.0° = 2\left[\dfrac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0°$ (2)

Equating equations (1) and (2), $2\left[\dfrac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$

$$q^2 = \frac{mg(0.300 \text{ m})^2 \tan 20.3°}{2k_e \cos 30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m})^2 \tan 20.3°}{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14} \text{ C}^2} = 2.05\times10^{-7} \text{ C} = \boxed{0.205\ \mu\text{C}}$$

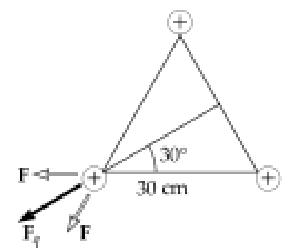


Figure C

23.57 Charge $Q/2$ resides on each block, which repel as point charges:

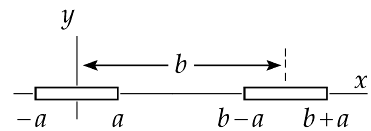
$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

$$Q = 2L\sqrt{\frac{k(L-L_i)}{k_e}} = 2(0.400 \text{ m})\sqrt{\frac{(100 \text{ N / m})(0.100 \text{ m})}{(8.99\times10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)}} = \boxed{26.7\ \mu\text{C}}$$

23.58 Charge $Q/2$ resides on each block, which repel as point charges: $F = \dfrac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$

Solving for Q, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$

***23.59** According to the result of Example 23.7, the lefthand rod creates this field at a distance d from its righthand end:



$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e Q Q}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

***23.60** The charge moves with acceleration of magnitude a given by $\sum F = ma = |q|E$

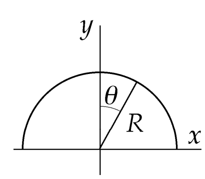
(a) $a = \frac{|q|E}{m} = \frac{1.60\times10^{-19}\text{ C}(1.00\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = 1.76\times10^{11}\text{ m/s}^2$

Then $v = v_i + at = 0 + at$ gives $t = \frac{v}{a} = \frac{3.00\times10^{7}\text{ m/s}}{1.76\times10^{11}\text{ m/s}^2} = \boxed{171\ \mu\text{s}}$

(b) $t = \frac{v}{a} = \frac{vm}{qE} = \frac{(3.00\times10^{7}\text{ m/s})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(1.00\text{ N/C})} = \boxed{0.313\text{ s}}$

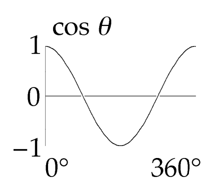
(c) From $t = \frac{vm}{qE}$, as E increases, t gets $\boxed{\text{shorter}}$ in inverse proportion.

23.61 $Q = \int \lambda\, d\ell = \int_{-90.0°}^{90.0°}\lambda_0\cos\theta\, R\, d\theta = \lambda_0 R\sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R[1-(-1)] = 2\lambda_0 R$

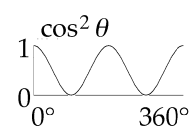


$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\text{ m} = 12.0\ \mu\text{C}$ so $\lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda\, d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})\left(\lambda_0\cos^2\theta R\, d\theta\right)}{R^2}\right)$$



$$F_y = \int_{-90.0°}^{90.0°}\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(3.00\times10^{-6}\text{ C})(10.0\times10^{-6}\text{ C/m})}{(0.600\text{ m})}\cos^2\theta\, d\theta$$



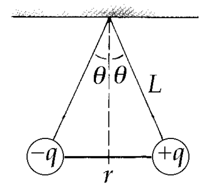
$$F_y = \frac{8.99(30.0)}{0.600}\left(10^{-3}\text{ N}\right)\int_{-\pi/2}^{\pi/2}\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = (0.450\text{ N})\left(\tfrac{1}{2}\pi + \tfrac{1}{4}\sin 2\theta\Big|_{-\pi/2}^{\pi/2}\right) = \boxed{0.707\text{ N}}\ \text{Downward.}$$

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

23.62 At equilibrium, the distance between the charges is $r = 2(0.100 \text{ m})\sin 10.0° = 3.47 \times 10^{-2} \text{ m}$

Now consider the forces on the sphere with charge $+q$, and use $\Sigma F_y = 0$:

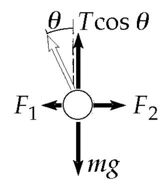


$\Sigma F_y = 0$: $\quad T\cos 10.0° = mg, \quad$ or $\quad T = \dfrac{mg}{\cos 10.0°} \qquad (1)$

$\Sigma F_x = 0$: $\quad F_{\text{net}} = F_2 - F_1 = T\sin 10.0° \qquad (2)$

F_{net} is the net electrical force on the charged sphere. Eliminate T from (2) by use of (1).

$$F_{\text{net}} = \frac{mg\sin 10.0°}{\cos 10.0°} = mg\tan 10.0° = \left(2.00 \times 10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\tan 10.0° = 3.46 \times 10^{-3} \text{ N}$$



F_{net} is the resultant of two forces, F_1 and F_2. F_1 is the attractive force on $+q$ exerted by $-q$, and F_2 is the force exerted on $+q$ by the external electric field.

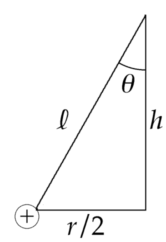
$F_{\text{net}} = F_2 - F_1$ or $F_2 = F_{\text{net}} + F_1$

$$F_1 = \left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.00 \times 10^{-8} \text{ C}\right)\left(5.00 \times 10^{-8} \text{ C}\right)}{\left(3.47 \times 10^{-3} \text{ m}\right)^2} = 1.87 \times 10^{-2} \text{ N}$$

Thus, $F_2 = F_{\text{net}} + F_1$ yields $F_2 = 3.46 \times 10^{-3} \text{ N} + 1.87 \times 10^{-2} \text{ N} = 2.21 \times 10^{-2} \text{ N}$

and $F_2 = qE$, or $\quad E = \dfrac{F_2}{q} = \dfrac{2.21 \times 10^{-2} \text{ N}}{5.00 \times 10^{-8} \text{ C}} = 4.43 \times 10^5 \text{ N/C} = \boxed{443 \text{ kN/C}}$

23.63 (a) From the $2Q$ charge we have $\quad F_e - T_2\sin\theta_2 = 0 \quad$ and $\quad mg - T_2\cos\theta_2 = 0$



Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$

From the Q charge we have $\quad F_e - T_1\sin\theta_1 = 0 \quad$ and $\quad mg - T_1\cos\theta_1 = 0$

Combining these we find $\quad \dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1 \quad$ or $\quad \boxed{\theta_2 = \theta_1}$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_eQ^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{(r/2)}{\ell}$. Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta \quad$ then $\quad \dfrac{2k_eQ^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell} \quad$ and solving for r we find $\quad r = \left[\dfrac{4k_eQ^2\ell}{mg}\right]^{1/3}$

23.64 At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium. In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P23.64, the charge at the origin exerts an attractive force $k_eQq/(s+\frac{1}{2}a\sqrt{3})^2$. The other two charges exert equal repulsive forces of magnitude k_eQq/r^2. The horizontal components of the two repulsive forces add, balancing the attractive force,

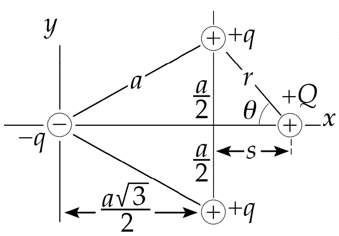
$$F_{\text{net}} = k_eQq\left\{\frac{2\cos\theta}{r^2} - \frac{1}{(s+\frac{1}{2}a\sqrt{3})^2}\right\} = 0$$

From Figure P23.64, $r = \dfrac{\frac{1}{2}a}{\sin\theta}$ $\quad s = \frac{1}{2}a\cot\theta$

The equilibrium condition, in terms of θ, is $F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_eQq\left(2\cos\theta\sin^2\theta - \dfrac{1}{(\sqrt{3}+\cot\theta)^2}\right) = 0$

Thus the equilibrium value of θ is $\quad 2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2 = 1.$

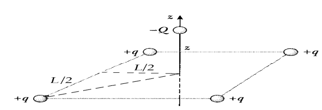
One method for solving for θ is to tabulate the left side. To three significant figures the value of θ corresponding to equilibrium is 81.7°. The distance from the origin to the equilibrium position is $x = \frac{1}{2}a(\sqrt{3}+\cot 81.7°) = \boxed{0.939a}$



θ	$2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

23.65 (a) The distance from each corner to the center of the square is

$$\sqrt{(L/2)^2 + (L/2)^2} = L/\sqrt{2}$$



The distance from each positive charge to $-Q$ is then $\sqrt{z^2 + L^2/2}$. Each positive charge exerts a force directed along the line joining q and $-Q$, of magnitude

$$\frac{k_eQq}{z^2 + L^2/2}$$

The line of force makes an angle with the z-axis whose cosine is $\dfrac{z}{\sqrt{z^2+L^2/2}}$

The four charges together exert forces whose x and y components add to zero, while the z-components add to

$$\mathbf{F} = \boxed{-\frac{4k_eQqz}{\left(z^2+L^2/2\right)^{3/2}}\mathbf{k}}$$

(b) For $z << L$, the magnitude of this force is $$F_z \approx -\frac{4k_eQqz}{\left(L^2/2\right)^{3/2}} = -\left(\frac{4(2)^{3/2}k_eQq}{L^3}\right)z = ma_z$$

Therefore, the object's vertical acceleration is of the form $\boxed{a_z = -\omega^2 z}$

with $$\omega^2 = \frac{4(2)^{3/2}k_eQq}{mL^3} = \frac{k_eQq\sqrt{128}}{mL^3}$$

Since the acceleration of the object is always oppositely directed to its excursion from equilibrium and in magnitude proportional to it, the object will execute simple harmonic motion with a period given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(128)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}} = \boxed{\frac{\pi}{(8)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}}$$

23.66 (a) The total non-contact force on the cork ball is: $$F = qE + mg = m\left(g + \frac{qE}{m}\right),$$

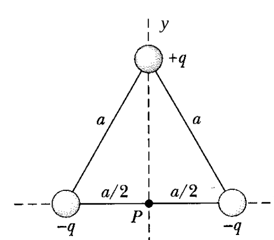
which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi\sqrt{\frac{L}{g + \frac{qE}{m}}} = 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m / s}^2 + \frac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)}{1.00\times10^{-3}\text{ kg}}}} = \boxed{0.307\text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $$T = 2\pi\sqrt{\frac{L}{qE/m}}$$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)/1.00\times10^{-3}\text{ kg}}} = 0.314\text{ s} \quad \text{(a 2.28\% difference).}$$

23.67 (a) Due to symmetry the field contribution from each negative charge is equal and opposite to each other. Therefore, their contribution to the net field is zero. The field contribution of the $+q$ charge is



$$E = \frac{k_eq}{r^2} = \frac{k_eq}{\left(3a^2/4\right)} = \frac{4k_eq}{3a^2}$$

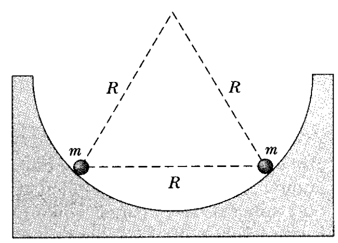
in the negative y direction, i.e., $\mathbf{E} = \boxed{-\frac{4k_eq}{3a^2}\mathbf{j}}$

(b) If $F_e = 0$, then E at P must equal zero. In order for the field to cancel at P, the $-4q$ must be above $+q$ on the y-axis.

Then, $E = 0 = -\dfrac{k_e q}{(1.00\text{ m})^2} + \dfrac{k_e(4q)}{y^2}$, which reduces to $y^2 = 4.00\text{ m}^2$.

Thus, $y = \pm 2.00\text{ m}$. Only the positive answer is acceptable since the $-4q$ must be located above $+q$. Therefore, the $-4q$ must be placed $\boxed{\text{2.00 meters above point } P \text{ along the } +y-\text{axis}}$.

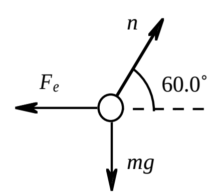
23.68 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram of the bead on the left:



$\Sigma F_y = n\sin 60.0° - mg = 0$,

or $n = \dfrac{mg}{\sin 60.0°}$

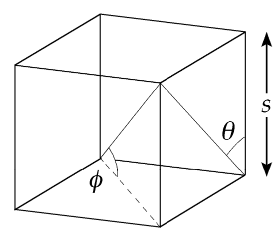
Also, $\Sigma F_x = -F_e + n\cos 60.0° = 0$,



or $\dfrac{k_e q^2}{R^2} = n\cos 60.0° = \dfrac{mg}{\tan 60.0°} = \dfrac{mg}{\sqrt{3}}$

Thus, $q = \boxed{R\left(\dfrac{mg}{k_e\sqrt{3}}\right)^{1/2}}$

23.69 (a) There are 7 terms which contribute:



3 are s away (along sides)

3 are $\sqrt{2}\ s$ away (face diagonals) and $\sin\theta = \dfrac{1}{\sqrt{2}} = \cos\theta$

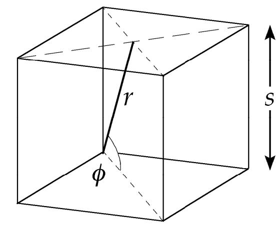
1 is $\sqrt{3}\ s$ away (body diagonal) and $\sin\phi = \dfrac{1}{\sqrt{3}}$

The component in each direction is the same by symmetry.

$$\mathbf{F} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right](\mathbf{i}+\mathbf{j}+\mathbf{k}) = \boxed{\frac{k_e q^2}{s^2}(1.90)(\mathbf{i}+\mathbf{j}+\mathbf{k})}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\dfrac{k_e q^2}{s^2} \text{ away from the origin}}$

23.70 (a) Zero contribution from the same face due to symmetry, opposite face contributes



$\sin\phi = s/r$

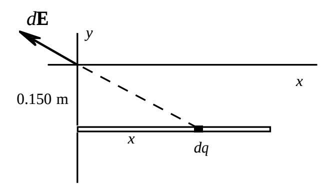
$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}\ s = 1.22\ s$$

$$E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\ \frac{k_e q}{s^2}}$$

(b) $\boxed{\text{The direction is the } \mathbf{k} \text{ direction.}}$

***23.71**

$$d\mathbf{E} = \frac{k_e\,dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right) = \frac{k_e\lambda\left(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$



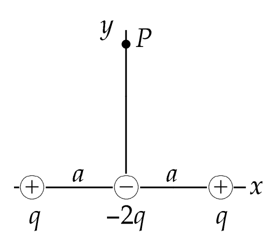
$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda\int_{x=0}^{0.400\text{ m}} \frac{\left(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\mathbf{i}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\,\mathbf{j}\,x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(35.0\times10^{-9}\ \frac{\text{C}}{\text{m}}\right)\left[\mathbf{i}(2.34 - 6.67)/\text{m} + \mathbf{j}(6.24 - 0)/\text{m}\right]$$

$$\mathbf{E} = \left(-1.36\,\mathbf{i} + 1.96\,\mathbf{j}\right)\times10^3\ \text{N/C} = \boxed{\left(-1.36\,\mathbf{i} + 1.96\,\mathbf{j}\right)\ \text{kN/C}}$$

23.72 By symmetry $\sum E_x = 0$. Using the distances as labeled,



$$\sum E_y = k_e\left[\frac{q}{(a^2+y^2)}\sin\theta + \frac{q}{(a^2+y^2)}\sin\theta - \frac{2q}{y^2}\right]$$

But $\sin\theta = \dfrac{y}{\sqrt{(a^2+y^2)}}$, so $E = \sum E_y = 2k_e q\left[\dfrac{y}{(a^2+y^2)^{3/2}} - \dfrac{1}{y^2}\right]$

Expand $(a^2+y^2)^{-3/2}$ as $(a^2+y^2)^{-3/2} = y^{-3} - (3/2)a^2y^{-5} + \ldots$

Therefore, for $a << y$, we can ignore terms in powers higher than 2,

and we have $E = 2k_e q\left[\dfrac{1}{y^2} - \left(\dfrac{3}{2}\right)\dfrac{a^2}{y^4} - \dfrac{1}{y^2}\right]$ or $\boxed{\mathbf{E} = \left[-\dfrac{k_e 3qa^2}{y^4}\right]\mathbf{j}}$

23.73 The field on the axis of the ring is calculated in Example 23.8, $E = E_x = \dfrac{k_e x Q}{(x^2 + a^2)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is

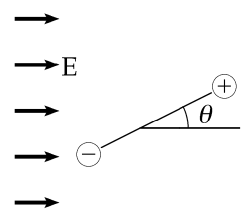
$F = -k_e Qq\left[\dfrac{x}{(x^2 + a^2)^{3/2}}\right]$ and when $x << a$, this becomes $F = \left(\dfrac{k_e Qq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law,

with an effective spring constant of $k = k_e Qq/a^3$

Since $\omega = 2\pi f = \sqrt{k/m}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$

23.74 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa \sin\theta = -2Eqa \sin\theta$.



For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$.

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\omega^2 = \dfrac{Ep}{I}$

This is the same form as Equation 13.17 and the frequency of oscillation is found by comparison with Equation 13.19, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$

Chapter 24 Solutions

24.1 (a) $\Phi_E = EA\cos\theta = (3.50\times10^3)(0.350\times0.700)\cos 0° = \boxed{858\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) $\theta = 90.0°$ $\boxed{\Phi_E = 0}$

(c) $\Phi_E = (3.50\times10^3)(0.350\times0.700)\cos 40.0° = \boxed{657\ \text{N}\cdot\text{m}^2/\text{C}}$

24.2 $\Phi_E = EA\cos\theta = (2.00\times10^4\ \text{N/C})(18.0\ \text{m}^2)\cos 10.0° = \boxed{355\ \text{kN}\cdot\text{m}^2/\text{C}}$

24.3 $\Phi_E = EA\cos\theta$

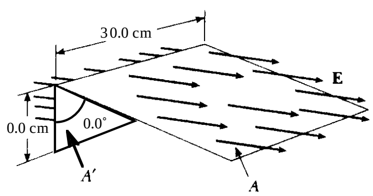
$A = \pi r^2 = \pi(0.200)^2 = 0.126\ \text{m}^2$

$5.20\times10^5 = E(0.126)\cos 0°$

$E = 4.14\times10^6\ \text{N/C} = \boxed{4.14\ \text{MN/C}}$

24.4 The uniform field enters the shell on one side and exits on the other so the total flux is $\boxed{\text{zero}}$.

24.5 (a) $A' = (10.0\ \text{cm})(30.0\ \text{cm})$



$A' = 300\ \text{cm}^2 = 0.0300\ \text{m}^2$

$\Phi_{E,\,A'} = EA'\cos\theta$

$\Phi_{E,\,A'} = (7.80\times10^4)(0.0300)\cos 180°$

$\Phi_{E,\,A'} = \boxed{-2.34\ \text{kN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\,A} = EA\cos\theta = (7.80\times10^4)(A)\cos 60.0°$

$A = (30.0\ \text{cm})(w) = (30.0\ \text{cm})\left(\dfrac{10.0\ \text{cm}}{\cos 60.0°}\right) = 600\ \text{cm}^2 = 0.0600\ \text{m}^2$

$\Phi_{E,\,A} = (7.80\times10^4)(0.0600)\cos 60° = \boxed{+2.34\ \text{kN}\cdot\text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to **E**, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{ total}} = -2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 2.34\ \text{kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

24.6 (a) $\Phi_E = \mathbf{E}\cdot\mathbf{A} = (a\mathbf{i} + b\mathbf{j})\cdot A\mathbf{i} = \boxed{aA}$

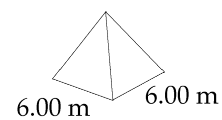
(b) $\Phi_E = (a\mathbf{i} + b\mathbf{j})\cdot A\mathbf{j} = \boxed{bA}$

(c) $\Phi_E = (a\mathbf{i} + b\mathbf{j})\cdot A\mathbf{k} = \boxed{0}$

24.7 Only the charge inside radius R contributes to the total flux.

$\Phi_E = \boxed{q / \epsilon_0}$

24.8 $\Phi_E = EA\cos\theta$ through the base



$\Phi_E = (52.0)(36.0)\cos 180° = -1.87\ \text{kN}\cdot\text{m}^2/\text{C}$

Note the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87\ \text{kN}\cdot\text{m}^2/\text{C}}$

24.9 The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \boxed{ERh}$. This is the same as the flux that exits the right side of the cone. Note that for a uniform field only the cross sectional area matters, not shape.

***24.10** (a) $E = \dfrac{k_e Q}{r^2}$

$8.90 \times 10^2 = \dfrac{(8.99\times 10^9)Q}{(0.750)^2}$, But Q is negative since $\mathbf{E}$ points inward.

$Q = -5.56\times 10^{-8}\ \text{C} = \boxed{-55.6\ \text{nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution.

24.11 (a) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89\times 10^6\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

24.12 $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$

Through S_1 $\Phi_E = \frac{-2Q+Q}{\epsilon_0} = \boxed{-\frac{Q}{\epsilon_0}}$

Through S_2 $\Phi_E = \frac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through S_3 $\Phi_E = \frac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\frac{2Q}{\epsilon_0}}$

Through S_4 $\Phi_E = \boxed{0}$

24.13 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\text{ plane}} = \frac{1}{2}\Phi_{E,\text{ total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

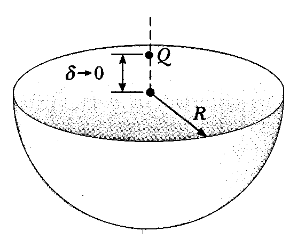
(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{ square}} \approx \Phi_{E,\text{ plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) The plane and the square look the same to the charge.

24.14 The flux through the curved surface is equal to the flux through the flat circle, $\boxed{E_0 \pi r^2}$.

24.15 (a) $\boxed{\frac{+Q}{2\epsilon_0}}$ Simply consider half of a closed sphere.



(b) $\boxed{\frac{-Q}{2\epsilon_0}}$ (from $\Phi_{E,\text{ total}} = \Phi_{E,\text{ dome}} + \Phi_{E,\text{ flat}} = 0$)

Goal Solution

A point charge Q is located just above the center of the flat face of a hemisphere of radius R, as shown in Figure P24.15. What is the electric flux (a) through the curved surface and (b) through the flat face?

G: From Gauss's law, the flux through a sphere with a point charge in it should be Q/ϵ_0, so we should expect the electric flux through a hemisphere to be half this value: $\Phi_{\text{curved}} = Q/2\epsilon_0$. Since the flat section appears like an infinite plane to a point just above its surface so that half of all the field lines from the point charge are intercepted by the flat surface, the flux through this section should also equal $Q/2\epsilon_0$.

O: We can apply the definition of electric flux directly for part (a) and then use Gauss's law to find the flux for part (b).

A: (a) With δ very small, all points on the hemisphere are nearly at distance R from the charge, so the field everywhere on the curved surface is $k_e Q / R^2$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \mathbf{E} \cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}} = \left(k_e \frac{Q}{R^2}\right)\left(\tfrac{1}{2}\right)\left(4\pi R^2\right) = \frac{1}{4\pi\epsilon_0} Q(2\pi) = \frac{Q}{2\epsilon_0}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \frac{-Q}{2\epsilon_0}$$

L: The direct calculations of the electric flux agree with our predictions, except for the negative sign in part (b), which comes from the fact that the area unit vector is defined as pointing outward from an enclosed surface, and in this case, the electric field has a component in the opposite direction (down).

24.16 (a) $\Phi_{E,\text{ shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.36 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C} = \boxed{1.36 \text{ MN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{ half shell}} = \frac{1}{2}(1.36 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}) = 6.78 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C} = \boxed{678 \text{ kN} \cdot \text{m}^2/\text{C}}$

(c) $\boxed{\text{No,}}$ the same number of field lines will pass through each surface, no matter how the radius changes.

24.17 From Gauss's Law, $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{q_{in}}{\epsilon_0}$.

Thus, $\Phi_E = \dfrac{Q}{\epsilon_0} = \dfrac{0.0462 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{5.22 \text{ kN} \cdot \text{m}^2/\text{C}}$

24.18 If $R \le d$, the sphere encloses no charge and $\Phi_E = q_{\text{in}} / \epsilon_0 = \boxed{0}$

If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{2\lambda\sqrt{R^2 - d^2}/\epsilon_0}$

24.19 The total charge is $Q - 6|q|$. The total outward flux from the cube is $(Q - 6|q|)/\epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6\epsilon_0} = \frac{(5.00 - 6.00)\times 10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times 8.85\times 10^{-12}\ \text{C}^2} = \boxed{-18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

24.20 The total charge is $Q - 6|q|$. The total outward flux from the cube is $(Q - 6|q|)/\epsilon_0$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q - 6|q|}{6\epsilon_0}}$$

24.21 When $R < d$, the cylinder contains no charge and $\Phi_E = \boxed{0}$.

When $R > d$, $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \boxed{\dfrac{\lambda L}{\epsilon_0}}$

24.22 $$\Phi_{E,\text{ hole}} = \mathbf{E}\cdot\mathbf{A}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right) = \left(\frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times 10^{-6}\ \text{C}\right)}{(0.100\ \text{m})^2}\right)\pi\left(1.00\times 10^{-3}\ \text{m}\right)^2$$

$$\Phi_{E,\text{ hole}} = \boxed{28.2\ \text{N}\cdot\text{m}^2/\text{C}}$$

24.23 $\Phi_E = \dfrac{q_{in}}{\epsilon_0} = \dfrac{170 \times 10^{-6}\text{ C}}{8.85 \times 10^{-12}\text{ C}^2/\text{N} \cdot \text{m}^2} = 1.92 \times 10^{7}\text{ N} \cdot \text{m}^2/\text{C}$

(a) $(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \dfrac{1.92 \times 10^{7}\text{ N} \cdot \text{m}^2/\text{C}}{6}$

$(\Phi_E)_{\text{one face}} = \boxed{3.20\text{ MN} \cdot \text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\text{ MN} \cdot \text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an unsymmetrical charge distribution. The sides of the cube nearer the charge would have more flux and the ones farther away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

24.24 (a) $\Phi_E = \dfrac{q_{in}}{\epsilon_0}$

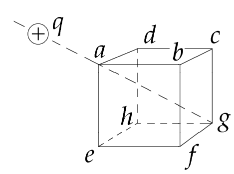
$8.60 \times 10^{4} = \dfrac{q_{in}}{8.85 \times 10^{-12}}$

$q_{in} = 7.61 \times 10^{-7}\text{ C} = \boxed{761\text{ nC}}$

(b) Since the net flux is positive, **the net charge must be positive** (boxed). It can have any distribution.

(c) **The net charge would have the same magnitude but be negative.** (boxed)

24.25 No charge is inside the cube. The net flux through the cube is zero. Positive flux comes out through the three faces meeting at *g*. These three faces together fill solid angle equal to one-eighth of a sphere as seen from *q*, and together pass flux $\frac{1}{8}(q/\epsilon_0)$. Each face containing *a* intercepts equal flux going into the cube:



$0 = \Phi_{E,\text{ net}} = 3\Phi_{E,\text{ abcd}} + q/8\epsilon_0$

$\Phi_{E,\text{ abcd}} = \boxed{-q/24\epsilon_0}$

24.26 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = k_e q / r^2$

$$E = \frac{(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(82 \times 1.60 \times 10^{-19} \text{ C})}{\left[(208)^{1/3} \; 1.20 \times 10^{-15} \text{ m}\right]^2}$$

$$E = \boxed{2.33 \times 10^{21} \text{ N/C}} \quad \text{away from the nucleus}$$

24.27 (a) $E = \dfrac{k_e Qr}{a^3} = \boxed{0}$

(b) $E = \dfrac{k_e Qr}{a^3} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365 \text{ kN/C}}$

(c) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46 \text{ MN/C}}$

(d) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649 \text{ kN/C}}$

The direction for each electric field is radially outward.

***24.28** (a) $E = \dfrac{2k_e \lambda}{r}$

$$3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{(0.190)}$$

$$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$$

(b) $\boxed{E = 0}$

24.29 $$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\text{e}_0} = \frac{\int \rho \, dV}{\text{e}_0} = \frac{\rho}{\text{e}_0} \, l \pi r^2$$

$$E 2\pi r l = \frac{\rho}{\text{e}_0} \, l \pi r^2$$

$$\boxed{\mathbf{E} = \frac{\rho}{2\epsilon_0} r \text{ away from the axis}}$$

Goal Solution

Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. Find the electric field at distance r from the axis where $r < R$.

G: According to Gauss's law, only the charge enclosed within the gaussian surface of radius r needs to be considered. The amount of charge within the gaussian surface will certainly increase as ρ and r increase, but the area of this gaussian surface will also increase, so it is difficult to predict which of these two competing factors will more strongly affect the electric field strength.

O: We can find the general equation for E from Gauss's law.

A: If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. The circular end caps have no electric flux through them; there $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 90.0° = 0$. The curved surface has $\mathbf{E}\cdot d\mathbf{A} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

Gauss's law, $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\substack{\text{Curved}\\ \text{Surface}}} dA = \frac{\rho\pi r^2 L}{\epsilon_0}$

Now the lateral surface area of the cylinder is $2\pi rL$: $E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$

Thus, $\mathbf{E} = \frac{\rho r}{2\epsilon_0}$ radially away from the cylinder axis

L: As we expected, the electric field will increase as ρ increases, and we can now see that E is also proportional to r. For the region outside the cylinder ($r > R$), we should expect the electric field to decrease as r increases, just like for a line of charge.

24.30 $\sigma = \left(8.60\times 10^{-6}\text{ C / cm}^2\right)\left(\frac{100\text{ cm}}{\text{m}}\right)^2 = 8.60\times 10^{-2}\text{ C / m}^2$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{8.60\times 10^{-2}}{2\left(8.85\times 10^{-12}\right)} = \boxed{4.86\times 10^{9}\text{ N / C}}$$

The field is essentially uniform as long as the distance from the center of the wall to the field point is much less than the dimensions of the wall.

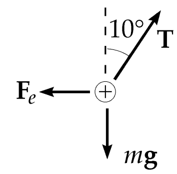
24.31 (a) $\boxed{E = 0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^{9})(32.0\times10^{-6})}{(0.200)^2} = \boxed{7.19\text{ MN/C}}$$

24.32 The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(46)^2 (1.60 \times 10^{-19} \text{ C})^2}{(2 \times 5.90 \times 10^{-15} \text{ m})^2} = 3.50 \times 10^3 \text{ N} = \boxed{3.50 \text{ kN}}$$

***24.33** Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.



(a) $\Sigma F_y = T\cos 10° - mg = 0 \;\Rightarrow\; T = \dfrac{mg}{\cos 10°}$

$\Sigma F_x = T\sin 10° - F_e = 0 \;\Rightarrow\; F_e = T\sin 10°$, so

$$F_e = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10° = (0.001 \text{ kg})(9.8 \text{ m/s}^2)\tan 10°$$

$F_e \approx 2 \times 10^{-3}$ N $\boxed{\sim 10^{-3} \text{ N or } 1 \text{ mN}}$

(b) $F_e = \dfrac{k_e q^2}{r^2}$

$$2 \times 10^{-3} \text{ N} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) q^2}{(0.25 \text{ m})^2}$$

$q \approx 1.2 \times 10^{-7}$ C $\boxed{\sim 10^{-7} \text{ C or } 100 \text{ nC}}$

(c) $$E = \frac{k_e q}{r^2} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.2 \times 10^{-7} \text{ C})}{(0.25 \text{ m})^2} \approx 1.7 \times 10^4 \text{ N/C} \quad \boxed{\sim 10 \text{ kN/C}}$$

(d) $$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2 \times 10^{-7} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 1.4 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C} \quad \boxed{\sim 10 \text{ kN} \cdot \text{m}^2/\text{C}}$$

24.34 (a) $$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{5.70 \times 10^{-6}}{\frac{4}{3}\pi (0.0400)^3} = 2.13 \times 10^{-2} \text{ C / m}^3$$

$$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = (2.13 \times 10^{-2})\left(\tfrac{4}{3}\pi\right)(0.0200)^3 = 7.13 \times 10^{-7} \text{ C} = \boxed{713 \text{ nC}}$$

(b) $$q_{\text{in}} = \rho\left(\tfrac{4}{3}\pi r^3\right) = (2.13 \times 10^{-2})\left(\tfrac{4}{3}\pi\right)(0.0400)^3 = \boxed{5.70\ \mu\text{C}}$$

24.35 (a) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\left(2.00\times10^{-6}\ \text{C}\right)/7.00\ \text{m}\right]}{0.100\ \text{m}}$

$E = \boxed{51.4\ \text{kN/C, radially outward}}$

(b) $\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos 0°$

$\Phi_E = \left(5.14\times10^4\ \text{N/C}\right)2\pi(0.100\ \text{m})(0.0200\ \text{m})(1.00) = \boxed{646\ \text{N}\cdot\text{m}^2/\text{C}}$

24.36 Note that the electric field in each case is directed radially inward, toward the filament.

(a) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{0.100\ \text{m}} = \boxed{16.2\ \text{MN/C}}$

(b) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{0.200\ \text{m}} = \boxed{8.09\ \text{MN/C}}$

(c) $E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C}\right)}{1.00\ \text{m}} = \boxed{1.62\ \text{MN/C}}$

24.37 $E = \frac{\sigma}{2\epsilon_0} = \frac{9.00\times10^{-6}\ \text{C}/\text{m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{508\ \text{kN/C, upward}}$

24.38 From Gauss's Law, $EA = \frac{Q}{\epsilon_0}$

$\sigma = \frac{Q}{A} = \epsilon_0 E = (8.85\times10^{-12})(130) = 1.15\times10^{-9}\ \text{C/m}^2 = \boxed{1.15\ \text{nC/m}^2}$

24.39 $\oint E\,dA = E(2\pi r l) = \frac{q_{in}}{\epsilon_0}$ $\quad E = \frac{q_{in}/l}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\boxed{E = 0}$ inside the conductor

(b) $r = 10.0$ cm $\quad E = \frac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(0.100)} = \boxed{5400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\quad E = \frac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(1.00)} = \boxed{540\ \text{N/C, outward}}$

***24.40** Just above the aluminum plate (a conductor), the electric field is $E = \sigma'/\epsilon_0$ where the charge Q is divided equally between the upper and lower surfaces of the plate:

Thus $\sigma' = \frac{(Q/2)}{A} = \frac{Q}{2A}$ and $E = \frac{Q}{2\epsilon_0 A}$

For the glass plate (an insulator), $E = \sigma / 2\epsilon_0$ where $\sigma = Q / A$ since the entire charge Q is on the upper surface.

Therefore, $E = \frac{Q}{2\epsilon_0 A}$

The electric field at a point just above the center of the upper surface is the same for each of the plates.

$\boxed{E = \frac{Q}{2\epsilon_0 A}\text{, vertically upward in each case (assuming } Q > 0)}$

***24.41** (a) $E = \sigma/\epsilon_0$ $\sigma = (8.00 \times 10^4)(8.85 \times 10^{-12}) = 7.08 \times 10^{-7}\ \text{C/m}^2$

$\boxed{\sigma = 708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \frac{Q}{A}$ $Q = \sigma A = (7.08 \times 10^{-7})\,(0.500)^2\ \text{C}$

$Q = 1.77 \times 10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$, positive on one face and negative on the other.

24.42 Use Gauss's Law to evaluate the electric field in each region, recalling that the electric field is zero everywhere within conducting materials. The results are:

$\boxed{E = 0 \text{ inside the sphere and inside the shell}}$

$\boxed{E = k_e \frac{Q}{r^2} \text{ between sphere and shell, directed radially inward}}$

$\boxed{E = k_e \frac{2Q}{r^2} \text{ outside the shell, directed radially inward}}$

Charge $\boxed{-Q \text{ is on the outer surface of the sphere}}$.

Charge $\boxed{+Q \text{ is on the inner surface of the shell}}$, and $+2Q$ is on the outer surface of the shell.

24.43 The charge divides equally between the identical spheres, with charge $Q/2$ on each. Then they repel like point charges at their centers:

$$F = \frac{k_e(Q/2)(Q/2)}{(L + R + R)^2} = \frac{k_e\ Q^2}{4(L + 2R)^2} = \frac{8.99 \times 10^9\ \mathrm{N \cdot m^2}(60.0 \times 10^{-6}\ \mathrm{C})^2}{4\ \mathrm{C^2}(2.01\ \mathrm{m})^2} = \boxed{2.00\ \mathrm{N}}$$

***24.44** The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vise-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{\epsilon_0} \quad \text{or} \quad \sigma = \epsilon_0 E$$

(a) Where the radius of curvature is the greatest,

$$\sigma = \epsilon_0 E_{\min} = \left(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\right)\left(2.80 \times 10^4\ \mathrm{N/C}\right) = \boxed{248\ \mathrm{nC/m^2}}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = \epsilon_0 E_{\max} = \left(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\right)\left(5.60 \times 10^4\ \mathrm{N/C}\right) = \boxed{496\ \mathrm{nC/m^2}}$$

24.45 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$$0 = \lambda\ell + q_{\text{in}} \Rightarrow \frac{q_{\text{in}}}{\ell} = \boxed{-\lambda}$$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell = q_{in} + q_{out}$.

$$q_{out} = 2\lambda\ell + \lambda\ell$$

so the outside charge/length $= \boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\lambda}{r} = \boxed{\frac{3\lambda}{2\pi\epsilon_0 r}}$$

24.46 (a) $E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\right)\left(6.40 \times 10^{-6}\right)}{(0.150)^2} = \boxed{2.56\ \text{MN/C, radially inward}}$

(b) $\boxed{E = 0}$

24.47 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{(4.00 \times 10^{-8}\ \text{C})}{(0.500\ \text{m})^2} = 8.00 \times 10^{-8}\ \text{C / m}^2 = \boxed{80.0\ \text{nC / m}^2}$$

(b) $\mathbf{E} = \left(\frac{\sigma}{\epsilon_0}\right)\mathbf{k} = \left(\frac{8.00 \times 10^{-8}\ \text{C / m}^2}{8.85 \times 10^{-12}\ \text{C}^2\ /\ \text{N} \cdot \text{m}^2}\right)\mathbf{k} = \boxed{(9.04\ \text{kN / C})\mathbf{k}}$

(c) $\mathbf{E} = \boxed{(-9.04\ \text{kN / C})\mathbf{k}}$

24. 48 (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is:

$$\sigma_a = \boxed{\frac{-q}{4\pi a^2}}$$

(b) The outer surface carries charge $Q + q$ with density

$$\sigma_b = \boxed{\frac{Q+q}{4\pi b^2}}$$

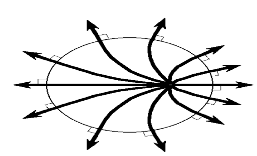
24.49 (a) $\boxed{E = 0}$

(b) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(8.00 \times 10^{-6})}{(0.0300)^2} = 7.99 \times 10^7\ \text{N / C} = \boxed{79.9\ \text{MN/C}}$

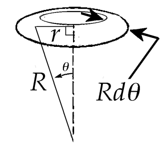
(c) $\boxed{E = 0}$

(d) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(4.00 \times 10^{-6})}{(0.0700)^2} = 7.34 \times 10^6\ \text{N / C} = \boxed{7.34\ \text{MN/C}}$

24.50 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



24.51 (a) Uniform **E**, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = R\,d\theta$, and the circumference is $2\pi r = 2\pi R \sin\theta$



$$A = \int 2\pi r\,ds = \int_0^\theta (2\pi R\sin\theta)R\,d\theta = 2\pi R^2 \int_0^\theta \sin\theta\,d\theta = 2\pi R^2(-\cos\theta)\Big|_0^\theta = 2\pi R^2(1-\cos\theta)$$

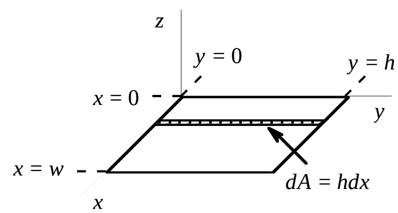
$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot 2\pi R^2(1-\cos\theta) = \boxed{\frac{Q}{2\epsilon_0}(1-\cos\theta)}$$ [independent of R!]

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 90°) = \boxed{\frac{Q}{2\epsilon_0}}$

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \frac{Q}{2\epsilon_0}(1-\cos 180°) = \boxed{\frac{Q}{\epsilon_0}}$ [Gauss's Law]

***24.52** In general, $\mathbf{E} = ay\,\mathbf{i} + bz\,\mathbf{j} + cx\,\mathbf{k}$

In the xy plane, $z = 0$ and $\mathbf{E} = ay\,\mathbf{i} + cx\,\mathbf{k}$



$$\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = \int (ay\,\mathbf{i} + cx\,\mathbf{k})\cdot\mathbf{k}\,dA$$

$$\Phi_E = ch\int_{x=0}^{w} x\,dx = ch\frac{x^2}{2}\Big|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

***24.53** (a) $q_{in} = +3Q - Q = \boxed{+2Q}$

(b) The charge distribution is spherically symmetric and $q_{in} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$.

(c) $E = \frac{k_e q_{in}}{r^2} = \boxed{\frac{2k_e Q}{r^2}}$ for $r \ge c$

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$ for $b < r < c$.

(e) $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A} = 0 \Rightarrow q_{in} = \epsilon_0\Phi_E = \boxed{0}$

(f) $q_{in} = \boxed{+3Q}$

(g) $E = \frac{k_e q_{in}}{r^2} = \boxed{\frac{3k_e Q}{r^2}}$ (radially outward) for $a \le r < b$

(h) $q_{\text{in}} = \rho V = \left(\dfrac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\dfrac{4}{3}\pi r^3\right) = \boxed{+3Q\dfrac{r^3}{a^3}}$

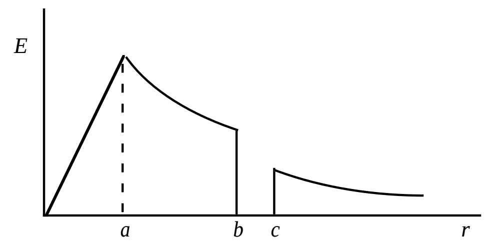
(i) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{k_e}{r^2}\left(+3Q\dfrac{r^3}{a^3}\right) = \boxed{3k_e Q\dfrac{r}{a^3}}$ (radially outward) for $0 \le r \le a$

(j) From part (d), $E = 0$ for $b < r < c$. Thus, for a spherical gaussian surface with $b < r < c$, $q_{\text{in}} = +3Q + q_{\text{inner}} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3Q}$

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -Q$, we have

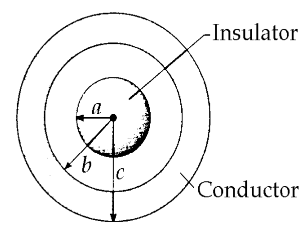
$q_{\text{outer}} = -Q - q_{\text{inner}} = -Q - (-3Q) = \boxed{+2Q}$

(l) This is shown in the figure to the right.



24.54 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

24.55 (a) $\oint \mathbf{E}\cdot d\mathbf{A} = E(4\pi r^2) = q_{in}/\epsilon_0$



For $r < a$, $q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$ so $\boxed{E = \dfrac{\rho r}{3\epsilon_0}}$

For $a < r < b$ and $c < r$, $q_{\text{in}} = Q$ so that $\boxed{E = \dfrac{Q}{4\pi r^2 \epsilon_0}}$

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$

Let q_2 = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require $q_1 + q_2 = 0$

and $\sigma_2 = \dfrac{q_1}{4\pi c^2} = \boxed{\dfrac{Q}{4\pi c^2}}$

24.56 $\oint \mathbf{E}\cdot d\mathbf{A} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) $\left(-3.60\times 10^3 \text{ N/C}\right)4\pi(0.100 \text{ m})^2 = \dfrac{Q}{8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2}$ $(a<r<b)$

$Q = -4.00\times 10^{-9} \text{ C} = \boxed{-4.00 \text{ nC}}$

(b) We take Q' to be the net charge on the hollow sphere. Outside c,

$\left(+2.00\times 10^2 \text{ N/C}\right)4\pi(0.500 \text{ m})^2 = \dfrac{Q+Q'}{8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2}$ $(r>c)$

$Q+Q' = +5.56\times 10^{-9} \text{ C}$, so $Q' = +9.56\times 10^{-9} \text{ C} = \boxed{+9.56 \text{ nC}}$

(c) For $b<r<c$: $E=0$ and $q_{\text{in}} = Q+Q_1 = 0$ where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00 \text{ nC}}$

Then, if Q_2 is the total charge on the outer surface of the hollow sphere, $Q_2 = Q' - Q_1 = 9.56 \text{ nC} - 4.00 \text{ nC} = \boxed{+5.56 \text{ nC}}$

24.57 The field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a Gaussian cylinder of radius r and length L. If $r<a$,

$\Phi_E = \dfrac{q_{in}}{\epsilon_0}$ and $E\left(2\pi rL\right) = \dfrac{\lambda L}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi r\epsilon_0}$ or $\boxed{\mathbf{E} = \dfrac{\lambda}{2\pi r\epsilon_0} \quad (r<a)}$

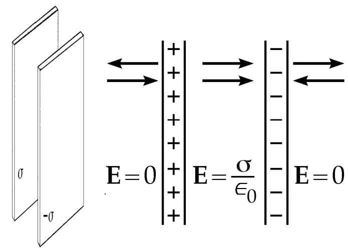
If $a<r<b$, $E\left(2\pi rL\right) = \dfrac{\lambda L + \rho\pi\left(r^2-a^2\right)L}{\epsilon_0}$

$\boxed{\mathbf{E} = \dfrac{\lambda + \rho\pi\left(r^2-a^2\right)}{2\pi r\epsilon_0} \quad (a<r<b)}$

If $r>b$, $E\left(2\pi rL\right) = \dfrac{\lambda L + \rho\pi\left(b^2-a^2\right)L}{\epsilon_0}$

$$\mathbf{E} = \frac{\lambda + \rho\pi\left(b^2 - a^2\right)}{2\pi r \epsilon_0} \qquad (r > b)$$

24.58 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 24.8:



$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

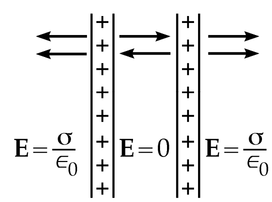
(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\boxed{E = 0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\boxed{E = \frac{\sigma}{\epsilon_0} \text{ toward the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\boxed{E = 0}$.

24.59 The magnitude of the field due to each sheet given by Equation 24.8 is



$$E = \frac{\sigma}{2\epsilon_0} \quad \text{directed perpendicular to the sheet.}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$E = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$E = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

Goal Solution

Repeat the calculations for Problem 58 when both sheets have **positive** uniform charge densities of value σ. **Note:** The new problem statement would be as follows: Two infinite, nonconducting sheets of charge are parallel to each other, as shown in Figure P24.58. Both sheets have positive uniform charge densities σ. Calculate the value of the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.

G: When both sheets have the same charge density, a positive test charge at a point midway between them will experience the same force in opposite directions from each sheet. Therefore, the electric field here will be zero. (We should ask: can we also conclude that the electron will experience equal and oppositely directed forces *everywhere* in the region between the plates?)

Outside the sheets the electric field will point away and should be twice the strength due to one sheet of charge, so $E = \sigma / \epsilon_0$ in these regions.

O: The principle of superposition can be applied to add the electric field vectors due to each sheet of charge.

A: For each sheet, the electric field at any point is $|\mathbf{E}| = \sigma/(2\epsilon_0)$ directed away from the sheet.

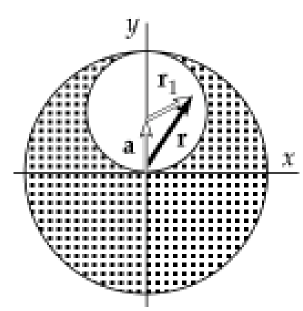
(a) At a point to the left of the two parallel sheets $\quad \mathbf{E} = E_1(-\mathbf{i}) + E_2(-\mathbf{i}) = 2E(-\mathbf{i}) = -\dfrac{\sigma}{\epsilon_0}\mathbf{i}$

(b) At a point between the two sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2(-\mathbf{i}) = 0$

(c) At a point to the right of the two parallel sheets $\quad \mathbf{E} = E_1\mathbf{i} + E_2\mathbf{i} = 2E\,\mathbf{i} = \dfrac{\sigma}{\epsilon_0}\mathbf{i}$

L: We essentially solved this problem in the Gather information step, so it is no surprise that these results are what we expected. A better check is to confirm that the results are complementary to the case where the plates are oppositely charged (Problem 58).

24.60 The resultant field within the cavity is the superposition of two fields, one $\mathbf{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\mathbf{E}_-$ due to a sphere of negative charge of radius a centered within the cavity.



$$\frac{4}{3}\frac{\pi r^3 \rho}{\epsilon_0} = 4\pi r^2 E_+ \quad \text{so} \quad \mathbf{E}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho \mathbf{r}}{3\epsilon_0}$$

$$-\frac{4}{3}\frac{\pi r_1^3 \rho}{\epsilon_0} = 4\pi r_1^2 E_- \quad \text{so} \quad \mathbf{E}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\mathbf{r}_1$$

Since $\mathbf{r} = \mathbf{a} + \mathbf{r}_1$, $\quad \mathbf{E}_- = \dfrac{-\rho(\mathbf{r} - \mathbf{a})}{3\epsilon_0}$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho \mathbf{r}}{3\epsilon_0} - \frac{\rho \mathbf{r}}{3\epsilon_0} + \frac{\rho \mathbf{a}}{3\epsilon_0} = \frac{\rho \mathbf{a}}{3\epsilon_0} = 0\,\mathbf{i} + \frac{\rho a}{3\epsilon_0}\mathbf{j}$$

Thus, $\boxed{E_x = 0}$ and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

24.61 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge $(Q = +e)$ with radius R.

$$\left(4\pi r^2\right)E = \frac{q_{in}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{\frac{4}{3}\pi r^3}{\epsilon_0} \quad \text{so} \quad E = \left(\frac{e}{4\pi\epsilon_0 R^3}\right)r \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\epsilon_0 R^3}\right)r = -\left(\frac{e^2}{4\pi\epsilon_0 R^3}\right)r = \boxed{-Kr}$$

(b) $K = \dfrac{e^2}{4\pi\epsilon_0 R^3} = \boxed{\dfrac{k_e e^2}{R^3}}$

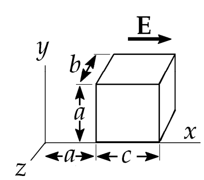
(c) $F_r = m_e a_r = -\left(\dfrac{k_e e^2}{R^3}\right)r$, so $a_r = -\left(\dfrac{k_e e^2}{m_e R^3}\right)r = -\omega^2 r$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}}$

(d) $f = 2.47 \times 10^{15}\text{ Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)R^3}}$

which yields $R^3 = 1.05\times10^{-30}\text{ m}^3$, or $R = 1.02\times10^{-10}\text{ m} = \boxed{102\text{ pm}}$

24.62 The electric field throughout the region is directed along x; therefore, **E** will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore,



$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3 + 2a^2\right)ab + \left(3 + 2(a+c)^2\right)ab = 2abc(2a+c)$$

Substituting the given values for a, b, and c, we find $\Phi_E = \boxed{0.269\text{ N}\cdot\text{m}^2/\text{C}}$

$Q = \epsilon_0\Phi_E = 2.38\times10^{-12}\text{ C} = \boxed{2.38\text{ pC}}$

24.63 $\oint \mathbf{E}\cdot d\mathbf{A} = E(4\pi r^2) = \dfrac{q_{in}}{\epsilon_0}$

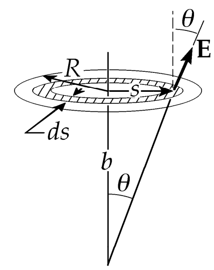
(a) For $r > R$, $q_{in} = \displaystyle\int_0^R Ar^2(4\pi r^2)dr = 4\pi\frac{AR^5}{5}$ and $E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$

(b) For $r < R$, $q_{in} = \displaystyle\int_0^r Ar^2(4\pi r^2)dr = \frac{4\pi Ar^5}{5}$ and $E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$

24.64 The total flux through a surface enclosing the charge Q is Q/ϵ_0. The flux through the disk is

$$\Phi_{\text{disk}} = \int \mathbf{E} \cdot d\mathbf{A}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{1}{4} Q/\epsilon_0$ to find how b and R are related. In the figure, take $d\mathbf{A}$ to be the area of an annular ring of radius s and width ds. The flux through $d\mathbf{A}$ is



$$\mathbf{E} \cdot d\mathbf{A} = E\,dA\cos\theta = E(2\pi\, s ds)\cos\theta$$

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \qquad \text{and} \qquad \cos\theta = \frac{b}{r} = \frac{b}{(s^2+b^2)^{1/2}}$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s\,ds}{(s^2+b^2)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-(s^2+b^2)^{1/2}\right]\Big|_0^R = \frac{Q}{2\epsilon_0}\left[1 - \frac{b}{(R^2+b^2)^{1/2}}\right]$$

The flux through the disk equals $Q/4\epsilon_0$ provided that $\dfrac{b}{(R^2+b^2)^{1/2}} = \dfrac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}\,b}$.

24.65

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{a}{r} 4\pi r^2 dr$$

$$E 4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r\,dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$$\boxed{E = \frac{a}{2\epsilon_0}} = \text{constant magnitude}$$

(The direction is radially outward from center for positive a; radially inward for negative a.)

24.66 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \mathbf{E}\cdot d\mathbf{A} = \dfrac{1}{\epsilon_0}\int \rho\, dV$.

We use a gaussian surface which is a cylinder of radius r, length ℓ, and is coaxial with the charge distribution.

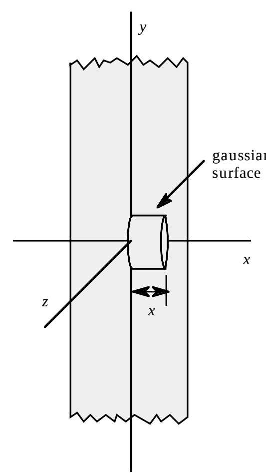
(a) When $r < R$, this becomes $E(2\pi r \ell) = \dfrac{\rho_0}{\epsilon_0}\displaystyle\int_0^r \left(a - \frac{r}{b}\right) dV$. The element of volume is a cylindrical shell of radius r, length ℓ, and thickness dr so that $dV = 2\pi r \ell\, dr$.

$$E(2\pi r\ell) = \left(\frac{2\pi r^2 \ell \rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right)$$ so inside the cylider, $$\boxed{E = \frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$$

(b) When $r > R$, Gauss's law becomes

$$E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^R \left(a - \frac{r}{b}\right)(2\pi r \ell\, dr)$$ or outside the cylinder, $$\boxed{E = \frac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \frac{2R}{3b}\right)}$$

24.67 (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with one end in the yz plane and the other end containing the point x:



Use Gauss's law: $\oint \mathbf{E}\cdot d\mathbf{A} = \dfrac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\mathbf{A}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x :

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance x from the mid-line of the slab, $\boxed{E = \dfrac{\rho x}{\epsilon_0}}$

(b) $$a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \epsilon_0}\right)x$$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\dfrac{\rho e}{m_e \epsilon_0}}$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{\rho e}{m_e \epsilon_0}}}$

24.68 Consider the gaussian surface described in the solution to problem 67.

(a) For $x > \frac{d}{2}$, $dq = \rho\, dV = \rho A\, dx = C\,Ax^2\, dx$

$$\int \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int dq$$

$$EA = \frac{CA}{\epsilon_0}\int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$E = \frac{Cd^3}{24\epsilon_0} \quad \text{or} \quad \boxed{\mathbf{E} = \frac{Cd^3}{24\epsilon_0}\mathbf{i} \ \text{ for } x > \frac{d}{2}; \qquad \mathbf{E} = -\frac{Cd^3}{24\epsilon_0}\mathbf{i} \ \text{ for } x < -\frac{d}{2}}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$ $\int \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int dq = \frac{CA}{\epsilon_0}\int_0^x x^2\, dx = \frac{CAx^3}{3\epsilon_0}$

$$\boxed{\mathbf{E} = \frac{Cx^3}{3\epsilon_0}\mathbf{i} \ \text{ for } x > 0; \qquad \mathbf{E} = -\frac{Cx^3}{3\epsilon_0}\mathbf{i} \ \text{ for } x < 0}$$

24.69 (a) A point mass m creates a gravitational acceleration $\mathbf{g} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$ at a distance r.

The flux of this field through a sphere is $\oint \mathbf{g}\cdot d\mathbf{A} = -\frac{Gm}{r^2}\left(4\pi r^2\right) = -4\pi Gm$

Since the r has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \mathbf{g}\cdot d\mathbf{A} = -4\pi G m_{\text{in}}}$$

(b) Take a spherical gaussian surface of radius r. The field is inward so

$$\oint \mathbf{g}\cdot d\mathbf{A} = g\,4\pi r^2 \cos 180° = -g\,4\pi r^2$$

and $-4\pi G m_{\text{in}} = -4G\frac{4}{3}\pi r^3\rho$

Then, $-g\,4\pi r^2 = -4\pi G\frac{4}{3}\pi r^3\rho$ and $g = \frac{4}{3}\pi r\rho G$

Or, since $\rho = M_E / \frac{4}{3}\pi R_E^3$, $g = \frac{M_E G r}{R_E^3}$ or $\boxed{\mathbf{g} = \frac{M_E G r}{R_E^3} \ \text{inward}}$

Chapter 25 Solutions

25.1 $\Delta V = -14.0$ V and

$$Q = -N_A e = -(6.02\times 10^{23})(1.60\times 10^{-19}\text{ C}) = -9.63\times 10^{4}\text{ C}$$

$$\Delta V = \frac{W}{Q}\ ,\quad \text{so}\quad W = Q(\Delta V) = (-9.63\times 10^{4}\text{ C})(-14.0\text{ J/C}) = \boxed{1.35\text{ MJ}}$$

25.2 $\Delta K = q|\Delta V|$ $\qquad 7.37\times 10^{-17} = q(115)$

$$\boxed{q = 6.41\times 10^{-19}\text{ C}}$$

25.3 $W = \Delta K = q|\Delta V|$

$$\tfrac{1}{2}mv^2 = e(120\text{ V}) = 1.92\times 10^{-17}\text{ J}$$

Thus, $v = \sqrt{\dfrac{3.84\times 10^{-17}\text{ J}}{m}}$

(a) For a proton, this becomes $\quad v = \sqrt{\dfrac{3.84\times 10^{-17}\text{ J}}{1.67\times 10^{-27}\text{ kg}}} = 1.52\times 10^{5}\text{ m/s} = \boxed{152\text{ km/s}}$

(b) If an electron, $\quad v = \sqrt{\dfrac{3.84\times 10^{-17}\text{ J}}{9.11\times 10^{-31}\text{ kg}}} = 6.49\times 10^{6}\text{ m/s} = \boxed{6.49\text{ Mm/s}}$

Goal Solution

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V.
(b) Calculate the speed of an electron that is accelerated through the same potential difference.

G: Since 120 V is only a modest potential difference, we might expect that the final speed of the particles will be substantially less than the speed of light. We should also expect the speed of the electron to be significantly greater than the proton because, with $m_e << m_p$, an equal force on both particles will result in a much greater acceleration for the electron.

O: Conservation of energy can be applied to this problem to find the final speed from the kinetic energy of the particles. (Review this work-energy theory of motion from Chapter 8 if necessary.)

A: (a) Energy is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$

$$0 + qV + 0 = \tfrac{1}{2} m v_p^{\,2} + 0$$

$$(1.60 \times 10^{-19}\ \text{C})(120\ \text{V})\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \tfrac{1}{2}(1.67 \times 10^{-27}\ \text{kg}) v_p^{\,2}$$

$$v_p = 1.52 \times 10^5\ \text{m / s}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V:

$$K_i + U_i + \Delta E_{nc} = K_f + U_f$$

$$0 + 0 + 0 = \tfrac{1}{2} m v_e^{\,2} + qV$$

$$0 = \tfrac{1}{2}(9.11 \times 10^{-31}\ \text{kg}) v_e^{\,2} + (-1.60 \times 10^{-19}\ \text{C})(120\ \text{J / C})$$

$$v_e = 6.49 \times 10^6\ \text{m / s}$$

L: Both of these speeds are significantly less than the speed of light as expected, which also means that we were justified in not using the relativistic kinetic energy formula. (For precision to three significant digits, the relativistic formula is only needed if *v* is greater than about 0.1 *c*.)

25.4 For speeds larger than one-tenth the speed of light, $\frac{1}{2}\, mv^2$ gives noticeably wrong answers for kinetic energy, so we use

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2 / c^2}} - 1\right) = \left(9.11 \times 10^{-31}\ \text{kg}\right)\left(3.00 \times 10^8\ \text{m / s}\right)^2\left(\frac{1}{\sqrt{1 - 0.400^2}} - 1\right) = 7.47 \times 10^{-15}\ \text{J}$$

Energy is conserved during acceleration: $K_i + U_i + \Delta E = K_f + U_f$

$0 + qV_i + 0 = 7.47 \times 10^{-15}\ \text{J} + qV_f$

The change in potential is $V_f - V_i$: $\quad V_f - V_i = \dfrac{-7.47 \times 10^{-15}\ \text{J}}{q} = \dfrac{-7.47 \times 10^{-15}\ \text{J}}{-1.60 \times 10^{-19}\ \text{C}} = \boxed{+\,46.7\ \text{kV}}$

The positive answer means that the electron speeds up in moving toward higher potential.

25.5 $W = \Delta K = -\,q\Delta V$

$$0 - \tfrac{1}{2}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(4.20 \times 10^5\ \text{m / s}\right)^2 = -\left(-\,1.60 \times 10^{-19}\ \text{C}\right)\Delta V$$

From which, $\Delta V = \boxed{-0.502\ \text{V}}$

***25.6** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$\Delta U = -$ (work done)

$\Delta U = -(\text{work from origin to } (20.0\text{ cm}, 0)) - (\text{work from } (20.0\text{ cm}, 0) \text{ to } (20.0\text{ cm}, 50.0\text{ cm}))$

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)(\Delta x) = -(12.0 \times 10^{-6}\text{ C})(250\text{ V/m})(0.200\text{ m}) = \boxed{-6.00 \times 10^{-4}\text{ J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00 \times 10^{-4}\text{ J}}{12.0 \times 10^{-6}\text{ C}} = -50.0\text{ J/C} = \boxed{-50.0\text{ V}}$$

***25.7** $$E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^{3}\text{ J/C}}{1.50 \times 10^{-2}\text{ m}} = 1.67 \times 10^{6}\text{ N/C} = \boxed{1.67\text{ MN/C}}$$

***25.8** (a) $$|\Delta V| = Ed = (5.90 \times 10^{3}\text{ V/m})(0.0100\text{ m}) = \boxed{59.0\text{ V}}$$

(b) $\frac{1}{2} mv_f^2 = |q(\Delta V)|$; $\quad \frac{1}{2}(9.11 \times 10^{-31})\ v_f^2 = (1.60 \times 10^{-19})(59.0)$

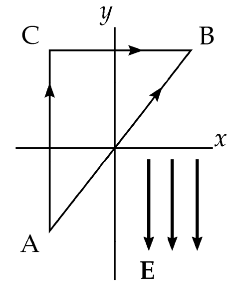
$$\boxed{v_f = 4.55 \times 10^{6}\text{ m/s}}$$

25.9 $$\Delta U = -\tfrac{1}{2} m\left(v_f^2 - v_i^2\right) = -\tfrac{1}{2}\left(9.11 \times 10^{-31}\text{ kg}\right)\left[\left(1.40 \times 10^{5}\text{ m/s}\right)^2 - \left(3.70 \times 10^{6}\text{ m/s}\right)^2\right] = 6.23 \times 10^{-18}\text{ J}$$

$\Delta U = q\Delta V$: $\quad +6.23 \times 10^{-18} = (-1.60 \times 10^{-19})\Delta V$

$\boxed{\Delta V = -38.9\text{ V}}$ The origin is at higher potential.

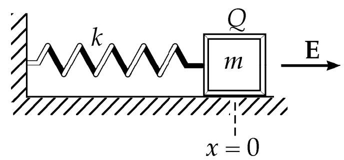
***25.10** $V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^C \mathbf{E}\cdot d\mathbf{s} - \int_C^B \mathbf{E}\cdot d\mathbf{s}$



$$V_B - V_A = (-E\cos 180°)\int_{-0.300}^{0.500} dy - (E\cos 90.0°)\int_{-0.200}^{0.400} dx$$

$$V_B - V_A = (325)(0.800) = \boxed{+260\text{ V}}$$

25.11 (a) Arbitrarily choose $V = 0$ at $x = 0$. Then at other points, $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

so the block comes to rest when the spring is stretched by an amount

$$x_{\max} = \frac{2QE}{k} = \frac{2\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ V/m}\right)}{100\text{ N/m}} = \boxed{0.500\text{ m}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. Thus, the equilibrium position is at

$$x = \frac{QE}{k} = \frac{\left(50.0\times10^{-6}\text{ C}\right)\left(5.00\times10^{5}\text{ N/C}\right)}{100\text{ N/m}} = \boxed{0.250\text{ m}}$$

(c) The equation of motion for the block is $\Sigma F_x = -kx + QE = m\frac{d^2x}{dt^2}$. Let $x' = x - \frac{QE}{k}$, or $x = x' + \frac{QE}{k}$ so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2}, \text{ or } \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2 x'\right)$, with $\omega = \sqrt{k/m}$. The period of the motion is then

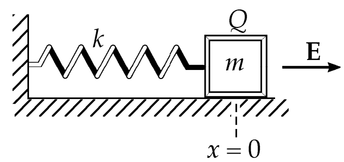
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.00\text{ kg}}{100\text{ N/m}}} = \boxed{1.26\text{ s}}$$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \frac{2(QE - \mu_k mg)}{k} = \frac{2\left[\left(50.0\times10^{-6}\ \text{C}\right)\left(5.00\times10^{5}\ \text{N/C}\right) - 0.200\left(4.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\right]}{100\ \text{N/m}} = \boxed{0.343\ \text{m}}$$

25.12 (a) Arbitrarily choose $V = 0$ at 0. Then at other points $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion,



$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$

$$0 + 0 + 0 = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max} \qquad \text{so} \qquad x_{\max} = \boxed{\frac{2QE}{k}}$$

(b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE$. So the equilibrium position is at $x = \boxed{\dfrac{QE}{k}}$

(c) The block's equation of motion is $\Sigma F_x = -kx + QE = m\dfrac{d^2x}{dt^2}$. Let $x' = x - \dfrac{QE}{k}$, or $x = x' + \dfrac{QE}{k}$, so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2\left(x' + QE/k\right)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $\left(a_{x'} = -\omega^2 x'\right)$, with $\omega = \sqrt{k/m}$

The period of the motion is then $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{m}{k}}}$

(d) $(K + U_s + U_e)_i + \Delta E = (K + U_s + U_e)_f$

$$0 + 0 + 0 - \mu_k mgx_{\max} = 0 + \tfrac{1}{2}kx_{\max}^2 - QEx_{\max}$$

$$x_{\max} = \boxed{\frac{2(QE - \mu_k mg)}{k}}$$

25.13 For the entire motion, $y - y_i = v_{yi}t + \tfrac{1}{2}a_y t^2$

$$0 - 0 = v_i t + \tfrac{1}{2}a_y t^2 \qquad \text{so} \qquad a_y = -\frac{2v_i}{t}$$

$\sum F_y = ma_y$: $\quad -mg - qE = -\dfrac{2mv_i}{t}$

$$E = \frac{m}{q}\left(\frac{2v_i}{t} - g\right) \qquad \text{and} \qquad \mathbf{E} = -\frac{m}{q}\left(\frac{2v_i}{t} - g\right)\mathbf{j}$$

For the upward flight: $v_{yf}^2 = v_{yi}^2 + 2a_y(y - y_i)$

$$0 = v_i^2 + 2\left(-\frac{2v_i}{t}\right)(y_{\max} - 0) \qquad \text{and} \qquad y_{\max} = \tfrac{1}{4}v_i t$$

$$\Delta V = \int_0^{y_{\max}} \mathbf{E} \cdot d\mathbf{y} = +\frac{m}{q}\left(\frac{2v_i}{t} - g\right)y\Big|_0^{y_{\max}} = \frac{m}{q}\left(\frac{2v_i}{t} - g\right)\left(\tfrac{1}{4}v_i t\right)$$

$$\Delta V = \frac{2.00 \text{ kg}}{5.00 \times 10^{-6} \text{ C}}\left(\frac{2(20.1 \text{ m/s})}{4.10 \text{ s}} - 9.80 \text{ m/s}^2\right)\left[\tfrac{1}{4}(20.1 \text{ m/s})(4.10 \text{ s})\right] = \boxed{40.2 \text{ kV}}$$

25.14 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$

(a) $(K+U)_i = (K+U)_f$

$$0 + 0 = \tfrac{1}{2}\mu L v^2 - \lambda LEd$$

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2(40.0\times10^{-6}\text{ C / m})(100\text{ N / C})(2.00\text{ m})}{(0.100\text{ kg / m})}} = \boxed{0.400\text{ m/s}}$$

(b) The same.

25.15 Arbitrarily take $V = 0$ at point P. Then (from Equation 25.8) the potential at the original position of the charge is $-\mathbf{E}\cdot\mathbf{s} = -EL\cos\theta$. At the final point a, $V = -EL$. Suppose the table is frictionless: $(K+U)_i = (K+U)_f$

$$0 - qEL\cos\theta = \tfrac{1}{2}mv^2 - qEL$$

$$v = = \sqrt{\frac{2qEL(1-\cos\theta)}{\text{m}}} = \sqrt{\frac{2(2.00\times10^{-6}\text{ C})(300\text{ N / C})(1.50\text{ m})(1-\cos 60.0°)}{0.0100\text{ kg}}} = \boxed{0.300\text{ m/s}}$$

***25.16** (a) The potential at 1.00 cm is

$$V_1 = k_e\frac{q}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{1.00\times10^{-2}\text{ m}} = \boxed{1.44\times10^{-7}\text{ V}}$$

(b) The potential at 2.00 cm is

$$V_2 = k_e\frac{q}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{2.00\times10^{-2}\text{ m}} = 0.719\times10^{-7}\text{ V}$$

Thus, the difference in potential between the two points is

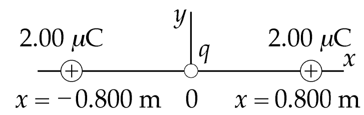
$$\Delta V = V_2 - V_1 = \boxed{-7.19\times10^{-8}\text{ V}}$$

(c) The approach is the same as above except the charge is -1.60×10^{-19} C. This changes the sign of all the answers, with the magnitudes remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44\times10^{-7}\text{ V}}$

The potential at 2.00 cm is -0.719×10^{-7} V, so $\Delta V = V_2 - V_1 = \boxed{7.19\times10^{-8}\text{ V}}$.

25.17 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$



(b) Since $F = qE = 0$, $\boxed{E = 0}$

(c) $V = 2k_e \dfrac{q}{r} = 2\left(8.99 \times 10^9 \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\dfrac{2.00 \times 10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$

$V = 4.50 \times 10^4\ \text{V} = \boxed{45.0\ \text{kV}}$

25.18 (a) $E_x = \dfrac{k_e q_1}{x^2} + \dfrac{k_e q_2}{(x - 2.00)^2} = 0$ becomes $E_x = k_e\left(\dfrac{+q}{x^2} + \dfrac{-2q}{(x - 2.00)^2}\right) = 0$

Dividing by k_e, $2qx^2 = q(x - 2.00)^2$

$x^2 + 4.00x - 4.00 = 0$

Therefore $E = 0$ when $x = \dfrac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = \boxed{-4.83\ \text{m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \dfrac{k_e q_1}{x} + \dfrac{k_e q_2}{(2.00 - x)} = 0$ or $V = k_e\left(\dfrac{+q}{x} - \dfrac{2q}{(2.00 - x)}\right) = 0$

Again solving for x, $2qx = q(2.00 - x)$

For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667\ \text{m}}$

and $\dfrac{q}{|x|} = \dfrac{-2q}{|2 - x|}$

For $x < 0$ $x = \boxed{-2.00\ \text{m}}$

25.19 (a) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{0.0529 \times 10^{-9}} = -4.35 \times 10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$

(b) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-(8.99 \times 10^9)(1.60 \times 10^{-19})^2}{2^2(0.0529 \times 10^{-9})} = \boxed{-6.80\ \text{eV}}$

(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{-k_e e^2}{\infty} = \boxed{0}$

Goal Solution

The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits around the proton. The radius of each Bohr orbit is $r = n^2(0.0529\text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron is in the (a) first allowed orbit, $n = 1$; (b) second allowed orbit, $n = 2$; and (c) when the electron has escaped from the atom ($r = \infty$). Express your answers in electron volts.

G: We may remember from chemistry that the lowest energy level for hydrogen is $E_1 = -13.6\text{ eV}$, and higher energy levels can be found from $E_n = E_1 / n^2$, so that $E_2 = -3.40\text{ eV}$ and $E_\infty = 0\text{ eV}$. (see section 42.2) Since these are the total energies (potential plus kinetic), the electric potential energy alone should be lower (more negative) because the kinetic energy of the electron must be positive.

O: The electric potential energy is given by $U = k_e \dfrac{q_1 q_2}{r}$

A: (a) For the first allowed Bohr orbit,

$$U = \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(-1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{(0.0529\times10^{-9}\text{ m})} = -4.35\times10^{-18}\text{ J} = \frac{-4.35\times10^{-18}\text{ J}}{1.60\times10^{-19}\text{ J / eV}} = -27.2\text{ eV}$$

(b) For the second allowed orbit,

$$U = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{2^2(0.0529\times10^{-9}\text{ m})} = -1.088\times10^{-18}\text{ J} = -6.80\text{ eV}$$

(c) When the electron is at $r = \infty$,

$$U = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(-1.60\times10^{-19}\text{ C}\right)\left(1.60\times10^{-19}\text{ C}\right)}{\infty} = 0\text{ J}$$

L: The potential energies appear to be twice the magnitude of the total energy values, so apparently the kinetic energy of the electron has the same absolute magnitude as the total energy.

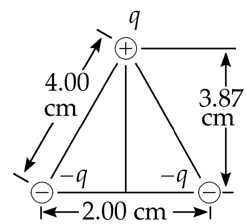
***25.20** (a) $U = \dfrac{qQ}{4\pi e_0 r} = \dfrac{(5.00\times10^{-9}\text{ C})(-3.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m/C})}{(0.350\text{ m})} = \boxed{-3.86\times10^{-7}\text{ J}}$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $V = \dfrac{Q_1}{4\pi e_0 r_1} + \dfrac{Q_2}{4\pi e_0 r_2} = \dfrac{(5.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}} + \dfrac{(-3.00\times10^{-9}\text{ C})(8.99\times10^9\text{ V}\cdot\text{m / C})}{0.175\text{ m}}$

$V = \boxed{103\text{ V}}$

25.21 $V = \sum_i k \frac{q_i}{r_i}$



$$V = (8.99 \times 10^9)(7.00 \times 10^{-6})\left[\frac{-1}{0.0100} - \frac{1}{0.0100} + \frac{1}{0.0387}\right]$$

$$V = \boxed{-1.10 \times 10^7 \text{ C} = -11.0 \text{ MV}}$$

***25.22**

$$U_e = q_4V_1 + q_4V_2 + q_4V_3 = q_4\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)$$

$$U_e = \left(10.0\times10^{-6}\text{ C}\right)^2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\text{ m}}+\frac{1}{0.150\text{ m}}+\frac{1}{\sqrt{(0.600\text{ m})^2+(0.150\text{ m})^2}}\right)$$

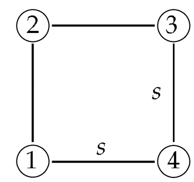
$$U_e = \boxed{8.95\text{ J}}$$

25.23

$$U = U_1 + U_2 + U_3 + U_4$$

$$U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})$$

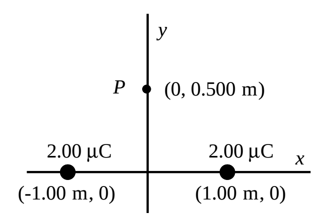
$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}}+1\right) + \frac{k_eQ^2}{s}\left(1+\frac{1}{\sqrt{2}}+1\right)$$



$$U = \frac{k_eQ^2}{s}\left(4+\frac{2}{\sqrt{2}}\right) = \boxed{5.41\frac{k_eQ^2}{s}}$$

An alternate way to get the term $\left(4+2/\sqrt{2}\right)$ is to recognize that there are 4 side pairs and 2 face diagonal pairs.

***25.24** (a) $$V = \frac{k_eq_1}{r_1}+\frac{k_eq_2}{r_2} = 2\left(\frac{k_eq}{r}\right) = 2\left(\frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\text{ C}\right)}{\sqrt{(1.00\text{ m})^2+(0.500\text{ m})^2}}\right)$$



$$V = 3.22\times10^4\text{ V} = \boxed{32.2\text{ kV}}$$

(b) $$U = qV = \left(-3.00\times10^{-6}\text{ C}\right)\left(3.22\times10^4\ \frac{\text{J}}{\text{C}}\right) = \boxed{-9.65\times10^{-2}\text{ J}}$$

***25.25** Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_eq}{R}}$$

***25.26** (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$ located at a finite distance from the charges, where this total potential is zero.

(b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{2k_e q}{a}}$

25.27 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2 (-\mathbf{i})$ or $v_2 = \frac{m_1 v_1}{m_2}$

By conservation of energy, $0 + \frac{k_e (-q_1) q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e (-q_1) q_2}{(r_1 + r_2)}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2 m_2 k_e q_1 q_2}{m_1 (m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d} \right)}$$

$$v_1 = \sqrt{\frac{2(0.700 \text{ kg})(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2 \times 10^{-6} \text{ C})(3 \times 10^{-6} \text{ C})}{(0.100 \text{ kg})(0.800 \text{ kg})} \left(\frac{1}{8 \times 10^{-3} \text{ m}} - \frac{1}{1.00 \text{ m}} \right)} = \boxed{10.8 \text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.100 \text{ kg})(10.8 \text{ m/s})}{0.700 \text{ kg}} = \boxed{1.55 \text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.28 (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2 (-\mathbf{i})$ or $v_2 = m_1 v_1 / m_2$

By conservation of energy, $0 + \frac{k_e (-q_1) q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e (-q_1) q_2}{(r_1 + r_2)}$

and $\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \frac{m_1^2 v_1^2}{m_2}$

$$v_1 = \boxed{\sqrt{\frac{2 m_2 k_e q_1 q_2}{m_1 (m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d} \right)}} \qquad v_2 = \left(\frac{m_1}{m_2} \right) v_1 = \boxed{\sqrt{\frac{2 m_1 k_e q_1 q_2}{m_2 (m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d} \right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

25.29 $V = \frac{k_e Q}{r}$ so $r = \frac{k_e Q}{V} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-9}\text{ C})}{V} = \frac{72.0 \text{ V}\cdot\text{m}}{V}$

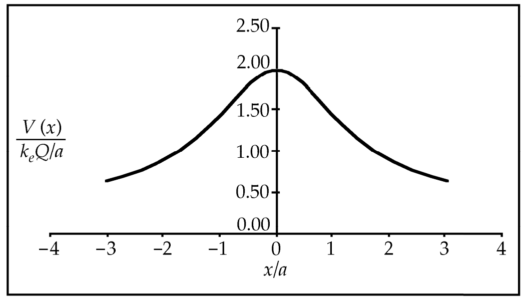
For $V = 100$ V, 50.0 V, and 25.0 V, $\boxed{r = 0.720 \text{ m}, 1.44 \text{ m, and } 2.88 \text{ m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

25.30 (a) $V(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{\sqrt{x^2+a^2}} + \frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

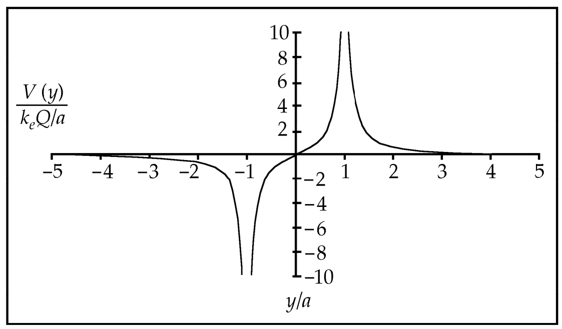
$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$



(b) $V(y) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{|y-a|} + \frac{k_e(-Q)}{|y+a|}$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$



25.31 Using conservation of energy, we have $K_f + U_f = K_i + U_i$.

But $U_i = \frac{k_e q_\alpha q_{\text{gold}}}{r_i}$, and $r_i \approx \infty$. Thus, $U_i = 0$.

Also $K_f = 0$ ($v_f = 0$ at turning point), so $U_f = K_i$, or $\frac{k_e q_\alpha q_{\text{gold}}}{r_{\min}} = \frac{1}{2} m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2} = \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\text{ C})^2}{(6.64\times10^{-27}\text{ kg})(2.00\times10^7 \text{ m/s})^2} = 2.74\times10^{-14}\text{ m} = \boxed{27.4 \text{ fm}}$$

25.32 Using conservation of energy

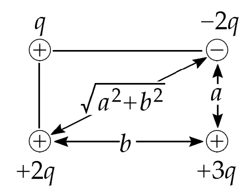
we have: $$\frac{k_e eQ}{r_1} = \frac{k_e eQ}{r_2} + \tfrac{1}{2}mv^2$$

which gives: $$v = \sqrt{\frac{2k_e eQ}{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

or $$v = \sqrt{\frac{(2)(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(10^{-9}\ \text{C})}{9.11\times10^{-31}\ \text{kg}}\left(\frac{1}{0.0300\ \text{m}} - \frac{1}{0.0200\ \text{m}}\right)}$$

Thus, $$v = \boxed{7.26\times10^6\ \text{m/s}}$$

25.33 $U = \sum \frac{k_e q_i q_j}{r_{ij}}$, summed over all pairs of (i, j) where $i \neq j$



$$U = k_e\left[\frac{q(-2q)}{b} + \frac{(-2q)(3q)}{a} + \frac{(2q)(3q)}{b} + \frac{q(2q)}{a} + \frac{q(3q)}{\sqrt{a^2+b^2}} + \frac{2q(-2q)}{\sqrt{a^2+b^2}}\right]$$

$$U = k_e q^2\left[\frac{-2}{0.400} - \frac{6}{0.200} + \frac{6}{0.400} + \frac{2}{0.200} + \frac{3}{0.447} - \frac{4}{0.447}\right]$$

$$U = \left(8.99\times10^9\right)\left(6.00\times10^{-6}\right)^2\left[\frac{4}{0.400} - \frac{4}{0.200} - \frac{1}{0.447}\right] = \boxed{-3.96\ \text{J}}$$

25.34 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum (K+U)_i = \sum (K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\tfrac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

$$\boxed{v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

25.35 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, $2 \times 6 = 12$ face diagonal pairs separated by $\sqrt{2}\, s$, and 4 interior diagonal pairs separated $\sqrt{3}\, s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\,\frac{k_e q^2}{s}}$$

25.36 $V = a + bx = 10.0\text{ V} + (-7.00\text{ V/m})x$

(a) At $x = 0$, $V = \boxed{10.0\text{ V}}$

At $x = 3.00\text{ m}$, $V = \boxed{-11.0\text{ V}}$

At $x = 6.00\text{ m}$, $V = \boxed{-32.0\text{ V}}$

(b) $E = -\dfrac{dV}{dx} = -b = -(-7.00\text{ V/m}) = \boxed{7.00\text{ N/C in } +x \text{ direction}}$

25.37 $V = 5x - 3x^2y + 2yz^2$ Evaluate E at $(1, 0 - 2)$

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5 + 6xy} = -5 + 6(1)(0) = -5$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2 - 2z^2} = 3(1)^2 - 2(-2)^2 = -5$$

$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$

$$E = \sqrt{E_x{}^2 + E_y{}^2 + E_z{}^2} = \sqrt{(-5)^2 + (-5)^2 + 0^2} = \boxed{7.07\text{ N/C}}$$

25.38 (a) For $r < R$ $V = \dfrac{k_e Q}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$ $V = \dfrac{k_e Q}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}$$

25.39 $$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

25.40 Inside the sphere, $E_x = E_y = E_z = 0$.

Outside, $$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$$

So $$E_x = -\left[0 + 0 + E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2x)\right] = \boxed{3E_0 a^3 xz(x^2 + y^2 + z^2)^{-5/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z(x^2 + y^2 + z^2)^{-3/2}\right)$$

$$E_y = -E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2} 2y = \boxed{3E_0 a^3 yz(x^2 + y^2 + z^2)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = E_0 - E_0 a^3 z(-3/2)(x^2 + y^2 + z^2)^{-5/2}(2z) - E_0 a^3 (x^2 + y^2 + z^2)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3 (2z^2 - x^2 - y^2)(x^2 + y^2 + z^2)^{-5/2}}$$

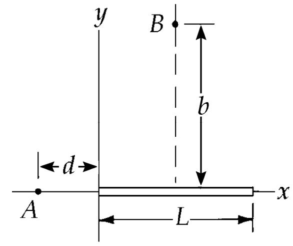
***25.41** $$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$$

***25.42** $$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

All bits of charge are at the same distance from O, so

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(-7.50 \times 10^{-6}\ \text{C}\right)}{(0.140\ \text{m}/\pi)} = \boxed{-1.51\ \text{MV}}$$

25.43 (a) $[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}} \cdot \left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$



(b) $$V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda\, dx}{r} = k_e \alpha \int_0^L \frac{x\, dx}{(d+x)} = \boxed{k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]}$$

25.44 $$V = \int \frac{k_e\, dq}{r} = k_e \int \frac{\alpha x\, dx}{\sqrt{b^2 + (L/2 - x)^2}}$$

Let $z = \frac{L}{2} - x$. Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = k_e \alpha \int \frac{(L/2 - z)(-dz)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e \alpha \int \frac{z\, dz}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \ln(z + \sqrt{z^2 + b^2}) + k_e \alpha \sqrt{z^2 + b^2}$$

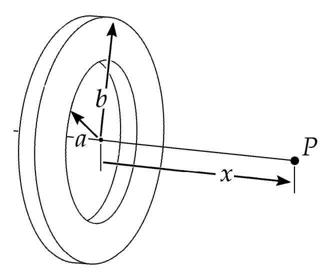
$$V = -\frac{k_e \alpha L}{2} \ln\left[(L/2 - x) + \sqrt{(L/2 - x)^2 + b^2}\,\right]\Big|_0^L + k_e \alpha \sqrt{(L/2 - x)^2 + b^2}\,\Big|_0^L$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2 + b^2}}\right] + k_e \alpha \left[\sqrt{(L/2 - L)^2 + b^2} - \sqrt{(L/2)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]}$$

25.45 $$dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}} \quad \text{where } dq = \sigma\, dA = \sigma 2\pi r\, dr$$

$$V = 2\pi \sigma k_e \int_a^b \frac{r\, dr}{\sqrt{r^2 + x^2}} = \boxed{2\pi k_e \sigma \left[\sqrt{x^2 + b^2} - \sqrt{x^2 + a^2}\right]}$$



25.46 $$V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda\, dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda\, ds}{R} + k_e \int_R^{3R} \frac{\lambda\, dx}{x}$$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x \Big|_R^{3R}$$

$$V = k_e \lambda \ln \frac{3R}{R} + k_e \lambda \pi + k_e \lambda \ln 3 = \boxed{k_e \lambda (\pi + 2 \ln 3)}$$

25.47 Substituting given values into $V = \frac{k_e q}{r}$, $7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\, q}{(0.300 \text{ m})}$

Substituting $q = 2.50 \times 10^{-7}$ C, $N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$

25.48 $q_1 + q_2 = 20.0\ \mu\text{C}$ so $q_1 = 20.0\ \mu\text{C} - q_2$

$\frac{q_1}{q_2} = \frac{r_1}{r_2}$ so $\frac{20.0\ \mu\text{C} - q_2}{q_2} = \frac{4.00 \text{ cm}}{6.00 \text{ cm}}$

Therefore $6.00(20.0\ \mu\text{C} - q_2) = 4.00 q_2$;

Solving, $q_2 = 12.0\ \mu\text{C}$ and $q_1 = 20.0\ \mu\text{C} - 12.0\ \mu\text{C} = 8.00\ \mu\text{C}$

(a) $$E_1 = \frac{k_e q_1}{r_1^2} = \frac{\left(8.99 \times 10^9\right)\left(8.00 \times 10^{-6}\right)}{(0.0400)^2} = 4.50 \times 10^7 \text{ V / m} = \boxed{45.0 \text{ MV/m}}$$

$$E_2 = \frac{k_e q_2}{r_2^2} = \frac{\left(8.99 \times 10^9\right)\left(12.0 \times 10^{-6}\right)}{(0.0600)^2} = 3.00 \times 10^7 \text{ V / m} = \boxed{30.0 \text{ MV/m}}$$

(b) $$V_1 = V_2 = \frac{k_e q_2}{r_2} = \boxed{1.80 \text{ MV}}$$

25.49 (a) $E = \boxed{0}$; $V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.140} = \boxed{1.67 \text{ MV}}$

(b) $$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)^2} = \boxed{5.84 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.140)^2} = \boxed{11.9 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

25.50 No charge stays on the inner sphere in equilibrium. If there were any, it would create an electric field in the wire to push more charge to the outer sphere. Charge Q is on the outer sphere. Therefore, $\boxed{\text{zero charge is on the inner sphere}}$ and $\boxed{10.0\ \mu\text{C is on the outer sphere}}$.

25.51 (a) $E_{\text{max}} = 3.00 \times 10^6 \text{ V / m} = \dfrac{k_e Q}{r^2} = \dfrac{k_e Q}{r}\dfrac{1}{r} = V_{\text{max}}\dfrac{1}{r}$

$$V_{\text{max}} = E_{\text{max}} r = 3.00 \times 10^6 (0.150) = \boxed{450 \text{ kV}}$$

(b) $\dfrac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}}$ $\left\{\text{or } \dfrac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\}$

$$Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{3.00 \times 10^6 (0.150)^2}{8.99 \times 10^9} = \boxed{7.51\ \mu\text{C}}$$

Goal Solution

Consider a Van de Graaff generator with a 30.0-cm-diameter dome operating in dry air. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

G: Van de Graaff generators produce voltages that can make your hair stand on end, somewhere on the order of about 100 kV (see the Puzzler at beginning of Chapter 25). With these high voltages, the maximum charge on the dome is probably more than typical point charge values of about 1 μC.

The maximum potential and charge will be limited by the electric field strength at which the air surrounding the dome will ionize. This critical value is determined by the **dielectric strength** of air which, from page 789 or from Table 26.1, is $E_{\text{critical}} = 3 \times 10^6 \text{ V / m}$. An electric field stronger than this will cause the air to act like a conductor instead of an insulator. This process is called dielectric breakdown and may be seen as a spark.

O: From the maximum allowed electric field, we can find the charge and potential that would create this situation. Since we are only given the diameter of the dome, we will assume that the conductor is spherical, which allows us to use the electric field and potential equations for a spherical conductor. With these equations, it will be easier to do part (b) first and use the result for part (a).

A: (b) For a spherical conductor with total charge Q, $|\mathbf{E}| = \dfrac{k_e Q}{r^2}$

$$Q = \frac{Er^2}{k_e} = \frac{\left(3.00 \times 10^6 \text{ V / m}\right)(0.150 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}(1 \text{ N} \cdot \text{m / V} \cdot \text{C}) = 7.51\ \mu\text{C}$$

(a) $V = \dfrac{k_e Q}{r} = \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(7.51 \times 10^{-6} \text{ C})}{0.150 \text{ m}} = 450 \text{ kV}$

L: These calculated results seem reasonable based on our predictions. The voltage is about 4000 times larger than the 120 V found from common electrical outlets, but the charge is similar in magnitude to many of the static charge problems we have solved earlier. This implies that most of these charge configurations would have to be in a vacuum because the electric field near these point charges would be strong enough to cause sparking in air. (Example: A charged ball with $Q = 1\ \mu\text{C}$ and $r = 1$ mm would have an electric field near its surface of

$$E = \frac{k_e Q}{r^2} = \frac{\left(9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(1 \times 10^{-6} \text{ C}\right)}{(0.001 \text{ m})^2} = 9 \times 10^9 \text{ V / m}$$

which is well beyond the dielectric breakdown of air!)

25.52 $V = \frac{k_e q}{r}$ and $E = \frac{k_e q}{r^2}$ Since $E = \frac{V}{r}$,

(b) $r = \frac{V}{E} = \frac{6.00 \times 10^5 \text{ V}}{3.00 \times 10^6 \text{ V/m}} = \boxed{0.200 \text{ m}}$ and

(a) $q = \frac{Vr}{k_e} = \boxed{13.3\ \mu\text{C}}$

25.53 $U = qV = k_e \frac{q_1 q_2}{r_{12}} = (8.99 \times 10^9) \frac{(38)(54)(1.60 \times 10^{-19})^2}{(5.50 + 6.20) \times 10^{-15}} = 4.04 \times 10^{-11} \text{ J} = \boxed{253 \text{ MeV}}$

***25.54** (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$V = Ed = \left(3.0 \times 10^6 \text{ V/m}\right)\left(5.0 \times 10^{-3} \text{ m}\right) = 1.5 \times 10^4 \text{ V} \quad \boxed{\sim 10^4 \text{ V}}$$

(b) Suppose your surface area is like that of a 70-kg cylinder with the density of water and radius 12 cm. Its length would be given by

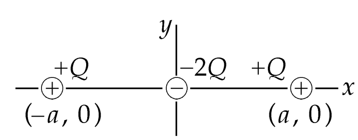
$$70 \times 10^3 \text{ cm}^3 = \pi(12 \text{ cm})^2 \ell \qquad \ell = 1.6 \text{ m}$$

The lateral surface area is $A = 2\pi r \ell = 2\pi(0.12 \text{ m})(1.6 \text{ m}) = 1.2 \text{ m}^2$

The electric field close to your skin is described by $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A \epsilon_0}$, so

$$Q = EA\epsilon_0 = \left(3.0 \times 10^6 \frac{\text{N}}{\text{C}}\right)\left(1.2 \text{ m}^2\right)\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right) \quad \boxed{\sim 10^{-5} \text{ C}}$$

25.55 (a) $V = k_e Q \left(\frac{1}{x+a} - \frac{2}{x} + \frac{1}{x-a}\right)$



$$V = k_e Q \left[\frac{x(x-a) - 2(x+a)(x-a) + x(x+a)}{x(x+a)(x-a)}\right] = \boxed{\frac{2k_e Q a^2}{x^3 - xa^2}}$$

(b) $V = \boxed{\frac{2k_e Q a^2}{x^3}}$ for $\frac{a}{x} << 1$

25.56 (a) $E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{2k_eQa^2}{x^3 - xa^2}\right) = \boxed{\frac{(2k_eQa^2)(3x^2 - a^2)}{(x^3 - xa^2)^2}}$ and $\boxed{E_y = E_z = 0}$

(b) $$E_x = \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3\times10^{-6}\text{ C})(2\times10^{-3}\text{ m})^2\left[3(6\times10^{-3}\text{ m})^2 - (2\times10^{-3}\text{ m})^2\right]}{\left[(6\times10^{-3}\text{ m})^3 - (6\times10^{-3}\text{ m})(2\times10^{-3}\text{ m})^2\right]^2}$$

$E_x = 609\times10^6 \text{ N/C} = \boxed{609 \text{ MN/C}}$

25.57 (a) $E = \frac{Q}{4\pi\epsilon_0 r^2}$

$V = \frac{Q}{4\pi\epsilon_0 r}$

$r = \frac{|V|}{|E|} = \frac{3000\text{ V}}{500\text{ V/m}} = \boxed{6.00\text{ m}}$

(b) $V = -3000\text{ V} = \frac{Q}{4\pi\epsilon_0(6.00\text{ m})}$

$Q = \frac{-3000\text{ V}}{(8.99\times10^9\text{ V}\cdot\text{m/C})}(6.00\text{ m}) = \boxed{-2.00\ \mu\text{C}}$

25.58 From Example 25.5, the potential created by the ring at the electron's starting point is

$$V_i = \frac{k_eQ}{\sqrt{x_i^2 + a^2}} = \frac{k_e(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$$

while at the center, it is $V_f = 2\pi k_e\lambda$. From conservation of energy,

$$0 + (-eV_i) = \tfrac{1}{2}m_e v_f^2 + (-eV_f)$$

$$v_f^2 = \frac{2e}{m_e}(V_f - V_i) = \frac{4\pi e k_e\lambda}{m_e}\left(1 - \frac{a}{\sqrt{x_i^2 + a^2}}\right)$$

$$v_f^2 = \frac{4\pi(1.60\times10^{-19})(8.99\times10^9)(1.00\times10^{-7})}{9.11\times10^{-31}}\left(1 - \frac{0.200}{\sqrt{(0.100)^2 + (0.200)^2}}\right)$$

$v_f = \boxed{1.45\times10^7\text{ m/s}}$

25.59 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $Q\,dx/h$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Q\,dx}{h\sqrt{x^2 + R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Q\,dx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x + \sqrt{x^2+R^2}\right)\Big|_d^{d+h} = \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $Q\,dx/h$ and charge-per-area $Q\,dx/\pi R^2 h$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Q\,dx}{\pi R^2 h}\left(\sqrt{x^2+R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2+R^2}\,dx - x\,dx\right) = \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2+R^2} + \frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\frac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2+R^2} - d\sqrt{d^2+R^2} - 2dh - h^2 + R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

25.60 The positive plate by itself creates a field $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{36.0\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.03\ \dfrac{\text{kN}}{\text{C}}$

away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field 4.07 kN/C in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})\,dx$$

The potential difference between the plates is $V = (4.07\times10^3\ \text{N/C})(0.120\ \text{m}) = \boxed{488\ \text{V}}$

(b) $\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$

$$qV = (1.60\times10^{-19}\ \text{C})(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\ \text{J}}$$

(c) $v_f = \boxed{306\ \text{km/s}}$

(d) $v_f^{\,2} = v_i^{\,2} + 2a(x - x_i)$

$(3.06 \times 10^5 \text{ m/s})^2 = 0 + 2a(0.120 \text{ m})$

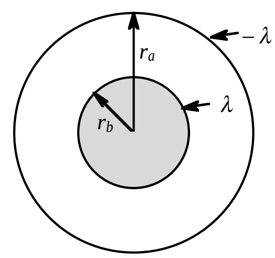
$a = \boxed{3.90 \times 10^{11} \text{ m/s}^2}$

(e) $\sum F = ma = (1.67 \times 10^{-27} \text{ kg})(3.90 \times 10^{11} \text{ m/s}^2) = \boxed{6.51 \times 10^{-16} \text{ N}}$

(f) $E = \dfrac{F}{q} = \dfrac{6.51 \times 10^{-16} \text{ N}}{1.60 \times 10^{-19} \text{ C}} = \boxed{4.07 \text{ kN/C}}$

25.61 $W = \int_0^Q V\,dq$ where $V = \dfrac{k_e q}{R}$; Therefore, $\boxed{W = \dfrac{k_e Q^2}{2R}}$

25.62 (a) $V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is



$$E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}\,dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right), \quad \text{or} \quad \boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by $E = -\dfrac{\partial V}{\partial r} = -2k_e\lambda\left(\dfrac{r}{r_a}\right)\left(-\dfrac{r_a}{r^2}\right) = \dfrac{2k_e\lambda}{r}$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$. Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$

25.63 $V_2 - V_1 = -\int_{r_1}^{r_2} \mathbf{E}\cdot d\mathbf{r} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r}\,dr$

$$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)}$$

25.64 For the given charge distribution, $V(x,y,z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x,y,z) = 0$

is given by $k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$, or $2r_1 = r_2$

This gives: $4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$

which may be written in the form: $x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0$ [1]

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or $x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + \left(x_0^2 + y_0^2 + z_0^2 - a^2\right) = 0$ [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R; \quad -2y_0 = 0; \quad -2z_0 = 0; \quad x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$.

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R, 0, 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$

25.65 (a) From Gauss's law, $\boxed{E_A = 0}$ (no charge within)

$$E_B = k_e \frac{q_A}{r^2} = (8.99\times10^9)\frac{(1.00\times10^{-8})}{r^2} = \boxed{\left(\frac{89.9}{r^2}\right)\text{V / m}}$$

$$E_C = k_e \frac{(q_A + q_B)}{r^2} = (8.99\times10^9)\frac{(-5.00\times10^{-9})}{r^2} = \boxed{\left(-\frac{45.0}{r^2}\right)\text{V / m}}$$

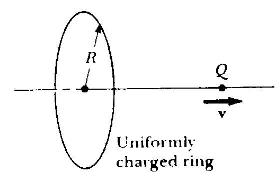
(b) $$V_C = k_e \frac{(q_A + q_B)}{r} = (8.99\times10^9)\frac{(-5.00\times10^{-9})}{r} = \boxed{\left(-\frac{45.0}{r}\right)\text{V}}$$

$$\therefore \text{ At } r_2,\ V = -\frac{45.0}{0.300} = -150\text{ V}$$

Inside r_2, $V_B = -150\text{ V} + \int_{r_2}^{r} \frac{89.9}{r^2}\,dr = -150 + 89.9\left(\frac{1}{r} - \frac{1}{0.300}\right) = \boxed{\left(-450 + \frac{89.9}{r}\right)\text{V}}$

$\therefore$ At r_1, $V = -450 + \dfrac{89.9}{0.150} = +150\text{ V}$ so $\boxed{V_A = +150\text{ V}}$

25.66 From Example 25.5, the potential at the center of the ring is $V_i = k_e Q/R$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge are:



$$U_i = QV_i = \frac{k_e Q^2}{R} \quad \text{and} \quad U_f = QV_f = 0$$

From conservation of energy, $K_f + U_f = K_i + U_i$

or $\frac{1}{2}Mv_f^2 + 0 = 0 + \dfrac{k_e Q^2}{R}$ giving $v_f = \boxed{\sqrt{\dfrac{2k_e Q^2}{MR}}}$

25.67 The sheet creates a field $\mathbf{E}_1 = \dfrac{\sigma}{2\in_0}\mathbf{i}$ for $x > 0$. Along the $x-$axis, the line of charge creates a field

$$\mathbf{E}_2 = \frac{\lambda}{2\pi r \in_0} \text{ away} = \frac{\lambda}{2\pi\in_0(3.00\text{ m} - x)}(-\mathbf{i}) \text{ for } x < 3.00\text{ m}$$

The total field along the $x-$axis in the region $0 < x < 3.00$ m is then

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \left[\frac{\sigma}{2\in_0} - \frac{\lambda}{2\pi\in_0(3.00 - x)}\right]\mathbf{i}$$

(a) The potential at point x follows from

$$V - V_0 = -\int_0^x \mathbf{E}\cdot\mathbf{i}dx = -\int_0^x\left[\frac{\sigma}{2\in_0} - \frac{\lambda}{2\pi\in_0(3.00-x)}\right]dx$$

$$V = V_0 - \frac{\sigma x}{2\in_0} - \frac{\lambda}{2\pi\in_0}\ln\left(1 - \frac{x}{3.00}\right)$$

$$V = 1.00\text{ kV} - \frac{(25.0\times10^{-9}\text{ C/m}^2)x}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} - \frac{80.0\times10^{-9}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\ln\left(1-\frac{x}{3.00}\right)$$

$$V = \boxed{1.00\text{ kV} - \left(1.41\ \frac{\text{kV}}{\text{m}}\right)x - (1.44\text{ kV})\ln\left(1.00 - \frac{x}{3.00\text{ m}}\right)}$$

(b) At $x = 0.800$ m, $V = 316$ V

and $U = QV = (2.00\times10^{-9}\text{ C})(316\text{ J/C}) = 6.33\times10^{-7}\text{ J} = \boxed{633\text{ nJ}}$

25.68 $$V = k_e\int_a^{a+L}\frac{\lambda\,dx}{\sqrt{x^2+b^2}} = k_e\lambda\ln\left[x+\sqrt{(x^2+b^2)}\right]\Big|_a^{a+L} = \boxed{k_e\lambda\ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

25.69 (a) $E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$

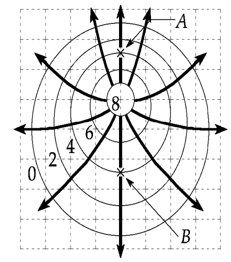
For $r >> a$, $E_r(0°) = \frac{2k_e p}{r^3}$ and $E_r(90°) = 0$, $E_\theta(0°) = 0$ and $E_\theta(90°) = \frac{k_e p}{r^3}$

These results are $\boxed{\text{reasonable for } r >> a}$.

However, for $\boxed{r \to 0,\ E(0)\to\infty}$.

(b) $V = \boxed{\frac{k_e py}{(x^2+y^2)^{3/2}}}$ and $\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e pxy}{(x^2+y^2)^{5/2}}}$

$E_y = -\frac{\partial V}{\partial y} = \boxed{\frac{k_e p(2y^2-x^2)}{(x^2+y^2)^{5/2}}}$



25.70 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$

(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.

25.71 For an element of area which is a ring of radius r and width dr, $dV = \dfrac{k_e\,dq}{\sqrt{r^2+x^2}}$

$dq = \sigma\,dA = Cr(2\pi r\,dr)$ and

$$V = C(2\pi k_e)\int_0^R \frac{r^2\,dr}{\sqrt{r^2+x^2}} = \boxed{C(\pi k_e)\left[R\sqrt{R^2+x^2} + x^2\ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

25.72 $dU = V\,dq$ where the potential $V = \dfrac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho(4\pi r^2\,dr)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2\,dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho(4\pi r^2\,dr) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4\,dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4\,dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$

***25.73** (a) From Problem 62, $E = \dfrac{\Delta V}{\ln(r_a/r_b)}\dfrac{1}{r}$

We require just outside the central wire $5.50\times10^6\ \dfrac{\text{V}}{\text{m}} = \dfrac{50.0\times10^3\ \text{V}}{\ln\left(\dfrac{0.850\ \text{m}}{r_b}\right)}\left(\dfrac{1}{r_b}\right)$

or $\left(110\ \text{m}^{-1}\right)r_b\ln\left(\dfrac{0.850\ \text{m}}{r_b}\right) = 1$

We solve by homing in on the required value

r_b (m)	0.0100	0.00100	0.00150	0.00145	0.00143	0.00142
$\left(110\ \text{m}^{-1}\right)r_b\ln\left(\dfrac{0.850\ \text{m}}{r_b}\right)$	4.89	0.740	1.05	1.017	1.005	0.999

Thus, to three significant figures, $\boxed{r_b = 1.42\ \text{mm}}$

(b) At r_a, $E = \dfrac{50.0\ \text{kV}}{\ln(0.850\ \text{m}/0.00142\ \text{m})}\left(\dfrac{1}{0.850\ \text{m}}\right) = \boxed{9.20\ \text{kV/m}}$

Chapter 26 Solutions

***26.1** (a) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(12.0\text{ V}) = 4.80 \times 10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00 \times 10^{-6}\text{ F})(1.50\text{ V}) = 6.00 \times 10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

26.2 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0 \times 10^{-6}\text{ C}}{10.0\text{ V}} = 1.00 \times 10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100 \times 10^{-6}\text{ C}}{1.00 \times 10^{-6}\text{ F}} = \boxed{100\text{ V}}$

26.3 $E = \dfrac{k_e q}{r^2}$: $\qquad q = \dfrac{(4.90 \times 10^4\text{ N / C})(0.210\text{ m})^2}{8.99 \times 10^9\text{ N} \cdot \text{m}^2 / \text{C}^2} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240 \times 10^{-6}}{4\pi(0.120)^2} = \boxed{1.33\ \mu\text{C} / \text{m}^2}$

(b) $C = 4\pi \epsilon_0 r = 4\pi(8.85 \times 10^{-12})(0.120) = \boxed{13.3\text{ pF}}$

26.4 (a) $C = 4\pi \epsilon_0 R$

$R = \dfrac{C}{4\pi \epsilon_0} = k_e C = (8.99 \times 10^9\text{ N} \cdot \text{m}^2 / \text{C}^2)(1.00 \times 10^{-12}\text{ F}) = \boxed{8.99\text{ mm}}$

(b) $C = 4\pi \epsilon_0 R = \dfrac{4\pi(8.85 \times 10^{-12}\text{ C}^2)(2.00 \times 10^{-3}\text{ m})}{\text{N} \cdot \text{m}^2} = \boxed{0.222\text{ pF}}$

(c) $Q = CV = (2.22 \times 10^{-13}\text{ F})(100\text{ V}) = \boxed{2.22 \times 10^{-11}\text{ C}}$

26.5 (a) $\dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

$Q_1 + Q_2 = \left(1 + \dfrac{R_1}{R_2}\right) Q_2 = 3.50 Q_2 = 7.00\ \mu\text{C}$

$\boxed{Q_2 = 2.00\ \mu\text{C}}$ $\quad \boxed{Q_1 = 5.00\ \mu\text{C}}$

(b) $V_1 = V_2 = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} = \dfrac{5.00\ \mu\text{C}}{\left(8.99 \times 10^9\text{ m/F}\right)^{-1}(0.500\text{ m})} = 8.99 \times 10^4\text{ V} = \boxed{89.9\text{ kV}}$

***26.6** $C = \frac{\kappa \epsilon_0 A}{d} = \frac{(1.00)(8.85 \times 10^{-12}\ \text{C}^2)(1.00 \times 10^3\ \text{m})^2}{\text{N} \cdot \text{m}^2(800\ \text{m})} = \boxed{11.1\ \text{nF}}$

The potential between ground and cloud is

$\Delta V = Ed = (3.00 \times 10^6\ \text{N/C})(800\ \text{m}) = 2.40 \times 10^9\ \text{V}$

$Q = C(\Delta V) = (11.1 \times 10^{-9}\ \text{C/V})(2.40 \times 10^9\ \text{V}) = \boxed{26.6\ \text{C}}$

26.7 (a) $\Delta V = Ed$

$E = \frac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$

(b) $E = \frac{\sigma}{\epsilon_0}$

$\sigma = (1.11 \times 10^4\ \text{N/C})(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) = \boxed{98.3\ \text{nC/m}^2}$

(c) $C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)\left(7.60\ \text{cm}^2\right)\left(1.00\ \text{m}/100\ \text{cm}\right)^2}{1.80 \times 10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$

(d) $\Delta V = \frac{Q}{C}$ $\quad Q = (20.0\ \text{V})(3.74 \times 10^{-12}\ \text{F}) = \boxed{74.7\ \text{pC}}$

26.8 $C = \frac{\kappa \epsilon_0 A}{d} = 60.0 \times 10^{-15}\ \text{F}$

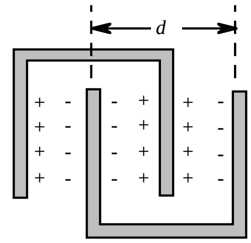
$d = \frac{\kappa \epsilon_0 A}{C} = \frac{(1)\left(8.85 \times 10^{-12}\right)\left(21.0 \times 10^{-12}\right)}{60.0 \times 10^{-15}}$

$d = 3.10 \times 10^{-9}\ \text{m} = \boxed{3.10\ \text{nm}}$

26.9 $Q = \frac{\epsilon_0 A}{d}(\Delta V)$ $\quad \frac{Q}{A} = \sigma = \frac{\epsilon_0(\Delta V)}{d}$

$d = \frac{\epsilon_0(\Delta V)}{\sigma} = \frac{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)(150\ \text{V})}{\left(30.0 \times 10^{-9}\ \text{C/cm}^2\right)\left(1.00 \times 10^4\ \text{cm}^2/\text{m}^2\right)} = \boxed{4.42\ \mu\text{m}}$

26.10 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\pi R^2/2$. By proportion, the effective area of a single sheet of charge is $(\pi - \theta)R^2/2$.



When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2} = \boxed{\frac{(2N-1)\epsilon_0(\pi-\theta)R^2}{d}}$$

26.11 (a) $C = \dfrac{l}{2k_e \ln\left(\frac{b}{a}\right)} = \dfrac{50.0}{2(8.99\times10^9)\ln\left(\frac{7.27}{2.58}\right)} = \boxed{2.68 \text{ nF}}$

(b) Method 1: $\Delta V = 2k_e \lambda \ln\left(\dfrac{b}{a}\right)$

$$\lambda = q / l = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99\times10^9)(1.62\times10^{-7})\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02\text{ kV}}$

26.12 Let the radii be b and a with $b = 2a$. Put charge Q on the inner conductor and $-Q$ on the outer. Electric field exists only in the volume between them. The potential of the inner sphere is $V_a = k_eQ/a$; that of the outer is $V_b = k_eQ/b$. Then

$$V_a - V_b = \frac{k_eQ}{a} - \frac{k_eQ}{b} = \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right) \quad \text{and} \quad C = \frac{Q}{V_a - V_b} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Here $C = \dfrac{4\pi\epsilon_0 2a^2}{a} = 8\pi\epsilon_0 a \qquad a = \dfrac{C}{8\pi\epsilon_0}$

The intervening volume is $\quad \text{Volume} = \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 = 7\left(\frac{4}{3}\pi a^3\right) = 7\left(\frac{4}{3}\pi\right)\dfrac{C^3}{8^3\pi^3\epsilon_0^3} = \dfrac{7C^3}{384\pi^2\epsilon_0^3}$

$$\text{Volume} = \frac{7(20.0\times10^{-6}\text{ C}^2/\text{N}\cdot\text{m})^3}{384\pi^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)^3} = \boxed{2.13\times10^{16}\text{ m}^3}$$

The outer sphere is 360 km in diameter.

26.13 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$

$$\Delta V = Ed = \frac{mgd\tan\theta}{q} = \frac{(350\times10^{-6}\text{ kg})(9.80\text{ m / s}^2)(4.00\times10^{-2}\text{ m})\tan 15.0°}{30.0\times10^{-9}\text{ C}} = \boxed{1.23\text{ kV}}$$

26.14 $\Sigma F_y = 0$: $T\cos\theta - mg = 0$ $\Sigma F_x = 0$: $T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$, so $E = \dfrac{mg}{q}\tan\theta$ and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$

26.15 (a) $C = \dfrac{ab}{k_e(b-a)} = \dfrac{(0.0700)(0.140)}{(8.99\times10^9)(0.140-0.0700)} = \boxed{15.6\text{ pF}}$

(b) $C = \dfrac{Q}{\Delta V}$ $\Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256\text{ kV}}$

<u>Goal Solution</u>

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

G: Since the separation between the inner and outer shells is much larger than a typical electronic capacitor with d ~ 0.1 mm and capacitance in the microfarad range, we might expect the capacitance of this spherical configuration to be on the order of picofarads, (based on a factor of about 700 times larger spacing between the conductors). The potential difference should be sufficiently low to prevent sparking through the air that separates the shells.

O: The capacitance can be found from the equation for spherical shells, and the voltage can be found from $Q = C\Delta V$.

A: (a) For a spherical capacitor with inner radius a and outer radius b,

$$C = \frac{ab}{k(b-a)} = \frac{(0.0700\text{ m})(0.140\text{ m})}{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(0.140-0.0700)\text{ m}} = 1.56\times10^{-11}\text{ F} = 15.6\text{ pF}$$

(b) $$\Delta V = \frac{Q}{C} = \frac{(4.00\times10^{-6}\text{ C})}{1.56\times10^{-11}\text{ F}} = 2.56\times10^5\text{ V} = 256\text{ kV}$$

L: The capacitance agrees with our prediction, but the voltage seems rather high. We can check this voltage by approximating the configuration as the electric field between two charged parallel plates separated by d = 7.00 cm, so

$$E \sim \frac{\Delta V}{d} = \frac{2.56\times10^5\text{ V}}{0.0700\text{ m}} = 3.66\times10^6\text{ V / m}$$

This electric field barely exceeds the dielectric breakdown strength of air $\left(3\times10^6\text{ V / m}\right)$, so it may not even be possible to place $4.00\ \mu$C of charge on this capacitor!

26.16 $C = 4\pi \epsilon_0 R = 4\pi\left(8.85\times10^{-12}\ \text{C/N}\cdot\text{m}^2\right)\left(6.37\times10^{6}\ \text{m}\right) = \boxed{7.08\times10^{-4}\ \text{F}}$

***26.17** (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$\Delta V = \boxed{9.00\ \text{V}}$

(c) $Q_5 = C(\Delta V) = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$ and $Q_{12} = C(\Delta V) = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

***26.18** (a) In series capacitors add as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}} \quad \text{and} \quad C_{eq} = \boxed{3.53\ \mu\text{F}}$$

(c) The charge on the equivalent capacitor is

$Q_{eq} = C_{eq}(\Delta V) = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$

Each of the series capacitors has this same charge on it. So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$

(b) The voltage across each is

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}} \quad \text{and} \quad \Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$$

26.19 $C_p = C_1 + C_2 \quad \dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Substitute $C_2 = C_p - C_1$ $\quad \dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_p - C_1} = \dfrac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

Simplifying, $C_1^2 - C_1C_p + C_pC_s = 0$

$$C_1 = \frac{C_p \pm \sqrt{C_p^{\,2} - 4C_pC_s}}{2} = \tfrac{1}{2}C_p \pm \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1 = \tfrac{1}{2}C_p + \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s} = \tfrac{1}{2}(9.00\ \text{pF}) + \sqrt{\tfrac{1}{4}(9.00\ \text{pF})^2 - (9.00\ \text{pF})(2.00\ \text{pF})} = \boxed{6.00\ \text{pF}}$$

$$C_2 = C_p - C_1 = \tfrac{1}{2}C_p - \sqrt{\tfrac{1}{4}C_p^{\,2} - C_pC_s} = \frac{1}{2}(9.00\ \text{pF}) - 1.50\ \text{pF} = \boxed{3.00\ \text{pF}}$$

26.20 $C_p = C_1 + C_2$ and $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Substitute $C_2 = C_p - C_1$: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_p - C_1} = \dfrac{C_p - C_1 + C_1}{C_1\left(C_p - C_1\right)}$

Simplifying, $C_1^2 - C_1 C_p + C_p C_s = 0$

and $C_1 = \dfrac{C_p \pm \sqrt{C_p^2 - 4C_p C_s}}{2} = \boxed{\dfrac{1}{2}C_p + \sqrt{\dfrac{1}{4}C_p^2 - C_p C_s}}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed). Then, from $C_2 = C_p - C_1$

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$$

26.21 (a) $\dfrac{1}{C_s} = \dfrac{1}{15.0} + \dfrac{1}{3.00}$ $C_s = 2.50\ \mu\text{F}$

$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$C_{eq} = \left(\dfrac{1}{8.50\ \mu\text{F}} + \dfrac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$

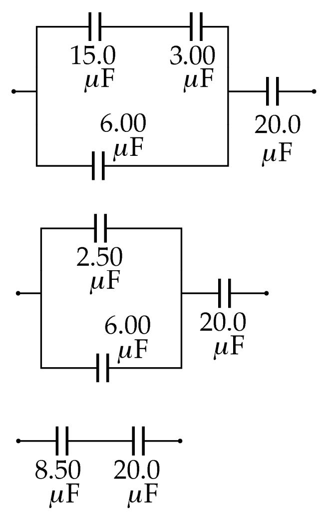
(b) $Q = (\Delta V)C = (15.0\text{ V})(5.96\ \mu\text{F}) = \boxed{89.5\ \mu\text{C}}$ on 20.0 μF

$\Delta V = \dfrac{Q}{C} = \dfrac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\text{ V}$

$15.0 - 4.47 = 10.53\text{ V}$

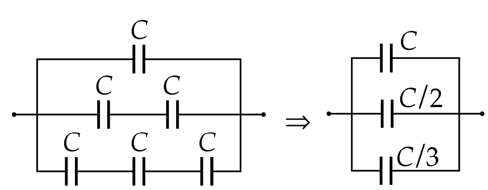
$Q = (\Delta V)C = (10.53)(6.00\ \mu\text{F}) = \boxed{63.2\ \mu\text{C}}$ on 6.00 μF

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on 15.0 μF and 3.00 μF



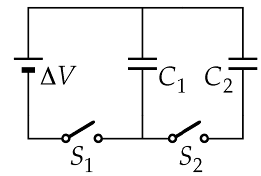
26.22 The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.

$C_{\text{eq}} = C\left(1 + \dfrac{1}{2} + \dfrac{1}{3}\right) = \dfrac{11}{6}C = \boxed{1.83C}$



26.23 $C = \dfrac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \dfrac{Q}{20.0}$ and $Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$ and $\Delta V = \dfrac{Q}{C}$



$\dfrac{120 - Q_2}{C_1} = \dfrac{Q_2}{C_2}$ or $\dfrac{120 - Q_2}{6.00} = \dfrac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \dfrac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

***26.24** (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s}$$

$$C_s = \frac{1}{2.51 \times 10^{-3}/\mu\text{F}} = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$

$$C_p = \boxed{2.20\ \mu\text{F}}$$

26.25 With switch closed, distance $d' = 0.500d$ and capacitance $C' = \dfrac{\epsilon_0 A}{d'} = \dfrac{2\epsilon_0 A}{d} = 2C$

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00 \times 10^{-6}\ \text{F})(100\ \text{V}) = \boxed{400\ \mu\text{C}}$

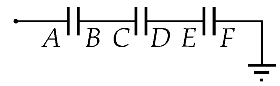
(b) The force stretching out one spring is

$$\mathbf{F} = \frac{Q^2}{2\epsilon_0 A} = \frac{4C^2(\Delta V)^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A / d)d} = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = d/4$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00 \times 10^{-6}\ \text{F})(100\ \text{V})^2}{(8.00 \times 10^{-3}\ \text{m})^2} = \boxed{2.50\ \text{kN/m}}$$

26.26 Positive charge on A will induce equal negative charges on B, D, and F, and equal positive charges on C and E. The nesting spheres form three capacitors in series. From Example 26.3,



$$C_{AB} = \frac{ab}{k_e(b-a)} = \frac{R(2R)}{k_e R} = \frac{2R}{k_e}$$

$$C_{CD} = \frac{(3R)(4R)}{k_e R} = \frac{12R}{k_e}$$

$$C_{EF} = \frac{(5R)(6R)}{k_e R} = \frac{30R}{k_e}$$

$$C_{\text{eq}} = \frac{1}{k_e/2R + k_e/12R + k_e/30R} = \boxed{\frac{60R}{37k_e}}$$

26.27

$$nC = \frac{100}{\underbrace{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}+\ldots}_{\text{n capacitors}}} = \frac{100}{n/C}$$

$$nC = \frac{100C}{n} \qquad \text{so} \qquad n^2 = 100 \qquad \text{and} \qquad n = \boxed{10}$$

Goal Solution

A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

G: Since capacitors in parallel add and ones in series add as inverses, 2 capacitors in parallel would have a capacitance 4 times greater than if they were in series, and 3 capacitors would give a ratio $C_p / C_s = 9$, so maybe $n = \sqrt{C_p / C_s} = \sqrt{100} = 10$**.**

O: The ratio reasoning above seems like an efficient way to solve this problem, but we should check the answer with a more careful analysis based on the general relationships for series and parallel combinations of capacitors.

A: Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

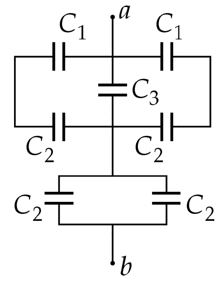
$$C_p = C_1 + C_2 + \ldots + C_n = nC$$

The relationship for n capacitors in series is $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} = \frac{n}{C}$

Therefore $\frac{C_p}{C_s} = \frac{nC}{C/n} = n^2$ or $n = \sqrt{\frac{C_p}{C_s}} = \sqrt{100} = 10$

L: Our prediction appears to be correct. A qualitative reason that $C_p / C_s = n^2$ is because the amount of charge that can be stored on the capacitors increases according to the area of the plates for a parallel combination, but the total charge remains the same for a series combination.

26.28



$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$$

C_{p1} = 2(3.33) + 2.00 = 8.66 μF

C_{p2} = 2(10.0) = 20.0 μF

$$C_{\text{eq}} = \left(\frac{1}{8.66} + \frac{1}{20.0}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

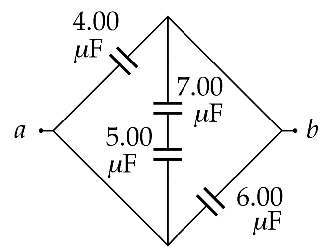
26.29

$$Q_{\text{eq}} = C_{\text{eq}}(\Delta V) = \left(6.04 \times 10^{-6}\ \text{F}\right)(60.0\ \text{V}) = 3.62 \times 10^{-4}\ \text{C}$$

$$Q_{p1} = Q_{\text{eq}}, \quad \text{so} \quad \Delta V_{p1} = \frac{Q_{\text{eq}}}{C_{p1}} = \frac{3.62 \times 10^{-4}\ \text{C}}{8.66 \times 10^{-6}\ \text{F}} = 41.8\ \text{V}$$

$$Q_3 = C_3\left(\Delta V_{p1}\right) = \left(2.00 \times 10^{-6}\ \text{F}\right)(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$

26.30



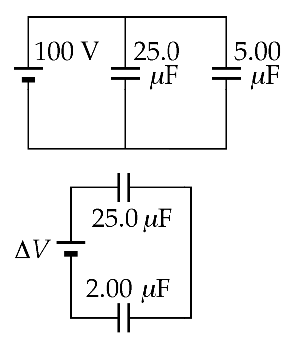
$$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

C_p = 2.92 + 4.00 + 6.00 = $\boxed{12.9\ \mu\text{F}}$

***26.31** (a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\ \text{V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$

***26.32** $U = \frac{1}{2}C(\Delta V)^2$



The circuit diagram is shown at the right.

(a) $C_p = C_1 + C_2$ = 25.0 μF + 5.00 μF = 30.0 μF

$U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = \boxed{0.150\ \text{J}}$

(b) $$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$$

$U = \frac{1}{2}C(\Delta V)^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{(0.150)(2)}{4.17 \times 10^{-6}}} = \boxed{268\ \text{V}}$$

***26.33** Use $U = \frac{1}{2}\frac{Q^2}{C}$ and $C = \frac{\epsilon_0 A}{d}$

If $d_2 = 2d_1$, $C_2 = \frac{1}{2} C_1$. Therefore, the $\boxed{\text{stored energy doubles}}$.

26.34 $u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$

$$\frac{1.00 \times 10^{-7}}{V} = \frac{1}{2}(8.85 \times 10^{-12})(3000)^2$$

$$V = \boxed{2.51 \times 10^{-3}\ \text{m}^3} = \left(2.51 \times 10^{-3}\ \text{m}^3\right)\left(\frac{1000\ \text{L}}{\text{m}^3}\right) = \boxed{2.51\ \text{L}}$$

26.35 $W = U = \int F\,dx$ so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2c}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\epsilon_0 A}\right) = \boxed{\frac{Q^2}{2\epsilon_0 A}}$

26.36 Plate a experiences force $-kx\mathbf{i}$ from the spring and force $QE\mathbf{i}$ due to the electric field created by plate b according to $E = \sigma / 2\epsilon_0 = Q / 2A\epsilon_0$. Then,

$$kx = \frac{Q^2}{2A\epsilon_0} \qquad x = \boxed{\frac{Q^2}{2A\epsilon_0 k}}$$

where A is the area of one plate.

26.37 The energy transferred is $W = \frac{1}{2}Q(\Delta V) = \frac{1}{2}(50.0\ \text{C})(1.00 \times 10^8\ \text{V}) = 2.50 \times 10^9\ \text{J}$ and 1% of this (or $W' = 2.50 \times 10^7\ \text{J}$) is absorbed by the tree. If m is the amount of water boiled away, then

$$W' = m(4186\ \text{J/kg °C})(100\ \text{°C} - 30.0\ \text{°C}) + m(2.26 \times 10^6\ \text{J/kg}) = 2.50 \times 10^7\ \text{J}$$

giving $\boxed{m = 9.79\ \text{kg}}$

26.38 $U = \frac{1}{2}C(\Delta V)^2$ where $C = 4\pi \in_0 R = \frac{R}{k_e}$ and $\Delta V = \frac{k_e Q}{R} - 0 = \frac{k_e Q}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_e Q}{R}\right)^2 = \boxed{\frac{k_e Q^2}{2R}}$$

26.39 $\frac{k_e Q^2}{2R} = mc^2$

$$R = \frac{k_e e^2}{2mc^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C})(1.60 \times 10^{-19} \text{ C})^2}{2(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2} = \boxed{1.40 \text{ fm}}$$

***26.40** $C = \frac{\kappa \in_0 A}{d} = \frac{4.90(8.85 \times 10^{-12} \text{ F/m})(5.00 \times 10^{-4} \text{ m}^2)}{2.00 \times 10^{-3} \text{ m}} = 1.08 \times 10^{-11} \text{ F} = \boxed{10.8 \text{ pF}}$

***26.41** (a) $C = \frac{\kappa \in_0 A}{d} = \frac{2.10(8.85 \times 10^{-12} \text{ F/m})(1.75 \times 10^{-4} \text{ m}^2)}{4.00 \times 10^{-5} \text{ m}} = 8.13 \times 10^{-11} \text{ F} = \boxed{81.3 \text{ pF}}$

(b) $\Delta V_{max} = E_{max} d = (60.0 \times 10^6 \text{ V/m})(4.00 \times 10^{-5} \text{ m}) = \boxed{2.40 \text{ kV}}$

***26.42** $Q_{max} = C(\Delta V_{max})$, but $\Delta V_{max} = E_{max} d$

Also, $C = \frac{\kappa \in_0 A}{d}$

Thus, $Q_{max} = \frac{\kappa \in_0 A}{d}(E_{max} d) = \kappa \in_0 A E_{max}$

(a) With air between the plates, $\kappa = 1.00$ and $E_{max} = 3.00 \times 10^6$ V/m. Therefore,

$Q_{max} = \kappa \in_0 A E_{max} = (8.85 \times 10^{-12} \text{ F/m})(5.00 \times 10^{-4} \text{ m}^2)(3.00 \times 10^6 \text{ V/m}) = \boxed{13.3 \text{ nC}}$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{max} = 24.0 \times 10^6$ V/m.

$Q_{max} = \kappa \in_0 A E_{max} = 2.56(8.85 \times 10^{-12} \text{ F/m})(5.00 \times 10^{-4} \text{ m}^2)(24.0 \times 10^6 \text{ V/m}) = \boxed{272 \text{ nC}}$

26.43 $C = \frac{\kappa \in_0 A}{d}$ or $95.0 \times 10^{-9} = \frac{3.70(8.85 \times 10^{-12})(0.0700)\ell}{(0.0250 \times 10^{-3})}$

= 1.04 m

***26.44** Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \cong 3$, $E_{\max} \sim 10^7$ V/m and thickness 1 mil = 2.54 cm/1000. Then,

$$C = \frac{\kappa \epsilon_0 A}{d} \sim \frac{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.4 \text{ m}^2)}{2.54 \times 10^{-5} \text{ m}} \sim \boxed{10^{-6} \text{ F}}$$

$$\Delta V_{\max} = E_{\max} d \sim \left(10^7 \ \frac{\text{V}}{\text{m}}\right)\left(2.54 \times 10^{-5} \text{ m}\right) \sim \boxed{10^2 \text{ V}}$$

***26.45** (a) With air between the plates, we find $C_0 = \dfrac{Q}{\Delta V} = \dfrac{48.0\ \mu\text{C}}{12.0\text{ V}} = \boxed{4.00\ \mu\text{F}}$

(b) When Teflon is inserted, the charge remains the same (48.0 μC) because the plates are isolated. However, the capacitance, and hence the voltage, changes. The new capacitance is

$$C' = \kappa C_0 = 2.10(4.00\ \mu\text{F}) = \boxed{8.40\ \mu\text{F}}$$

(c) The voltage on the capacitor now is $\Delta V' = \dfrac{Q}{C'} = \dfrac{48.0\ \mu\text{C}}{8.40\ \mu\text{F}} = \boxed{5.71\text{ V}}$

and the charge is $\boxed{48.0\ \mu\text{C}}$

26.46 Originally, $C = \epsilon_0 A / d = Q / (\Delta V)_i$

(a) The charge is the same before and after immersion, with value $Q = \epsilon_0 A (\Delta V)_i / d$.

$$Q = \frac{(8.85 \times 10^{-12} \text{ C}^2)(25.0 \times 10^{-4} \text{ m}^2)(250 \text{ V})}{\text{N} \cdot \text{m}^2 \ (1.50 \times 10^{-2} \text{ m})} = \boxed{369 \text{ pC}}$$

(b) Finally, $C_f = \kappa \epsilon_0 A / d = Q / (\Delta V)_f$

$$C_f = \frac{80.0(8.85 \times 10^{-12} \text{ C}^2)(25.0 \times 10^{-4} \text{ m}^2)}{\text{N} \cdot \text{m}^2 \ (1.50 \times 10^{-2} \text{ m})} = \boxed{118 \text{ pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A (\Delta V)_i d}{\kappa \epsilon_0 A d} = \frac{(\Delta V)_i}{\kappa} = \frac{250 \text{ V}}{80.0} = \boxed{3.12 \text{ V}}$$

(c) Originally, $U = \frac{1}{2} C (\Delta V)_i^2 = \dfrac{\epsilon_0 A (\Delta V)_i^2}{2d}$

Finally, $U_f = \frac{1}{2} C_f (\Delta V)_f^2 = \dfrac{\kappa \epsilon_0 A (\Delta V)_i^2}{2d \ \kappa^2} = \dfrac{\epsilon_0 A (\Delta V)_i^2}{2d \kappa}$

So, $\Delta U = U_f - U = -\dfrac{\epsilon_0 A (\Delta V)_i^2 (\kappa - 1)}{2d \ \kappa}$

$$\Delta U = \frac{(-8.85 \times 10^{-12} \text{ C}^2)(25.0 \times 10^{-4} \text{ m}^2)(250 \text{ V})^2 (79.0)}{\text{N} \cdot \text{m}^2 \, 2(1.50 \times 10^{-2} \text{ m})80} = \boxed{-45.5 \text{ nJ}}$$

26.47 $$\frac{1}{C} = \frac{1}{\left(\frac{\kappa_1 ab}{k_e(b-a)}\right)} + \frac{1}{\left(\frac{\kappa_2 bc}{k_e(c-b)}\right)} = \frac{k_e(b-a)}{\kappa_1 ab} + \frac{k_e(c-b)}{\kappa_2 bc}$$

$$C = \frac{1}{\frac{k_e(b-a)}{\kappa_1 ab} + \frac{k_e(c-b)}{\kappa_2 bc}} = \frac{\kappa_1\kappa_2\, abc}{k_e\,\kappa_2(bc-ac) + k_e\,\kappa_1(ac-ab)} = \boxed{\frac{4\pi\,\kappa_1\kappa_2\, abc\epsilon_0}{\kappa_2 bc - \kappa_1 ab + (\kappa_1-\kappa_2)ac}}$$

26.48 (a) $$C = \kappa C_0 = \frac{\kappa\epsilon_0 A}{d} = \frac{(173)(8.85\times10^{-12})(1.00\times10^{-4}\ \text{m}^2)}{0.100\times10^{-3}\ \text{m}} = \boxed{1.53\ \text{nF}}$$

(b) The battery delivers the free charge

$$Q = C(\Delta V) = (1.53\times10^{-9}\ \text{F})(12.0\ \text{V}) = \boxed{18.4\ \text{nC}}$$

(c) The surface density of free charge is

$$\sigma = \frac{Q}{A} = \frac{18.4\times10^{-9}\ \text{C}}{1.00\times10^{-4}\ \text{m}^2} = \boxed{1.84\times10^{-4}\ \text{C/m}^2}$$

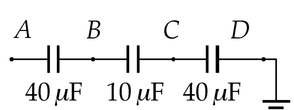
The surface density of polarization charge is

$$\sigma_p = \sigma\left(1-\frac{1}{\kappa}\right) = \sigma\left(1-\frac{1}{173}\right) = \boxed{1.83\times10^{-4}\ \text{C/m}^2}$$

(d) We have $E = E_0/\kappa$ and $E_0 = \Delta V/d$; hence,

$$E = \frac{\Delta V}{\kappa d} = \frac{12.0\ \text{V}}{(173)(1.00\times10^{-4}\ \text{m})} = \boxed{694\ \text{V/m}}$$

26.49 The given combination of capacitors is equivalent to the circuit diagram shown to the right.



Put charge Q on point A. Then,

$$Q = (40.0\ \mu\text{F})\Delta V_{AB} = (10.0\ \mu\text{F})\Delta V_{BC} = (40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \tfrac{1}{4}(\Delta V_{BC}) = 3.75\ \text{V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75\ \text{V} + 15.0\ \text{V} + 3.75\ \text{V} = \boxed{22.5\ \text{V}}$

***26.50** (a) The displacement from negative to positive charge is

$$2\mathbf{a} = (-1.20\,\mathbf{i} + 1.10\,\mathbf{j})\text{ mm} - (1.40\,\mathbf{i} - 1.30\,\mathbf{j})\text{ mm} = (-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\text{ m}$$

The electric dipole moment is

$$\mathbf{p} = 2\mathbf{a}q = (3.50 \times 10^{-9}\text{ C})(-2.60\,\mathbf{i} + 2.40\,\mathbf{j}) \times 10^{-3}\text{ m} = \boxed{(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\text{ C}\cdot\text{m}}$$

(b) $$\tau = \mathbf{p} \times \mathbf{E} = \left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\text{ C}\cdot\text{m}\right] \times \left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\text{ N/C}\right]$$

$$\tau = (+44.6\,\mathbf{k} - 65.5\,\mathbf{k}) \times 10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09 \times 10^{-8}\text{ N}\cdot\text{m}\,\mathbf{k}}$$

(c) $$U = -\mathbf{p}\cdot\mathbf{E} = -\left[(-9.10\,\mathbf{i} + 8.40\,\mathbf{j}) \times 10^{-12}\text{ C}\cdot\text{m}\right]\cdot\left[(7.80\,\mathbf{i} - 4.90\,\mathbf{j}) \times 10^{3}\text{ N/C}\right]$$

$$U = (71.0 + 41.2) \times 10^{-9}\text{ J} = \boxed{112\text{ nJ}}$$

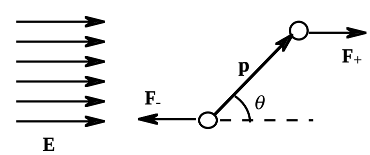
(d) $$|\mathbf{p}| = \sqrt{(9.10)^2 + (8.40)^2} \times 10^{-12}\text{ C}\cdot\text{m} = 12.4 \times 10^{-12}\text{ C}\cdot\text{m}$$

$$|\mathbf{E}| = \sqrt{(7.80)^2 + (4.90)^2} \times 10^{3}\text{ N C} = 9.21 \times 10^{3}\text{ N/C}$$

$$U_{\max} = |\mathbf{p}|\,|\mathbf{E}| = 114\text{ nJ}, \qquad U_{\min} = -114\text{ nJ}$$

$$U_{\max} - U_{\min} = \boxed{228\text{ nJ}}$$

***26.51** (a) Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\mathbf{F}_- = -qE(x)\mathbf{i}$. The force on the positive charge is



$$\mathbf{F}_+ = +qE(x + 2a\cos\theta)\mathbf{i} \cong qE(x)\mathbf{i} + q\frac{dE}{dx}(2a\cos\theta)\mathbf{i}$$

The force on the dipole is altogether $$\mathbf{F} = \mathbf{F}_- + \mathbf{F}_+ = q\frac{dE}{dx}(2a\cos\theta)\mathbf{i} = \boxed{p\frac{dE}{dx}\cos\theta\,\mathbf{i}}$$

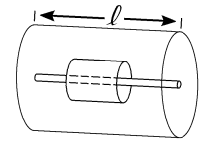
(b) The balloon creates field along the x – axis of $\dfrac{k_e q}{x^2}\mathbf{i}$.

Thus, $$\frac{dE}{dx} = \frac{(-2)k_e q}{x^3}$$

At $x = 16.0$ cm, $$\frac{dE}{dx} = \frac{(-2)(8.99 \times 10^9)(2.00 \times 10^{-6})}{(0.160)^3} = \boxed{-8.78\ \frac{\text{MN}}{\text{C}\cdot\text{m}}}$$

$$\mathbf{F} = (6.30 \times 10^{-9}\text{ C}\cdot\text{m})\left(-8.78 \times 10^{6}\ \frac{\text{N}}{\text{C}\cdot\text{m}}\right)\cos 0°\ \mathbf{i} = \boxed{-55.3\,\mathbf{i}\text{ mN}}$$

26.52 $2\pi r \ell E = \dfrac{q_{\text{in}}}{\epsilon_0}$ so $E = \dfrac{\lambda}{2\pi r \epsilon_0}$



$$\Delta V = -\int_{r_1}^{r_2} \mathbf{E}\cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r\,\epsilon_0}dr = \frac{\lambda}{2\pi\,\epsilon_0}\ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{\lambda_{\max}}{2\pi\epsilon_0} = E_{\max} r_{\text{inner}}$$

$$\Delta V = \left(1.20\times10^{6}\ \frac{\text{V}}{\text{m}}\right)\left(0.100\times10^{-3}\ \text{m}\right)\left(\ln\frac{25.0}{0.200}\right)$$ $$\Delta V_{\max} = \boxed{579\ \text{V}}$$

***26.53** (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$EA' + EA' = \dfrac{Q}{\in A}A'$, so $\boxed{E = \dfrac{Q}{2\in A}}$ directed away from the positive sheet.

(b) In the space between the sheets, each creates field $Q/2\in A$ away from the positive and toward the negative sheet. Together, they create a field of

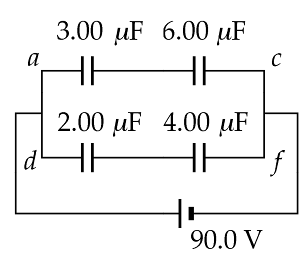
$$\boxed{E = \frac{Q}{\in A}}$$

(c) Assume that the field is in the positive x – direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \mathbf{E}\cdot d\mathbf{s} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A}\mathbf{i}\cdot(-\mathbf{i}\,dx) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \dfrac{Q}{\Delta V} = \dfrac{Q}{Qd/\in A} = \boxed{\dfrac{\in A}{d} = \dfrac{\kappa\epsilon_0 A}{d}}$

26.54 (a) $C = \left[\dfrac{1}{3.00} + \dfrac{1}{6.00}\right]^{-1} + \left[\dfrac{1}{2.00} + \dfrac{1}{4.00}\right]^{-1} = \boxed{3.33\ \mu\text{F}}$



(c) $Q_{ac} = C_{ac}\,(\Delta V_{ac}) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}$

Therefore, $Q_3 = Q_6 = \boxed{180\ \mu\text{C}}$

$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33\ \mu F)(90.0\ \text{V}) = \boxed{120\ \mu\text{C}}$

(b) $\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\text{ V}}$

$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$

$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\text{ V}}$

$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$

(d) $U_T = \frac{1}{2} C_{\text{eq}} (\Delta V)^2 = \frac{1}{2}(3.33 \times 10^{-6})(90.0\text{ V})^2 = \boxed{13.4\text{ mJ}}$

***26.55** The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \mathbf{E} \cdot d\mathbf{r} = -\frac{\lambda}{2\pi \in_0} \int_{D-d}^{d} \frac{dr}{r} = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{D-d}{d}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D-d}{d}\right)$$

and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{\left(\frac{\lambda}{\pi \in_0}\right) \ln\left(\frac{D-d}{d}\right)} = \frac{\pi \in_0 \ell}{\ln\left(\frac{D-d}{d}\right)}$$

The capacitance per unit length is: $\boxed{\frac{C}{\ell} = \frac{\pi \in_0}{\ln\left(\frac{D-d}{d}\right)}}$

26.56 (a) We use Equation 26.11 to find the potential energy. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are

$$U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right) \quad \text{and} \quad U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$$

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00\times10^{-9}\text{ F})(100\text{ V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\text{ J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives $\Delta V_f = \kappa\Delta V_i = (5.00)(100\text{ V}) = \boxed{500\text{ V}}$

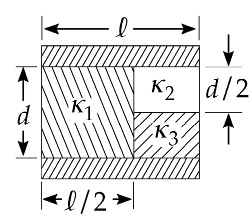
Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

26.57 $\kappa = 3.00$, $E_{\max} = 2.00\times10^8\text{ V/m} = \Delta V_{\max}/d$

For $C = \frac{\kappa e_0 A}{d} = 0.250\times10^{-6}\text{ F}$,

$$A = \frac{Cd}{\kappa e_0} = \frac{C(\Delta V_{\max})}{\kappa e_0 E_{\max}} = \frac{(0.250\times10^{-6})(4000)}{(3.00)(8.85\times10^{-12})(2.00\times10^8)} = \boxed{0.188\text{ m}^2}$$

26.58 (a) $C_1 = \frac{\kappa_1 e_0 A/2}{d}$; $C_2 = \frac{\kappa_2 e_0 A/2}{d/2}$; $C_3 = \frac{\kappa_3 e_0 A/2}{d/2}$

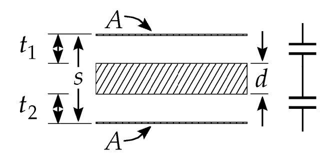


$$\left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{e_0 A}{d}\left(\frac{\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right)$$

$$C = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \boxed{\frac{e_0 A}{d}\left(\frac{\kappa_1}{2} + \frac{\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right)}$$

(b) Using the given values we find: $C_{\text{total}} = 1.76 \times 10^{-12}\ \text{F} = \boxed{1.76\ \text{pF}}$

26.59 The system may be considered to be two capacitors in series:



$$C_1 = \frac{\epsilon_0 A}{t_1} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{t_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t_1 + t_2}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{t_1 + t_2} = \boxed{\frac{\epsilon_0 A}{s - d}}$$

Goal Solution

A conducting slab of a thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as shown in Figure P26.59. The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

G: It is difficult to predict an exact relationship for the capacitance of this system, but we can reason that C should increase if the distance between the slab and plates were decreased (until they touched and formed a short circuit). So maybe $C \propto 1/(s-d)$. Moving the metal slab does not change the amount of charge the system can store, so the capacitance should therefore be independent of the slab position. The slab must have zero net charge, with each face of the plate holding the same magnitude of charge as the outside plates, regardless of where the slab is between the plates.

O: If the capacitor is charged with $+Q$ on the top plate and $-Q$ on the bottom plate, then free charges will move across the conducting slab to neutralize the electric field inside it, with the top face of the slab carrying charge $-Q$ and the bottom face carrying charge $+Q$. Then the capacitor and slab combination is electrically equivalent to two capacitors in series. (We are neglecting the slight fringing effect of the electric field near the edges of the capacitor.) Call x the upper gap, so that $s-d-x$ is the distance between the lower two surfaces.

A: For the upper capacitor, $C_1 = \epsilon_0 A/x$

and the lower has $C_2 = \dfrac{\epsilon_0 A}{s-d-x}$

So the combination has
$$C = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}} = \frac{1}{\dfrac{x}{\epsilon_0 A} + \dfrac{s-d-x}{\epsilon_0 A}} = \frac{\epsilon_0 A}{s-d}$$

L: The equivalent capacitance is inversely proportional to $(s-d)$ as expected, and is also proportional to A. This result is the same as for the special case in Example 26.9 when the slab is just halfway between the plates; the only critical factor is the thickness of the slab relative to the plate spacing.

26.60 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity,

the potential at the surface of a is $V_a = \frac{k_e Q}{a} - \frac{k_e Q}{d}$

and the potential of b is $V_b = \frac{-k_e Q}{b} + \frac{k_e Q}{d}$

The difference in potential is $V_a - V_b = \frac{k_e Q}{a} + \frac{k_e Q}{b} - \frac{k_e Q}{d} - \frac{k_e Q}{d}$

and $C = \frac{Q}{V_a - V_b} = \boxed{\left(\frac{4\pi\epsilon_0}{(1/a)+(1/b)-(2/d)}\right)}$

(b) As $d \to \infty$, $1/d$ becomes negligible compared to $1/a$. Then,

$$C = \frac{4\pi\epsilon_0}{1/a + 1/b} \quad \text{and} \quad \frac{1}{C} = \boxed{\frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}}$$

as for two spheres in series.

26.61 Note that the potential difference between the plates is held constant at ΔV_i by the battery.

$$C_i = \frac{q_0}{\Delta V_i} \quad \text{and} \quad C_f = \frac{q_f}{\Delta V_i} = \frac{q_0 + q}{\Delta V_i}$$

But $C_f = \kappa C_i$, so $\frac{q_0 + q}{\Delta V_i} = \kappa\left(\frac{q_0}{\Delta V_i}\right)$

Thus, $\kappa = \frac{q_0 + q}{q_0}$ or $\kappa = \boxed{1 + \frac{q}{q_0}}$

26.62 (a) $C = \frac{\epsilon_0}{d}\left[(\ell - x)\ell + \kappa\ell x\right] = \boxed{\frac{\epsilon_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \boxed{\frac{1}{2}\left(\frac{\epsilon_0(\Delta V)^2}{d}\right)\left[\ell^2 + \ell x(\kappa - 1)\right]}$

(c) $|\mathbf{F}| = \left|-\frac{dU}{dx}\right| =$ to the left

(d) $$F = \frac{(2000)^2(8.85\times10^{-12})(0.0500)(4.50-1)}{2(2.00\times10^{-3})} = \boxed{1.55\times10^{-3}\ \text{N}}$$

***26.63** The portion of the capacitor nearly filled by metal has capacitance $\kappa \in_0 (\ell x)/d \to \infty$ and stored energy $Q^2/2C \to 0$. The unfilled portion has capacitance $\in_0 \ell \; (\ell - x)/d$. The charge on this portion is $Q = (\ell - x)Q_0 / \ell$.

(a) The stored energy is $U = \frac{Q^2}{2C} = \frac{[(\ell - x)Q_0/\ell]^2}{2\in_0 \ell(\ell - x)/d} = \boxed{\frac{Q_0^2 d(\ell - x)}{2\in_0 \ell^3}}$

(b) $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2 d(\ell - x)}{2\in_0 \ell^3}\right) = +\frac{Q_0^2 d}{2\in_0 \ell^3}$

F = to the right

(c) $Stress = \frac{F}{\ell d} =$

(d) $u = \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\left(\frac{\sigma}{\in_0}\right)^2 = \frac{1}{2}\in_0\left(\frac{Q_0}{\in_0 \ell^2}\right)^2 =$

26.64 Gasoline: $\left(126\,000\ \frac{\text{Btu}}{\text{gal}}\right)\left(1054\ \frac{\text{J}}{\text{Btu}}\right)\left(\frac{1.00\ \text{gal}}{3.786\times 10^{-3}\ \text{m}^3}\right)\left(\frac{1.00\ \text{m}^3}{670\ \text{kg}}\right) = 5.25\times 10^7\ \frac{\text{J}}{\text{kg}}$

Battery: $\frac{(12.0\ \text{J/C})(100\ \text{C/s})(3600\ \text{s})}{16.0\ \text{kg}} = 2.70\times 10^5\ \text{J/kg}$

Capacitor: $\frac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}} = 72.0\ \text{J/kg}$

Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor

26.65 Call the unknown capacitance C_u

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

Goal Solution

An isolated capacitor of unknown capacitance has been charged to a potential difference of 100 V. When the charged capacitor is then connected in parallel to an uncharged 10.0-μF capacitor, the voltage across the combination is 30.0 V. Calculate the unknown capacitance.

G: The voltage of the combination will be reduced according to the size of the added capacitance. (Example: If the unknown capacitance were $C = 10.0\ \mu\text{F}$, then $\Delta V_1 = 50.0$ V because the charge is now distributed evenly between the two capacitors.) Since the final voltage is less than half the original, we might guess that the unknown capacitor is about $5.00\ \mu\text{F}$.

O: We can use the relationships for capacitors in parallel to find the unknown capacitance, along with the requirement that the charge on the unknown capacitor must be the same as the total charge on the two capacitors in parallel.

A: We name our ignorance and call the unknown capacitance C_u. The charge originally deposited on **each** plate, + on one, – on the other, is

$$Q = C_u \Delta V = C_u(100\ \text{V})$$

Now in the new connection this same conserved charge redistributes itself between the two capacitors according to $Q = Q_1 + Q_2$.

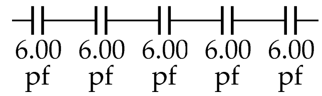
$$Q_1 = C_u(30.0\ \text{V}) \text{ and } \quad Q_2 = (10.0\ \mu\text{F})(30.0\ \text{V}) = 300\ \mu\text{C}$$

We can eliminate Q and Q_1 by substitution:

$$C_u(100\ \text{V}) = C_u(30.0\ \text{V}) + 300\ \mu\text{C} \qquad \text{so} \qquad C_u = \frac{300\ \mu\text{C}}{70.0\ \text{V}} = 4.29\ \mu\text{F}$$

L: The calculated capacitance is close to what we expected, so our result seems reasonable. In this and other capacitance combination problems, it is important not to confuse the charge and voltage of the system with those of the individual components, especially if they have different values. Careful attention must be given to the subscripts to avoid this confusion. It is also important to not confuse the variable "*C*" for capacitance with the unit of charge, "C" for coulombs.

26.66 Put five 6.00 pF capacitors in series.



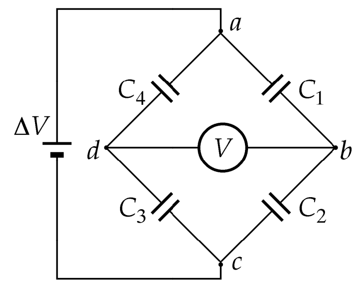
The potential difference across any one of the capacitors will be:

$$\Delta V = \frac{\Delta V_{\text{max}}}{5} = \frac{1000\ \text{V}}{5} = 200\ \text{V}$$

and the equivalent capacitance is:

$$\frac{1}{C_{\text{eq}}} = 5\left(\frac{1}{6.00\ \text{pF}}\right) \quad \text{or} \quad C_{\text{eq}} = \frac{6.00\ \text{pF}}{5} = 1.20\ \text{pF}$$

26.67 When $\Delta V_{db} = 0$, $\Delta V_{bc} = \Delta V_{dc}$, and $\dfrac{Q_2}{C_2} = \dfrac{Q_3}{C_3}$



Also, $\Delta V_{ba} = \Delta V_{da}$ or $\dfrac{Q_1}{C_1} = \dfrac{Q_4}{C_4}$

From these equations we have $C_2 = \left(\dfrac{C_3}{C_4}\right)\left(\dfrac{Q_2}{Q_1}\right)\left(\dfrac{Q_4}{Q_3}\right)C_1$

However, from the properties of capacitors in series, we have $Q_1 = Q_2$ and $Q_3 = Q_4$

Therefore, $$C_2 = \left(\frac{C_3}{C_4}\right)C_1 = \frac{9.00}{12.0}(4.00\ \mu F) = \boxed{3.00\ \mu\text{F}}$$

26.68 Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\,\Delta V_{\text{chg}} = (5.00 \times 10^{-4}\ \text{F})(800\ \text{V}) = 0.400\ \text{C}$$

While being discharged in series,

$$\Delta V_{\text{disch}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\ \text{C}}{5.00 \times 10^{-5}\ \text{F}} = \boxed{8.00\ \text{kV}}$$

or 10 times the original voltage.

26.69 (a) $C_0 = \dfrac{\epsilon_0 A}{d} = \dfrac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}; \qquad U_0 = \frac{C_0(\Delta V_0)^2}{2}$$

$$U = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0)^2}{2} \quad \text{and} \quad \frac{U}{U_0} = \kappa$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0 = C_0 \Delta V_0$ and $Q = C \Delta V_0 = \kappa C_0 \Delta V_0$ so $\boxed{Q / Q_0 = \kappa}$

26.70 (a) A slice of width (dx) at coordinate x in $0 \le x \le L$ has thickness xd/L filled with dielectric κ_2, and $d - xd/L$ is filled with the material having constant κ_1. This slice has a capacitance given by

$$\frac{1}{dC} = \frac{1}{\left(\frac{\kappa_2 \epsilon_0 (dx) W}{xd/L}\right)} + \frac{1}{\left(\frac{\kappa_1 \epsilon_0 (dx) W}{d - xd/L}\right)} = \frac{xd}{\kappa_2 \epsilon_0 WL(dx)} + \frac{dL - xd}{\kappa_1 \epsilon_0 WL(dx)} = \frac{\kappa_1 xd + \kappa_2 dL - \kappa_2 xd}{\kappa_1 \kappa_2 \epsilon_0 WL(dx)}$$

$$dC = \frac{\kappa_1 \kappa_2 \epsilon_0 WL(dx)}{\kappa_2 dL + (\kappa_1 - \kappa_2) xd}$$

The whole capacitor is all the slices in parallel:

$$C = \int dC = \int_{x=0}^{L} \frac{\kappa_1 \kappa_2 \epsilon_0 WL(dx)}{\kappa_2 Ld + (\kappa_1 - \kappa_2) xd} = \frac{\kappa_1 \kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2) d} \int_{x=0}^{L} \left(\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right)^{-1} (\kappa_1 - \kappa_2) d(dx)$$

$$C = \frac{\kappa_1 \kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2) d} \ln\left[\kappa_2 Ld + (\kappa_1 - \kappa_2) xd\right]\Big|_0^L = \frac{\kappa_1 \kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2) d} \left[\ln \kappa_1 Ld - \ln \kappa_2 Ld\right] = \boxed{\frac{\kappa_1 \kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2) d} \ln \frac{\kappa_1}{\kappa_2}}$$

(b) To take the limit $\kappa_1 \to \kappa_2$, write $\kappa_1 = \kappa_2 (1 + x)$ and let $x \to 0$. Then

$$C = \frac{\kappa_2^2 (1 + x) \epsilon_0 WL}{(\kappa_2 + \kappa_2 x - \kappa_2) d} \ln(1 + x)$$

Use the expansion of $\ln(1 + x)$ from Appendix B.5.

$$C = \frac{\kappa_2^2 (1 + x) \epsilon_0 WL}{\kappa_2 xd}\left(x - \tfrac{1}{2} x^2 + \tfrac{1}{3} x^3 \ldots\right) = \frac{\kappa_2 (1 + x) \epsilon_0 WL}{d}\left(1 - \tfrac{1}{2} x + \ldots\right)$$

$$\lim_{x \to 0} C = \frac{\kappa_2 \epsilon_0 WL}{d} = \boxed{\frac{\kappa \epsilon_0 A}{d}}$$

26.71 The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_p = \frac{\epsilon_0 (A/2)}{d} + \frac{\kappa \epsilon_0 (A/2)}{d} = \left(\frac{\kappa + 1}{2}\right)\frac{\epsilon_0 A}{d}$$

where A is the area of either plate and d is the separation of the plates. The horizontal orientation produces two capacitors in series. If f is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa \epsilon_0 A} + \frac{(1 - f) d}{\epsilon_0 A} = \left[\frac{f + \kappa(1 - f)}{\kappa}\right]\frac{d}{\epsilon_0 A}, \quad \text{or} \quad C_s = \left[\frac{\kappa}{f + \kappa(1 - f)}\right]\frac{\epsilon_0 A}{d}$$

Requiring that $C_p = C_s$ gives $\dfrac{\kappa + 1}{2} = \dfrac{\kappa}{f + \kappa(1 - f)}$, or $(\kappa + 1)\left[f + \kappa(1 - f)\right] = 2\kappa$

For $\kappa = 2.00$, this yields $3.00\left[2.00 - (1.00) f\right] = 4.00$, with the solution $f = \boxed{2/3}$.

26.72 Initially (capacitors charged in parallel),

$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1500\ \mu\text{C}$

$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$

After reconnection (positive plate to negative plate),

$q'_{\text{total}} = q_1 - q_2 = 1000\ \mu\text{C}$ and $\Delta V' = \dfrac{q'_{\text{total}}}{C_{\text{total}}} = \dfrac{1000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$

Therefore,

$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$

$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$

26.73 E_{max} occurs at the inner conductor's surface.

$E_{\text{max}} = \dfrac{2k_e\lambda}{a}$ from Equation 24.7.

$\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$ from Example 26.2

$$E_{\text{max}} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\text{max}} = E_{\text{max}} a \ln\left(\frac{b}{a}\right) = (18.0\times 10^{6}\ \text{V/m})(0.800\times 10^{-3}\ \text{m})\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0\ \text{kV}}$$

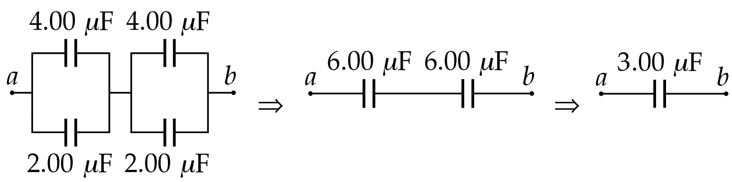
26.74 $E = \dfrac{2\kappa\lambda}{a}$; $\Delta V = 2\kappa\lambda \ln\left(\dfrac{b}{a}\right)$

$$\Delta V_{\text{max}} = E_{\text{max}} a \ln\left(\frac{b}{a}\right)$$

$$\frac{dV_{\text{max}}}{da} = E_{\text{max}}\left[\ln\left(\frac{b}{a}\right) + a\left(\frac{1}{b/a}\right)\left(-\frac{b}{a^2}\right)\right] = 0$$

$\ln\left(\dfrac{b}{a}\right) = 1$ or $\dfrac{b}{a} = e^1$ so $\boxed{a = \dfrac{b}{e}}$

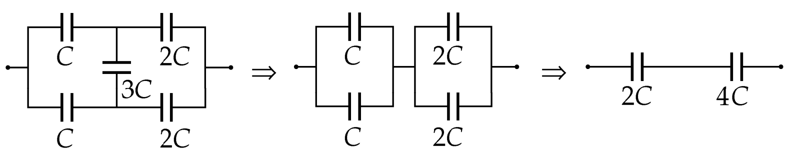
26.75 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:



$C_{ab} = \boxed{3.00\ \mu\text{F}}$

26.76 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to

$$C_{\text{eq}} = \frac{(2C)(4C)}{2C + 4C} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$



Chapter 27 Solutions

27.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\qquad \Delta Q = I\,\Delta t = (30.0 \times 10^{-6}\text{ A})(40.0\text{ s}) = 1.20 \times 10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20 \times 10^{-3}\text{ C}}{1.60 \times 10^{-19}\text{ C/electron}} = \boxed{7.50 \times 10^{15}\text{ electrons}}$$

***27.2** The atomic weight of silver = 107.9, and the volume V is

$V = (\text{area})(\text{thickness}) = (700 \times 10^{-4}\text{ m}^2)(0.133 \times 10^{-3}\text{ m}) = 9.31 \times 10^{-6}\text{ m}^3$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5 \times 10^{3}\text{ kg / m}^3\right)\left(9.31 \times 10^{-6}\text{ m}^3\right) = 9.78 \times 10^{-2}\text{ kg}.$

and the number of silver atoms deposited is

$$N = (9.78 \times 10^{-2}\text{ kg})\frac{6.02 \times 10^{26}\text{ atoms}}{107.9\text{ kg}} = 5.45 \times 10^{23}$$

$$I = \frac{V}{R} = \frac{12.0\text{ V}}{1.80\ \Omega} = 6.67\text{ A} = 6.67\text{ C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{(5.45 \times 10^{23})(1.60 \times 10^{-19}\text{ C})}{6.67\text{ C/s}} = 1.31 \times 10^{4}\text{ s} = \boxed{3.64\text{ h}}$$

27.3 $Q(t) = \int_0^t I\,dt = I_0\tau(1 - e^{-t/\tau})$

(a) $Q(\tau) = I_0\tau(1 - e^{-1}) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau(1 - e^{-10}) = \boxed{(0.99995)I_0\tau}$

(c) $Q(\infty) = I_0\tau(1 - e^{-\infty}) = \boxed{I_0\tau}$

27.4 (a) Using $\dfrac{k_e e^2}{r^2} = \dfrac{mv^2}{r}$, we get: $\quad v = \sqrt{\dfrac{k_e e^2}{mr}} = \boxed{2.19 \times 10^{6}\text{ m / s}}$.

(b) The time for the electron to revolve around the proton once is:

$$t = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11}\text{ m})}{(2.19 \times 10^{6}\text{ m / s})} = 1.52 \times 10^{-16}\text{ s}$$

The total charge flow in this time is 1.60×10^{-19} C, so the current is

$$I = \frac{1.60 \times 10^{-19}\ \text{C}}{1.52 \times 10^{-16}\ \text{s}} = 1.05 \times 10^{-3}\ \text{A} = \boxed{1.05\ \text{mA}}$$

27.5 $\omega = \dfrac{2\pi}{T}$ where T is the period.

$$I = \frac{q}{T} = \frac{q\omega}{2\pi} = \frac{(8.00\times10^{-9}\text{ C})(100\pi\text{ rad / s})}{2\pi} = 4.00\times10^{-7}\text{ A} = \boxed{400\text{ nA}}$$

27.6 The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

27.7 $q = 4t^3 + 5t + 6$ $\quad A = (2.00\text{ cm}^2)\left(\dfrac{1.00\text{ m}}{100\text{ cm}}\right)^2 = 2.00\times10^{-4}\text{ m}^2$

(a) $I(1.00\text{ s}) = \left.\dfrac{dq}{dt}\right|_{t=1.00\text{ s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\text{ s}} = \boxed{17.0\text{ A}}$

(b) $J = \dfrac{I}{A} = \dfrac{17.0\text{ A}}{2.00\times10^{-4}\text{ m}^2} = \boxed{85.0\text{ kA / m}^2}$

27.8 $I = \dfrac{dq}{dt}$

$$q = \int dq = \int I\,dt = \int_0^{1/240\text{ s}} (100\text{ A})\sin(120\pi t\text{ / s})\,dt$$

$$q = \frac{-100\text{ C}}{120\pi}\left[\cos(\pi / 2) - \cos 0\right] = \frac{+100\text{ C}}{120\pi} = \boxed{0.265\text{ C}}$$

27.9 (a) $J = \dfrac{I}{A} = \dfrac{5.00\text{ A}}{\pi(4.00\times10^{-3}\text{ m})^2} = \boxed{99.5\text{ kA/m}^2}$

(b) $J_2 = \dfrac{1}{4}J_1$; $\quad \dfrac{I}{A_2} = \dfrac{1}{4}\dfrac{I}{A_1}$

$A_1 = \dfrac{1}{4}A_2$ so $\quad \pi(4.00\times10^{-3})^2 = \dfrac{1}{4}\pi r_2^2$

$r_2 = 2(4.00\times10^{-3}) = 8.00\times10^{-3}\text{ m} = \boxed{8.00\text{ mm}}$

27.10 (a) The speed of each deuteron is given by $K = \frac{1}{2} mv^2$

$(2.00 \times 10^6)(1.60 \times 10^{-19}\ \text{J}) = \frac{1}{2}(2 \times 1.67 \times 10^{-27}\ \text{kg})\, v^2$ and $v = 1.38 \times 10^7\ \text{m/s}$

The time between deuterons passing a stationary point is t in $I = q/t$

$10.0 \times 10^{-6}\ \text{C/s} = 1.60 \times 10^{-19}\ \text{C}/t$ or $t = 1.60 \times 10^{-14}\ \text{s}$

So the distance between them is $vt = \left(1.38 \times 10^7\ \text{m/s}\right)\left(1.60 \times 10^{-14}\ \text{s}\right) = \boxed{2.21 \times 10^{-7}\ \text{m}}$

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})}{2.21 \times 10^{-7}\ \text{m}} = \boxed{6.49 \times 10^{-3}\ \text{V}}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

27.11 (a) $$J = \frac{I}{A} = \frac{8.00 \times 10^{-6}\ \text{A}}{\pi\left(1.00 \times 10^{-3}\ \text{m}\right)^2} = \boxed{2.55\ \text{A/m}^2}$$

(b) From $J = nev_d$, we have $$n = \frac{J}{ev_d} = \frac{2.55\ \text{A/m}^2}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(3.00 \times 10^8\ \text{m/s}\right)} = \boxed{5.31 \times 10^{10}\ \text{m}^{-3}}$$

(c) From $I = \Delta Q/\Delta t$, we have $$\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)}{8.00 \times 10^{-6}\ \text{A}} = \boxed{1.20 \times 10^{10}\ \text{s}}$$

(This is about 381 years!)

***27.12** We use $I = nqAv_d$ where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume). We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molecular weight of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\ \text{g}}{N_A} = \frac{27.0\ \text{g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23}\ \text{g/atom}$$

Thus, $$n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\ \text{g/cm}^3}{4.49 \times 10^{-23}\ \text{g/atom}}$$

$$n = 6.02 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3} = 6.02 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}$$

Therefore, $$v_d = \frac{I}{nqA} = \frac{5.00\ \text{A}}{(6.02 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(4.00 \times 10^{-6}\ \text{m}^2)} = 1.30 \times 10^{-4}\ \text{m/s}$$

or, $\boxed{v_d = 0.130\ \text{mm/s}}$

***27.13** $I = \dfrac{\Delta V}{R} = \dfrac{120 \text{ V}}{240\ \Omega} = 0.500 \text{ A} = \boxed{500 \text{ mA}}$

27.14 (a) Applying its definition, we find the resistance of the rod,

$$R = \frac{\Delta V}{I} = \frac{15.0 \text{ V}}{4.00 \times 10^{-3} \text{ A}} = 3750\ \Omega = \boxed{3.75 \text{ k}\Omega}$$

(b) The length of the rod is determined from Equation 27.11: $R = \rho\ell / A$. Solving for ℓ and substituting numerical values for R, A, and the values of ρ given for carbon in Table 27.1, we obtain

$$\ell = \frac{RA}{\rho} = \frac{(3.75 \times 10^{3}\ \Omega)(5.00 \times 10^{-6} \text{ m}^2)}{(3.50 \times 10^{-5}\ \Omega \cdot \text{m})} = \boxed{536 \text{ m}}$$

27.15 $\Delta V = IR$ and $R = \dfrac{\rho l}{A}$: $A = 0.600 \text{ mm}^2 \left(\dfrac{1.00 \text{ m}}{1000 \text{ mm}}\right)^2 = 6.00 \times 10^{-7} \text{ m}^2$

$\Delta V = \dfrac{I\rho l}{A}$: $I = \dfrac{\Delta V A}{\rho l} = \dfrac{(0.900 \text{ V})(6.00 \times 10^{-7} \text{ m}^2)}{(5.60 \times 10^{-8}\ \Omega \cdot \text{m})(1.50 \text{ m})}$

$I = \boxed{6.43 \text{ A}}$

27.16 $J = \dfrac{I}{\pi r^2} = \sigma E = \dfrac{3.00 \text{ A}}{\pi (0.0120 \text{ m})^2} = \sigma (120 \text{ N/C})$

$\sigma = 55.3(\Omega \cdot \text{m})^{-1}$ $\rho = \dfrac{1}{\sigma} = \boxed{0.0181\ \Omega \cdot \text{m}}$

27.17 (a) Given $M = \rho_d V = \rho_d A l$ where $\rho_d \equiv$ mass density, we obtain: $A = \dfrac{M}{\rho_d l}$

Taking $\rho_r \equiv$ resistivity, $R = \dfrac{\rho_r l}{A} = \dfrac{\rho_r l}{\left(\dfrac{M}{\rho_d l}\right)} = \dfrac{\rho_r \rho_d l^2}{M}$

Thus, $l = \sqrt{\dfrac{MR}{\rho_r \rho_d}} = \sqrt{\dfrac{\left(1.00 \times 10^{-3}\right)(0.500)}{(1.70 \times 10^{-8})(8.92 \times 10^{3})}} = \boxed{1.82 \text{ m}}$

(b) $V = \frac{M}{\rho_d}$, or $\pi r^2 \ell = \frac{M}{\rho_d}$

Thus, $r = \sqrt{\frac{M}{\pi \rho_d \ell}} = \sqrt{\frac{1.00 \times 10^{-3}}{\pi (8.92 \times 10^3)(1.82)}} = 1.40 \times 10^{-4}\ \text{m}$

The diameter is twice this distance: $\boxed{\text{diameter} = 280\ \mu\text{m}}$

***27.18** (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho \ell}{A} = \frac{4\rho \ell}{\pi d^2} \sim \frac{4\left(10^{13}\ \Omega \cdot \text{m}\right)\left(10^{-1}\ \text{m}\right)}{\pi \left(10^{-3}\ \text{m}\right)^2} = \boxed{\sim 10^{18}\ \Omega}$$

(b) $R = \frac{4\rho \ell}{\pi d^2} \sim \frac{4(1.7 \times 10^{-8}\ \Omega \cdot \text{m})(10^{-3}\ \text{m})}{\pi (2 \times 10^{-2}\ \text{m})^2}\ \boxed{\sim 10^{-7}\ \Omega}$

(c) $I = \frac{\Delta V}{R} \sim \frac{10^2\ \text{V}}{10^{18}\ \Omega}\ \boxed{\sim 10^{-16}\ \text{A}}$

$I \sim \frac{10^2\ \text{V}}{10^{-7}\ \Omega}\ \boxed{\sim 10^9\ \text{A}}$

27.19 The distance between opposite faces of the cube is $\ell = \left(\frac{90.0\ \text{g}}{10.5\ \text{g/cm}^3}\right)^{1/3} = 2.05\ \text{cm}$

(a) $R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\ell^2} = \frac{\rho}{\ell} = \frac{1.59 \times 10^{-8}\ \Omega \cdot \text{m}}{2.05 \times 10^{-2}\ \text{m}} = 7.77 \times 10^{-7}\ \Omega = \boxed{777\ \text{n}\Omega}$

(b) $I = \frac{\Delta V}{R} = \frac{1.00 \times 10^{-5}\ \text{V}}{7.77 \times 10^{-7}\ \Omega} = 12.9\ \text{A}$

$$n = \frac{10.5\ \text{g/cm}^3}{107.87\ \text{g/mol}}\left(6.02 \times 10^{23}\ \frac{\text{electrons}}{\text{mol}}\right)$$

$$n = \left(5.86 \times 10^{22}\ \frac{\text{electrons}}{\text{cm}^3}\right)\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1.00\ \text{m}^3}\right) = 5.86 \times 10^{28}/\text{m}^3$$

$I = nqvA$ and $v = \frac{I}{nqA} = \frac{12.9\ \text{C/s}}{\left(5.86 \times 10^{28}/\text{m}^3\right)\left(1.60 \times 10^{-19}\ \text{C}\right)(0.0205\ \text{m})^2} = \boxed{3.28\ \mu\text{m/s}}$

27.20 Originally, $R = \frac{\rho \ell}{A}$

Finally, $R_f = \frac{\rho(\ell / 3)}{3A} = \frac{\rho \ell}{9A} = \boxed{\frac{R}{9}}$

27.21 The total volume of material present does not change, only its shape. Thus,

$A_f \ell_f = A_f(1.25 \ell_i) = A_i \ell_i$ giving $A_f = A_i / 1.25$

The final resistance is then: $R_f = \frac{\rho \ell_f}{A_f} = \frac{\rho(1.25 \ell_i)}{A_i / 1.25} = 1.56\left(\frac{\rho \ell_i}{A_i}\right) = \boxed{1.56R}$

27.22 $\frac{\rho_{Al} \ell}{\pi(r_{Al})^2} = \frac{\rho_{Cu} \ell}{\pi(r_{Cu})^2}$

$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82 \times 10^{-8}}{1.70 \times 10^{-8}}} = \boxed{1.29}$

27.23 $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00 \times 10^{-13} \text{ A / m}^2}{100 \text{ V / m}} = \boxed{6.00 \times 10^{-15} \ (\Omega \cdot \text{m})^{-1}}$

27.24 $R = \frac{\rho_1 \ell_1}{A_1} + \frac{\rho_2 \ell_2}{A_2} = (\rho_1 \ell_1 + \rho_2 \ell_2) / d^2$

$$R = \frac{(4.00 \times 10^{-3} \ \Omega \cdot \text{m})(0.250 \text{ m}) + (6.00 \times 10^{-3} \ \Omega \cdot \text{m})(0.400 \text{ m})}{(3.00 \times 10^{-3} \text{ m})^2} = \boxed{378 \ \Omega}$$

27.25 $\rho = \frac{m}{nq^2 \tau}$ so $\tau = \frac{m}{\rho n q^2} = \frac{9.11 \times 10^{-31}}{(1.70 \times 10^{-8})(8.49 \times 10^{28})(1.60 \times 10^{19})^2} = 2.47 \times 10^{-14} \text{ s}$

$v_d = \frac{qE}{m} \tau$ so $7.84 \times 10^{-4} = \frac{(1.60 \times 10^{-19}) E (2.47 \times 10^{-14})}{9.11 \times 10^{-31}}$

Therefore $\boxed{E = 0.181 \text{ V/m}}$

Goal Solution

If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m / s, what is the electric field in the conductor?

G: For electrostatic cases, we learned that the electric field inside a conductor is always zero. On the other hand, if there is a current, a non-zero electric field must be maintained by a battery or other source to make the charges flow. Therefore, we might expect the electric field to be small, but definitely **not** zero.

O: The drift velocity of the electrons can be used to find the current density, which can be used with Ohm's law to find the electric field inside the conductor.

A: We first need the electron density in copper, which from Example 27.1 is $n=8.49\times10^{28}\ \text{e}^-/\text{m}^3$. The current density in this wire is then

$$J=nqv_d=(8.49\times10^{28}\ \text{e}^-/\text{m}^3)(1.60\times10^{-19}\ \text{C}/\text{e}^-)(7.84\times10^{-4}\ \text{m/s})=1.06\times10^{7}\ \text{A/m}^2$$

Ohm's law can be stated as $J=\sigma E=E/\rho$ where $\rho=1.7\times10^{-8}\ \Omega\cdot\text{m}$ for copper, so then

$$E=\rho J=(1.70\times10^{-8}\ \Omega\cdot\text{m})(1.06\times10^{7}\ \text{A/m}^2)=0.181\ \text{V/m}$$

L: This electric field is certainly smaller than typical static values outside charged objects. The direction of the electric field should be along the length of the conductor, otherwise the electrons would be forced to leave the wire! The reality is that excess charges arrange themselves on the surface of the wire to create an electric field that "steers" the free electrons to flow along the length of the wire from low to high potential (opposite the direction of a positive test charge). It is also interesting to note that when the electric field is being established it travels at the speed of light; but the drift velocity of the electrons is literally at a "snail's pace"!

27.26 (a) n is $\boxed{\text{unaffected}}$

(b) $|J|=\dfrac{I}{A}\propto I$ so it $\boxed{\text{doubles}}$

(c) $J=nev_d$ so v_d $\boxed{\text{doubles}}$

(d) $\tau=\dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to heating.

27.27 From Equation 27.17,

$$\tau=\frac{m_e}{nq^2\rho}=\frac{9.11\times10^{-31}}{\left(8.49\times10^{28}\right)\left(1.60\times10^{-19}\right)^2\left(1.70\times10^{-8}\right)}=2.47\times10^{-14}\ \text{s}$$

$$l=v\tau=\left(8.60\times10^{5}\ \text{m/s}\right)\left(2.47\times10^{-14}\ \text{s}\right)=2.12\times10^{-8}\ \text{m}=\boxed{21.2\ \text{nm}}$$

27.28 At the low temperature T_C we write $R_C = \dfrac{\Delta V}{I_C} = R_0\left[1+\alpha(T_C - T_0)\right]$ where $T_0 = 20.0°\text{C}$

At the high temperature T_h, $R_h = \dfrac{\Delta V}{I_h} = \dfrac{\Delta V}{1\ \text{A}} = R_0\left[1+\alpha(T_h - T_0)\right]$

Then $$\frac{(\Delta V)/(1.00\ \text{A})}{(\Delta V)/I_C} = \frac{1 + (3.90\times10^{-3})(38.0)}{1 + (3.90\times10^{-3})(-108)}$$

and $I_C = (1.00\ \text{A})(1.15/0.579) = \boxed{1.98\ \text{A}}$

***27.29** $R = R_0\left[1+\alpha(\Delta T)\right]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+(4.50\times10^{-3}/°\text{C})\Delta T\right]$

Solving, $\Delta T = 1.42\times10^3\ °\text{C} = T - 20.0\ °\text{C}$

And, the final temperature is $\boxed{T = 1.44\times10^3\ °\text{C}}$

27.30 $R = R_c + R_n = R_c\left[1+\alpha_c(T-T_0)\right] + R_n\left[1+\alpha_n(T-T_0)\right]$

$0 = R_c\alpha_c\,(T-T_0) + R_n\alpha_n\,(T-T_0)$ so $R_c = -R_n\dfrac{\alpha_n}{\alpha_c}$

$$R = -R_n\frac{\alpha_n}{\alpha_c} + R_n$$

$R_n = R(1-\alpha_n/\alpha_c)^{-1}$ $R_c = R(1-\alpha_c/\alpha_n)^{-1}$

$$R_n = 10.0\ \text{k}\Omega\left[1 - \frac{(0.400\times10^{-3}/\text{C}°)}{(-0.500\times10^{-3}/\text{C}°)}\right]^{-1}$$

$\boxed{R_n = 5.56\ \text{k}\Omega}$ and $\boxed{R_c = 4.44\ \text{k}\Omega}$

27.31 (a) $\rho = \rho_0\left[1+\alpha(T-T_0)\right] = (2.82\times10^{-8}\ \Omega\cdot\text{m})\left[1+3.90\times10^{-3}(30.0°)\right] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^6\ \text{A/m}^2}$

(c) $I = JA = \dfrac{\pi d^2}{4}J = \dfrac{\pi(1.00\times10^{-4}\ \text{m})^2}{4}(6.35\times10^6\ \text{A/m}^2) = \boxed{49.9\ \text{mA}}$

(d) $$n = \frac{6.02\times10^{23}\ \text{electrons}}{\left(\dfrac{26.98\ \text{g}}{2.70\times10^6\ \text{g/m}^3}\right)} = 6.02\times10^{28}\ \text{electrons/m}^3$$

$$v_d = \frac{J}{ne} = \frac{(6.35\times10^6\ \text{A/m}^2)}{(6.02\times10^{28}\ \text{electrons/m}^3)(1.60\times10^{-19}\ \text{C})} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$

***27.32** For aluminum, $\alpha_E = 3.90 \times 10^{-3} / °C$ (Table 27.1) $\alpha = 24.0 \times 10^{-6} / °C$ (Table 19.2)

$$R = \frac{\rho l}{A} = \frac{\rho_0(1+\alpha_E \Delta T)(1+\alpha \Delta T)}{A(1+\alpha \Delta T)^2} = R_0 \frac{(1+\alpha_E \Delta T)}{(1+\alpha \Delta T)} = (1.234\ \Omega)\frac{(1.39)}{(1.0024)} = \boxed{1.71\ \Omega}$$

27.33 $R = R_0[1 + \alpha \Delta T]$

$R - R_0 = R_0 \alpha \Delta T$

$$\frac{R - R_0}{R_0} = \alpha \Delta T = (5.00 \times 10^{-3})25.0 = \boxed{0.125}$$

27.34 Assuming linear change of resistance with temperature, $R = R_0(1 + \alpha \Delta T)$

$$R_{77\,K} = (1.00\ \Omega)\left[1 + \left(3.92 \times 10^{-3}\right)(-216°\text{C})\right] = \boxed{0.153\ \Omega}$$

27.35 $\rho = \rho_0(1 + \alpha \Delta T)$ or $\Delta T_W = \dfrac{1}{\alpha_W}\left(\dfrac{\rho_W}{\rho_{0W}} - 1\right)$

Require that $\rho_W = 4\rho_{0_{\text{Cu}}}$ so that $\Delta T_W = \left(\dfrac{1}{4.50 \times 10^{-3}\ /°\text{C}}\right)\left(\dfrac{4\left(1.70 \times 10^{-8}\right)}{5.60 \times 10^{-8}} - 1\right) = 47.6\ °\text{C}$

Therefore, $T_W = 47.6\ °\text{C} + T_0 = \boxed{67.6°\text{C}}$

27.36

$$\alpha = \frac{1}{R_0}\left(\frac{\Delta R}{\Delta T}\right) = \left(\frac{1}{R_0}\right)\frac{2R_0 - R_0}{T - T_0} = \frac{1}{T - T_0}$$

so, $T = \left(\dfrac{1}{\alpha}\right) + T_0$ and $T = \left(\dfrac{1}{0.400 \times 10^{-3}\ \text{C°}^{-1}}\right) + 20.0\ °\text{C}$ so $T = \boxed{2.52 \times 10^3\ °\text{C}}$

***27.37**

$$I = \frac{P}{\Delta V} = \frac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}$

27.38 $\mathcal{P} = 0.800(1500\text{ hp})(746\text{ W/hp}) = 8.95 \times 10^5\text{ W}$

$\mathcal{P} = I(\Delta V)$

$8.95 \times 10^5 = I(2000)$

$\boxed{I = 448\text{ A}}$

27.39 The heat that must be added to the water is

$Q = mc\,\Delta T = (1.50\text{ kg})(4186\text{ J/kg°C})(40.0°\text{C}) = 2.51 \times 10^5\text{ J}$

Thus, the power supplied by the heater is

$$\mathcal{P} = \frac{W}{t} = \frac{Q}{t} = \frac{2.51 \times 10^5\text{ J}}{600\text{ s}} = 419\text{ W}$$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(110\text{ V})^2}{419\text{ W}} = \boxed{28.9\ \Omega}$

27.40 The heat that must be added to the water is $Q = mc(T_2 - T_1)$

Thus, the power supplied by the heat is $\mathcal{P} = \dfrac{W}{\Delta t} = \dfrac{Q}{\Delta t} = \dfrac{mc(T_2 - T_1)}{t}$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \boxed{\dfrac{(\Delta V)^2 t}{mc(T_2 - T_1)}}$

27.41 $$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2 / R}{(\Delta V_0)^2 / R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P} - \mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0} - 1\right)(100\%) = (1.361 - 1)100 = \boxed{36.1\%}$$

Goal Solution

Suppose that a voltage surge produces 140 V for a moment. By what percentage does the power output of a 120-V, 100-W light bulb increase? (Assume that its resistance does not change.)

G: The voltage increases by about 20%, but since $\mathcal{P} = (\Delta V)^2 / R$, the power will increase as the square of the voltage:

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \frac{(\Delta V_f)^2/R}{(\Delta V_i)^2/R} = \frac{(140\text{ V})^2}{(120\text{ V})^2} = 1.361 \text{ or a } 36.1\% \text{ increase.}$$

O: We have already found an answer to this problem by reasoning in terms of ratios, but we can also calculate the power explicitly for the bulb and compare with the original power by using Ohm's law and the equation for electrical power. To find the power, we must first find the resistance of the bulb, which should remain relatively constant during the power surge (we can check the validity of this assumption later).

A: From $\mathcal{P} = (\Delta V)^2 / R$, we find that $R = \frac{(\Delta V_i)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\ \Omega$

The final current is, $I_f = \frac{\Delta V_f}{R} = \frac{140\text{ V}}{144\ \Omega} = 0.972\text{ A}$

The power during the surge is $\mathcal{P}_f = \frac{(\Delta V_f)^2}{R} = \frac{(140\text{ V})^2}{144\ \Omega} = 136\text{ V}$

So the percentage increase is $\frac{136\text{ W} - 100\text{ W}}{100\text{ W}} = 0.361 = 36.1\%$

L: Our result tells us that this 100-W light bulb momentarily acts like a 136-W light bulb, which explains why it would suddenly get brighter. Some electronic devices (like computers) are sensitive to voltage surges like this, which is the reason that **surge protectors** are recommended to protect these devices from being damaged.

In solving this problem, we assumed that the resistance of the bulb did not change during the voltage surge, but we should check this assumption. Let us assume that the filament is made of tungsten and that its resistance will change linearly with temperature according to equation 27.21. Let us further assume that the increased voltage lasts for a time long enough so that the filament comes to a new equilibrium temperature. The temperature change can be estimated from the power surge according to Stefan's law (equation 20.18), assuming that all the power loss is due to radiation. By this law, $T \propto \sqrt[4]{\mathcal{P}}$ so that a 36% change in power should correspond to only about a 8% increase in temperature. A typical operating temperature of a white light bulb is about 3000 °C, so $\Delta T \approx 0.08(3273\ °\text{C}) = 260\ °\text{C}$. Then the increased resistance would be roughly

$$R = R_0\left(1 + \alpha(T - T_0)\right) = (144\ \Omega)\left(1 + 4.5 \times 10^{-3}(260)\right) \cong 310\ \Omega$$

It appears that the resistance could change double from 144 Ω. On the other hand, if the voltage surge lasts only a very short time, the 136 W we calculated originally accurately describes the conversion of electrical into internal energy in the filament.

27.42 $\mathcal{P} = I(\Delta V) = \dfrac{(\Delta V)^2}{R} = 500\text{ W}$

$$R = \frac{(110\text{ V})^2}{(500\text{ W})} = 24.2\ \Omega$$

(a) $R = \dfrac{\rho}{A}\ell$ so $\ell = \dfrac{RA}{\rho} = \dfrac{(24.2\ \Omega)\pi(2.50\times10^{-4}\text{ m})^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}} = \boxed{3.17\text{ m}}$

(b) $R = R_0[1 + \alpha\Delta T] = 24.2\ \Omega\left[1+(0.400\times10^{-3})(1180)\right] = 35.6\ \Omega$

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{(110)^2}{35.6} = \boxed{340\text{ W}}$$

27.43 $R = \dfrac{\rho\ell}{A} = \dfrac{(1.50\times10^{-6}\ \Omega\cdot\text{m})25.0\text{ m}}{\pi(0.200\times10^{-3}\text{ m})^2} = 298\ \Omega$

$$\Delta V = IR = (0.500\text{ A})(298\ \Omega) = 149\text{ V}$$

(a) $E = \dfrac{\Delta V}{\ell} = \dfrac{149\text{ V}}{25.0\text{ m}} = \boxed{5.97\text{ V/m}}$

(b) $\mathcal{P} = (\Delta V)I = (149\text{ V})(0.500\text{ A}) = \boxed{74.6\text{ W}}$

(c) $R = R_0\left[1+\alpha(T-T_0)\right] = 298\ \Omega\left[1+(0.400\times10^{-3}\text{ / C°})320\text{ C°}\right] = 337\ \Omega$

$$I = \frac{\Delta V}{R} = \frac{(149\text{ V})}{(337\ \Omega)} = 0.443\text{ A}$$

$$\mathcal{P} = (\Delta V)I = (149\text{ V})(0.443\text{ A}) = \boxed{66.1\text{ W}}$$

27.44 (a) $\Delta U = q(\Delta V) = It(\Delta V) = (55.0\text{ A}\cdot\text{h})(12.0\text{ V})\left(\dfrac{1\text{ C}}{1\text{ A}\cdot\text{s}}\right)\left(\dfrac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right)\left(\dfrac{1\text{ W}\cdot\text{s}}{1\text{ J}}\right) = 660\text{ W}\cdot\text{h} = \boxed{0.660\text{ kWh}}$

(b) $\text{Cost} = 0.660\text{ kWh}\left(\dfrac{\$0.0600}{1\text{ kWh}}\right) = \boxed{3.96¢}$

27.45 $\mathcal{P} = I(\Delta V)$ $\Delta V = IR$

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{(10.0)^2}{120} = \boxed{0.833\text{ W}}$$

27.46 The total clock power is $\left(270\times10^{6}\text{ clocks}\right)\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)=2.43\times10^{12}\text{ J/h}$

From $e=\frac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{t}=\frac{W_{\text{out}}/t}{e}=\frac{2.43\times10^{12}\text{ J/h}}{0.250}=9.72\times10^{12}\text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate}=\left(9.72\times10^{12}\text{ J/h}\right)\left(\frac{1.00\text{ kg coal}}{33.0\times10^{6}\text{ J}}\right)=2.95\times10^{5}\ \frac{\text{kg coal}}{\text{h}}=\boxed{295\ \frac{\text{metric ton}}{\text{h}}}$$

27.47 $\mathcal{P}=I(\Delta V)=(1.70\text{ A})(110\text{ V})=187\text{ W}$

Energy used in a 24-hour day = (0.187 kW)(24.0 h) = 4.49 kWh

$$\therefore\ \text{cost}=4.49\text{ kWh}\left(\frac{\$0.0600}{\text{kWh}}\right)=\$0.269=\boxed{26.9¢}$$

27.48 $\mathcal{P}=I(\Delta V)=(2.00\text{ A})(120\text{ V})=240\text{ W}$

$\Delta U=(0.500\text{ kg})(4186\text{ J/kg°C})(77.0\text{°C})=161\text{ kJ}$

$$t=\frac{\Delta U}{\mathcal{P}}=\frac{1.61\times10^{5}\text{ J}}{240\text{ W}}=\boxed{672\text{ s}}$$

27.49 At operating temperature,

(a) $\mathcal{P}=I(\Delta V)=(1.53\text{ A})(120\text{ V})=\boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R=R_0(1+\alpha\,\Delta T)$

$$\frac{120}{1.53}=\frac{120}{1.80}\left[1+(0.400\times10^{-3})\Delta T\right]$$

$\Delta T=441\text{°C}$

$T=20.0\text{°C}+441\text{°C}=\boxed{461\text{°C}}$

Goal Solution

A certain toaster has a heating element made of Nichrome resistance wire. When the toaster is first connected to a 120-V source of potential difference (and the wire is at a temperature of 20.0 °C), the initial current is 1.80 A. However, the current begins to decrease as the resistive element warms up. When the toaster has reached its final operating temperature, the current has dropped to 1.53 A. (a) Find the power the toaster consumes when it is at its operating temperature. (b) What is the final temperature of the heating element?

G: Most toasters are rated at about 1000 W (usually stamped on the bottom of the unit), so we might expect this one to have a similar power rating. The temperature of the heating element should be hot enough to toast bread but low enough that the nickel-chromium alloy element does not melt. (The melting point of nickel is 1455 °C, and chromium melts at 1907 °C.)

O: The power can be calculated directly by multiplying the current and the voltage. The temperature can be found from the linear conductivity equation for Nichrome, with $\alpha = 0.4 \times 10^{-3}\ °\text{C}^{-1}$ from Table 27.1.

A: (a) $\mathcal{P} = (\Delta V)I = (120\text{ V})(1.53\text{ A}) = 184\text{ W}$

(b) The resistance at 20.0 °C is $R_0 = \dfrac{\Delta V}{I} = \dfrac{120\text{ V}}{1.80\text{ A}} = 66.7\ \Omega$

At operating temperature, $R = \dfrac{120\text{ V}}{1.53\text{ A}} = 78.4\ \Omega$

Neglecting thermal expansion, $R = \dfrac{\rho l}{A} = \dfrac{\rho_0(1+\alpha(T-T_0))l}{A} = R_0(1+\alpha(T-T_0))$

$$T = T_0 + \frac{R/R_0 - 1}{\alpha} = 20.0\ °\text{C} + \frac{78.4\ \Omega/66.7\ \Omega - 1}{0.4\times 10^{-3}\ °\text{C}^{-1}} = 461\ °\text{C}$$

L: Although this toaster appears to use significantly less power than most, the temperature seems high enough to toast a piece of bread in a reasonable amount of time. In fact, the temperature of a typical 1000-W toaster would only be slightly higher because Stefan's radiation law (Eq. 20.18) tells us that (assuming all power is lost through radiation) $T \propto \sqrt[4]{\mathcal{P}}$, so that the temperature might be about 700 °C. In either case, the operating temperature is well below the melting point of the heating element.

27.50 $\mathcal{P} = (10.0\text{ W/ft}^2)(10.0\text{ ft})(15.0\text{ ft}) = 1.50\text{ kW}$

$\text{Energy} = \mathcal{P}\,t = (1.50\text{ kW})(24.0\text{ h}) = 36.0\text{ kWh}$

$\text{Cost} = (36.0\text{ kWh})(\$0.0800/\text{kWh}) = \boxed{\$2.88}$

***27.51** Consider a 400-W blow dryer used for ten minutes daily for a year. The energy converted is

$$\mathcal{P}\,t = (400\text{ J/s})(600\text{ s/d})(365\text{ d}) \cong 9\times 10^{7}\text{ J}\left(\frac{1\text{ kWh}}{3.6\times 10^{6}\text{ J}}\right) \cong 20\text{ kWh}$$

We suppose that electrical energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$\text{Cost} \cong (20\text{ kWh})(\$0.100/\text{kWh}) = \$2$ $\boxed{\sim\$1}$

***27.52** (a) $I = \dfrac{\Delta V}{R}$ so $\mathcal{P} = (\Delta V)I = \dfrac{(\Delta V)^2}{R}$

$R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega}$ and $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

(b) $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \dfrac{Q}{t} = \dfrac{1.00\text{ C}}{t}$

$t = \dfrac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$ The charge has $\boxed{\text{lower potential energy}}$.

(c) $\mathcal{P} = 25.0\text{ W} = \dfrac{\Delta U}{t} = \dfrac{1.00\text{ J}}{t}$

$t = \dfrac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.0400\text{ s}}$ The energy $\boxed{\text{changes from electrical to heat and light}}$.

(d) $\Delta U = \mathcal{P}t = (25.0\text{ J/s})(86400\text{ s/d})(30.0\text{ d}) = 64.8 \times 10^6\text{ J}$

The energy company sells $\boxed{\text{energy}}$.

$$\text{Cost} = 64.8 \times 10^6\text{ J}\left(\frac{\$0.0700}{\text{kWh}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.0700}{\text{kWh}}\left(\frac{\text{kWh}}{3.60 \times 10^6\text{ J}}\right) = \boxed{\$1.94 \times 10^{-8}/\text{J}}$$

***27.53** We find the drift velocity from $I = nqv_d A = nqv_d \pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1000\text{ A}}{8.00 \times 10^{28}\text{ m}^{-3}(1.60 \times 10^{-19}\text{ C})\pi(10^{-2}\text{ m})^2} = 2.49 \times 10^{-4}\text{ m/s}$$

$$v = \frac{x}{t} \qquad t = \frac{x}{v} = \frac{200 \times 10^3\text{ m}}{2.49 \times 10^{-4}\text{ m/s}} = 8.04 \times 10^8\text{ s} = \boxed{25.5\text{ yr}}$$

***27.54** The resistance of one wire is $\left(\dfrac{0.500\ \Omega}{\text{mi}}\right)(100\text{ mi}) = 50.0\ \Omega$

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$IR = (1000\text{ A})(50.0\ \Omega) = 50.0\text{ kV}$

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = (50.0 \times 10^3\text{ V})(1000\text{ A}) = \boxed{50.0\text{ MW}}$

27.55 We begin with the differential equation $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$

(a) Separating variables, $\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_{T_0}^{T} \alpha\, dT$

$\ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T - T_0)}}$

(b) From the series expansion $e^x \cong 1 + x$, $(x << 1)$,

$\boxed{\rho \cong \rho_0[1 + \alpha(T - T_0)]}$

***27.56** Consider a 1.00-m length of cable. The potential difference between its ends is

$$\Delta V = \frac{\mathcal{P}}{I} = \frac{2.00\text{ W}}{300\text{ A}} = 6.67\text{ mV}$$

The resistance is $R = \frac{\Delta V}{I} = \frac{6.67 \times 10^{-3}\text{ V}}{300\text{ A}} = 22.2\ \mu\Omega$

Then $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi r^2}$ gives $r = \sqrt{\frac{\rho\ell}{\pi R}} = \sqrt{\frac{(1.70 \times 10^{-8}\ \Omega\cdot\text{m})(1.00\text{ m})}{\pi(22.2 \times 10^{-6}\ \Omega)}} = \boxed{1.56\text{ cm}}$

27.57 $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$

ℓ (m)	$R(\Omega)$	$\rho(\Omega\cdot\text{m})$
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47 \times 10^{-6}\ \Omega\cdot\text{m}}$ (in agreement with tabulated value)

$\rho = \frac{RA}{\ell} = \boxed{1.50 \times 10^{-6}\ \Omega\cdot\text{m}}$ (Table 27.1)

27.58 2 wires $\rightarrow \ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\ \text{m}}(100\ \text{m}) = 0.0360\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.0360) = \boxed{116\ \text{V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2R = (110\ \text{A})^2(0.0360\ \Omega) = \boxed{436\ \text{W}}$

***27.59** (a) $\mathbf{E} = -\frac{dV}{dx}\mathbf{i} = -\frac{(0 - 4.00)\ \text{V}}{(0.500 - 0)\ \text{m}} = \boxed{8.00\mathbf{i}\ \text{V/m}}$

(b) $R = \frac{\rho \ell}{A} = \frac{(4.00 \times 10^{-8}\ \Omega \cdot \text{m})(0.500\ \text{m})}{\pi (1.00 \times 10^{-4}\ \text{m})^2} = \boxed{0.637\ \Omega}$

(c) $I = \frac{\Delta V}{R} = \frac{4.00\ \text{V}}{0.637\ \Omega} = \boxed{6.28\ \text{A}}$

(d) $\mathbf{J} = \frac{I}{A}\mathbf{i} = \frac{6.28\ \text{A}}{\pi (1.00 \times 10^{-4}\ \text{m})^2} = 2.00 \times 10^8\ \mathbf{i}\ \text{A/m}^2 = \boxed{200\mathbf{i}\ \text{MA/m}^2}$

(e) $\rho \mathbf{J} = (4.00 \times 10^{-8}\ \Omega \cdot \text{m})(2.00 \times 10^8\ \mathbf{i}\ \text{A/m}^2) = 8.00\mathbf{i}\ \text{V/m} = \mathbf{E}$

***27.60** (a) $\mathbf{E} = -\frac{dV(x)}{dx}\ \mathbf{i} = \boxed{\frac{V}{L}\mathbf{i}}$

(b) $R = \frac{\rho \ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$

(c) $I = \frac{\Delta V}{R} = \boxed{\frac{V \pi d^2}{4\rho L}}$

(d) $\mathbf{J} = \frac{I}{A}\ \mathbf{i} = \boxed{\frac{V}{\rho L}\mathbf{i}}$

(e) $\rho\, \mathbf{J} = \frac{V}{L}\ \mathbf{i} = \boxed{\mathbf{E}}$

27.61 $R = R_0[1+\alpha(T-T_0)]$ so $T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$

In this case, $I = \frac{I_0}{10}$, so $T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.00450/°\text{C}} = \boxed{2020\ °\text{C}}$

27.62 $R = \frac{\Delta V}{I} = \frac{12.0}{I} = \frac{6.00}{(I-3.00)}$ thus $12.0I - 36.0 = 6.00I$ and $I = 6.00\text{ A}$

Therefore, $R = \frac{12.0\text{ V}}{6.00\text{ A}} = \boxed{2.00\ \Omega}$

27.63 (a) $\mathcal{P} = I(\Delta V)$ so $I = \frac{\mathcal{P}}{\Delta V} = \frac{8.00\times10^3\text{ W}}{12.0\text{ V}} = \boxed{667\text{ A}}$

(b) $t = \frac{\Delta U}{\mathcal{P}} = \frac{2.00\times10^7\text{ J}}{8.00\times10^3\text{ W}} = 2.50\times10^3\text{ s}$ and $d = vt = (20.0\text{ m/s})(2.50\times10^3\text{ s}) = \boxed{50.0\text{ km}}$

27.64 (a) We begin with $R = \frac{\rho \ell}{A} = \frac{\rho_0[1+\alpha(T-T_0)]\,\ell_0[1+\alpha'(T-T_0)]}{A_0[1+2\alpha'(T-T_0)]}$,

which reduces to $R = \boxed{\frac{R_0[1+\alpha(T-T_0)][1+\alpha'(T-T_0)]}{[1+2\alpha'(T-T_0)]}}$

(b) For copper: $\rho_0 = 1.70\times10^{-8}\ \Omega\cdot\text{m}$, $\alpha = 3.90\times10^{-3}\ °\text{C}^{-1}$, and $\alpha' = 17.0\times10^{-6}\ °\text{C}^{-1}$

$$R_0 = \frac{\rho_0 \ell_0}{A_0} = \frac{(1.70\times10^{-8})(2.00)}{\pi(0.100\times10^{-3})^2} = \boxed{1.08\ \Omega}$$

The simple formula for R gives:

$$R = (1.08\ \Omega)[1+(3.90\times10^{-3}\ °\text{C}^{-1})(100°\text{C}-20.0°\text{C})] = \boxed{1.420\ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08\ \Omega)[1+(3.90\times10^{-3}\ °\text{C}^{-1})(80.0°\text{C})][1+(17.0\times10^{-6}\ °\text{C}^{-1})(80.0°\text{C})]}{[1+2(17.0\times10^{-6}\ °\text{C}^{-1})(80.0°\text{C})]} = \boxed{1.418\ \Omega}$$

27.65 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0 °C. Then $\rho = \rho_0[1+\alpha(T-20.0°\text{C})]$, and $\rho = \rho'[1+\alpha'(T-0°\text{C})]$ must both give the correct resistivity at any temperature *T*. That is, we must have:

$$\rho_0[1+\alpha(T-20.0°\text{C})] = \rho'[1+\alpha'(T-0°\text{C})] \quad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0[1-\alpha(20.0°\text{C})]$,

and setting $T = 20.0°\text{C}$ in equation (1) gives: $\rho_0 = \rho'[1+\alpha'(20.0°\text{C})]$

Put ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0[1-\alpha(20.0°\text{C})][1+\alpha'(20.0°\text{C})]$$

Therefore $$1+\alpha'(20.0°\text{C}) = \frac{1}{1-\alpha(20.0°\text{C})}$$

which simplifies to $$\alpha' = \frac{\alpha}{[1-\alpha(20.0\ °\text{C})]}$$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, Table 27.1 becomes at 0°C :

Material	Temp Coefficients at 0°C
Silver	$4.1\times10^{-3}/°\text{C}$
Copper	$4.2\times10^{-3}/°\text{C}$
Gold	$3.6\times10^{-3}/°\text{C}$
Aluminum	$4.2\times10^{-3}/°\text{C}$
Tungsten	$4.9\times10^{-3}/°\text{C}$
Iron	$5.6\times10^{-3}/°\text{C}$
Platinum	$4.25\times10^{-3}/°\text{C}$
Lead	$4.2\times10^{-3}/°\text{C}$
Nichrome	$0.4\times10^{-3}/°\text{C}$
Carbon	$-0.5\times10^{-3}/°\text{C}$
Germanium	$-24\times10^{-3}/°\text{C}$
Silicon	$-30\times10^{-3}/°\text{C}$

27.66 (a) $$R = \frac{\rho \ell}{A} = \boxed{\frac{\rho L}{\pi(r_b^2 - r_a^2)}}$$

(b) $$R = \frac{(3.50\times10^5\ \Omega\cdot\text{m})(0.0400\ \text{m})}{\pi[(0.0120\ \text{m})^2-(0.00500\ \text{m})^2]} = 3.74\times10^7\ \Omega = \boxed{37.4\ \text{M}\Omega}$$

(c) $$dR = \frac{\rho\, d\ell}{A} = \frac{\rho\, dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}, \quad \text{so} \quad R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$$

(d) $$R = \frac{(3.50\times10^5\ \Omega\cdot\text{m})}{2\pi(0.0400\ \text{m})}\ln\left(\frac{1.20}{0.500}\right) = 1.22\times10^6\ \Omega = \boxed{1.22\ \text{M}\Omega}$$

27.67 Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2 R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

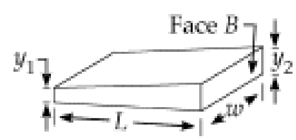
27.68 $\Delta V = -E \cdot \ell$ or $dV = -E \cdot dx$

$\Delta V = -IR = -E \cdot \ell$

$$I = \frac{dq}{dt} = \frac{E \cdot \ell}{R} = \frac{A}{\rho \ell} E \cdot \ell = \frac{A}{\rho} E = -\sigma A \frac{dV}{dx} = \boxed{\sigma A \left| \frac{dV}{dx} \right|}$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

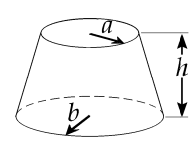
27.69 $R = \int \frac{\rho\, dx}{A} = \int \frac{\rho\, dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L} x$



$$R = \frac{\rho}{w} \int_0^L \frac{dx}{y_1 + \frac{y_2 - y_1}{L} x} = \frac{\rho L}{w(y_2 - y_1)} \ln\left[y_1 + \frac{y_2 - y_1}{L} x \right] \Bigg|_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1} \right)}$$

27.70 From the geometry of the longitudinal section of the resistor shown in the figure, we see that



$$\frac{(b - r)}{y} = \frac{(b - a)}{h}$$

From this, the radius at a distance y from the base is $r = (a - b)\frac{y}{h} + b$

For a disk-shaped element of volume $dR = \frac{\rho\, dy}{\pi r^2}$: $R = \frac{\rho}{\pi} \int_0^h \frac{dy}{\left[(a - b)(y / h) + b \right]^2}$.

Using the integral formula $\int \frac{du}{(au + b)^2} = -\frac{1}{a(au + b)}$, $\boxed{R = \frac{\rho}{\pi} \frac{h}{ab}}$

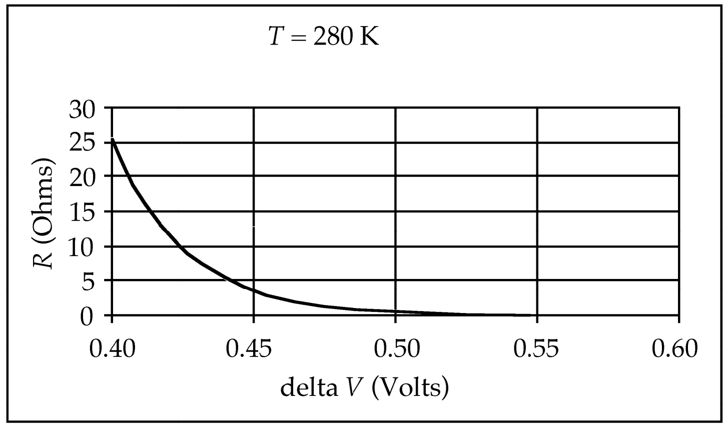
27.71 $I = I_0\left[\exp\left(e\Delta V / k_B T\right) - 1\right]$ and $R = \frac{\Delta V}{I}$

with $I_0 = 1.00 \times 10^{-9}$ A, $e = 1.60 \times 10^{-19}$ C, and $k_B = 1.38 \times 10^{-23}$ J/K.

The following includes a partial table of calculated values and a graph for each of the specified temperatures.

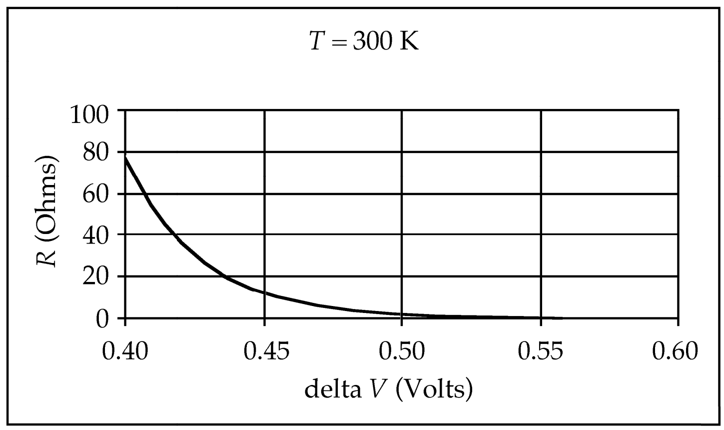
(i) For $T = 280$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0156	25.6
0.440	0.0818	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.0476
0.600	61.6	0.0097



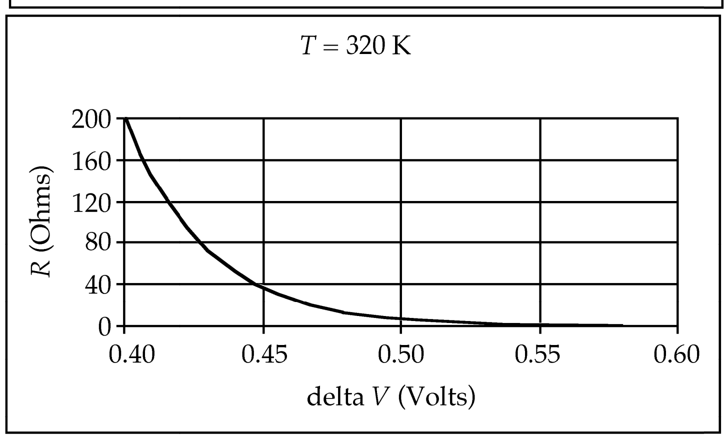
(ii) For $T = 300$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051



(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.0020	203
0.440	0.0084	52.5
0.480	0.0357	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217



Chapter 28 Solutions

28.1 (a) $\mathcal{P} = \frac{(\Delta V)^2}{R}$ becomes $20.0\text{ W} = \frac{(11.6\text{ V})^2}{R}$ so $R = \boxed{6.73\ \Omega}$

(b) $\Delta V = IR$ so $11.6\text{ V} = I(6.73\ \Omega)$ and $I = 1.72\text{ A}$

$\mathcal{E} = IR + Ir$ so $15.0\text{ V} = 11.6\text{ V} + (1.72\text{ A})r$

$r = \boxed{1.97\ \Omega}$

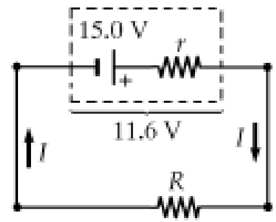


Figure for Goal Solution

Goal Solution

A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor *R*. (a) What is the value of *R*? (b) What is the internal resistance of the battery?

G: The internal resistance of a battery usually is less than 1 Ω, with physically larger batteries having less resistance due to the larger anode and cathode areas. The voltage of this battery drops significantly (23%), when the load resistance is added, so a sizable amount of current must be drawn from the battery. If we assume that the internal resistance is about 1 Ω, then the current must be about 3 A to give the 3.4 V drop across the battery's internal resistance. If this is true, then the load resistance must be about $R \approx 12\text{ V} / 3\text{ A} = 4\ \Omega$.

O: We can find *R* exactly by using Joule's law for the power delivered to the load resistor when the voltage is 11.6 V. Then we can find the internal resistance of the battery by summing the electric potential differences around the circuit.

A: (a) Combining Joule's law, $\mathcal{P} = \Delta VI$, and the definition of resistance, $\Delta V = IR$, gives

$$R = \frac{\Delta V^2}{\mathcal{P}} = \frac{(11.6\text{ V})^2}{20.0\ W} = 6.73\ \Omega$$

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V/R$.

$$r = \frac{\mathcal{E} - IR}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V} = \frac{(15.0\text{ V} - 11.6\text{ V})(6.73\ \Omega)}{11.6\text{ V}} = 1.97\ \Omega$$

L: The resistance of the battery is larger than 1 Ω, but it is reasonable for an old battery or for a battery consisting of several small electric cells in series. The load resistance agrees reasonably well with our prediction, despite the fact that the battery's internal resistance was about twice as large as we assumed. Note that in our initial guess we did not consider the power of the load resistance; however, there is not sufficient information to accurately solve this problem without this data.

28.2 (a) $\Delta V_{\text{term}} = IR$

becomes $10.0 \text{ V} = I(5.60\ \Omega)$

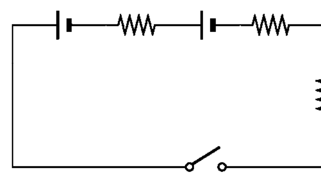
so $I = \boxed{1.79 \text{ A}}$

(b) $\Delta V_{\text{term}} = \mathcal{E} - Ir$

becomes $10.0 \text{ V} = \mathcal{E} - (1.79 \text{ A})(0.200\ \Omega)$

so $\mathcal{E} = \boxed{10.4 \text{ V}}$

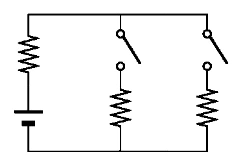
28.3 The total resistance is $R = \dfrac{3.00 \text{ V}}{0.600 \text{ A}} = 5.00\ \Omega$



(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

28.4 (a) Here $\mathcal{E} = I(R + r)$, so $I = \dfrac{\mathcal{E}}{R + r} = \dfrac{12.6 \text{ V}}{(5.00\ \Omega + 0.0800\ \Omega)} = 2.48 \text{ A}$



Then, $\Delta V = IR = (2.48 \text{ A})(5.00\ \Omega) = \boxed{12.4 \text{ V}}$

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

Then, $I_1 = I_2 + 35.0 \text{ A}$, and $\mathcal{E} - I_1 r - I_2 R = 0$

so $\mathcal{E} = (I_2 + 35.0 \text{ A})(0.0800\ \Omega) + I_2(5.00\ \Omega) = 12.6 \text{ V}$

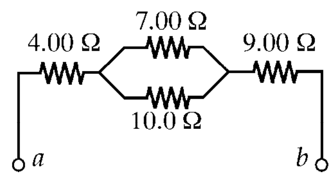
giving $I_2 = 1.93 \text{ A}$

Thus, $\Delta V_2 = (1.93 \text{ A})(5.00\ \Omega) = \boxed{9.65 \text{ V}}$

28.5 $\Delta V = I_1 R_1 = (2.00 \text{ A})R_1$ and $\Delta V = I_2(R_1 + R_2) = (1.60 \text{ A})(R_1 + 3.00\ \Omega)$

Therefore, $(2.00 \text{ A})R_1 = (1.60 \text{ A})(R_1 + 3.00\ \Omega)$ or $R_1 = \boxed{12.0\ \Omega}$

28.6 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$



$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for 4.00 Ω, 9.00 Ω resistors

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

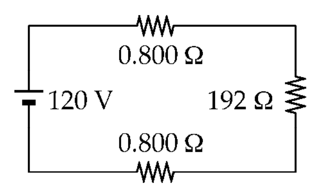
$8.18\ \text{V} = I(7.00\ \Omega)$ so $I = \boxed{1.17\ \text{A}}$ for 7.00 Ω resistor

$8.18\ \text{V} = I(10.0\ \Omega)$ so $I = \boxed{0.818\ \text{A}}$ for 10.0 Ω resistor

***28.7** If $\boxed{\text{all 3 resistors are placed in parallel}}$,

$\dfrac{1}{R} = \dfrac{1}{500} + \dfrac{2}{250} = \dfrac{5}{500}$ and $R = 100\ \Omega$

***28.8** For the bulb in use as intended,



$I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{75.0\ \text{W}}{120\ \text{V}} = 0.625\ \text{A}$ and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{0.625\ \text{A}} = 192\ \Omega$

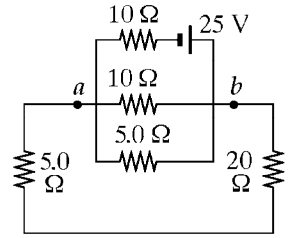
Now, presuming the bulb resistance is unchanged,

$I = \dfrac{120\ \text{V}}{193.6\ \Omega} = 0.620\ \text{A}$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\ \text{A}) = 119\ \text{V}$

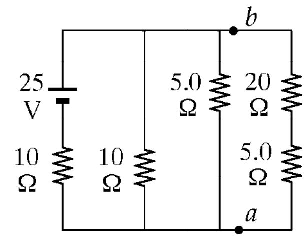
so its power is $\mathcal{P} = (\Delta V)I = 119\ \text{V}(0.620\ \text{A}) = \boxed{73.8\ \text{W}}$

28.9 If we turn the given diagram on its side, we find that it is the same as Figure (a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is as shown in (b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:



$$R_{eq} = \frac{1}{\left(\frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega$$

This is shown in Figure (c), which in turn reduces to the circuit shown in (d).

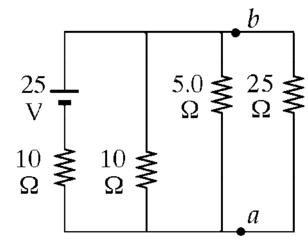


(a)

Next, we work backwards through the diagrams applying $I = \Delta V / R$ and $\Delta V = IR$. The 12.94-Ω resistor is connected across 25.0-V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In Figure (c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:

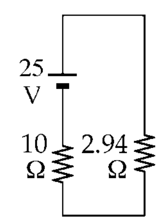


(b)

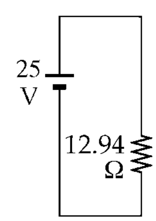
$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From Figure (b), we see that this potential difference is the same across V_{ab}, the 10-Ω resistor, and the 5.00-Ω resistor.

(b) Therefore, $V_{ab} = \boxed{5.68\ \text{V}}$



(c)



(d)

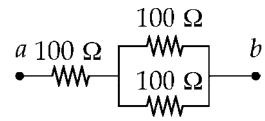
(a) Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

$$I = \frac{V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

28.10 $$120\ \text{V} = IR_{eq} = I\left(\frac{\rho l}{A_1} + \frac{\rho l}{A_2} + \frac{\rho l}{A_3} + \frac{\rho l}{A_4}\right),\ \text{or}\quad I\rho l = \frac{(120\ \text{V})}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)}$$

$$\Delta V_2 = \frac{I\rho l}{A_2} = \frac{(120\ \text{V})}{A_2\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)} = \boxed{29.5\ \text{V}}$$

28.11 (a) Since all the current flowing in the circuit must pass through the series 100-Ω resistor, $\mathcal{P} = RI^2$



$$\mathcal{P}_{max} = RI_{max}^2 \quad \text{so} \quad I_{max} = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{max} = R_{eq} I_{max} = \boxed{75.0\text{ V}}$$

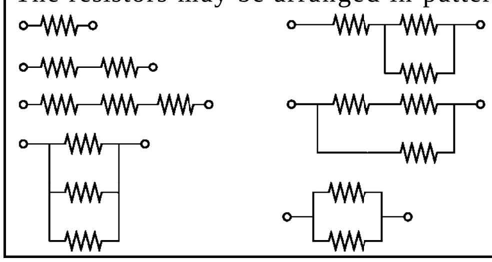
(b) $\mathcal{P} = (\Delta V)I = (75.0\text{ V})(0.500\text{ A}) = \boxed{37.5\text{ W}}$ total power

$\mathcal{P}_1 = \boxed{25.0\text{ W}}$ $\quad \mathcal{P}_2 = \mathcal{P}_3 = RI^2 = (100\ \Omega)(0.250\text{ A})^2 = \boxed{6.25\text{ W}}$

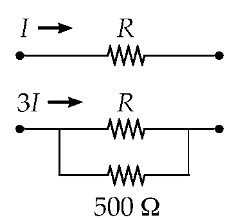
28.12 Using 2.00-Ω, 3.00-Ω, 4.00-Ω resistors, there are 7 series, 4 parallel, and 6 mixed combinations:

Series		Parallel	Mixed
2.00 Ω	6.00 Ω	0.923 Ω	1.56 Ω
3.00 Ω	7.00 Ω	1.20 Ω	2.00 Ω
4.00 Ω	9.00 Ω	1.33 Ω	2.22 Ω
5.00 Ω		1.71 Ω	3.71 Ω
			4.33 Ω
			5.20 Ω

The resistors may be arranged in patterns:



28.13 The potential difference is the same across either combination.



$$\Delta V = IR = 3I\frac{1}{\left(\frac{1}{R} + \frac{1}{500}\right)} \quad \text{so} \quad R\left(\frac{1}{R} + \frac{1}{500}\right) = 3$$

$$1 + \frac{R}{500} = 3 \quad \text{and} \quad R = 1000\ \Omega = \boxed{1.00\text{ k}\Omega}$$

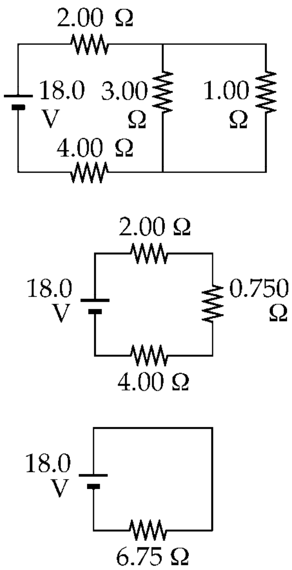
28.14 If the switch is open, $I = \mathcal{E}/(R' + R)$ and $\mathcal{P} = \mathcal{E}^2 R'/(R' + R)^2$

If the switch is closed, $I = \mathcal{E}/(R + R'/2)$ and $\mathcal{P} = \mathcal{E}^2 (R'/2)/(R + R'/2)^2$

Then,
$$\frac{\mathcal{E}^2 R'}{(R' + R)^2} = \frac{\mathcal{E}^2 R'}{2(R + R'/2)^2}$$

$$2R^2 + 2RR' + R'^2/2 = R'^2 + 2RR' + R^2$$

The condition becomes $R^2 = R'^2/2$ so $R' = \sqrt{2}R = \sqrt{2}(1.00\ \Omega) = \boxed{1.41\Omega}$



28.15 $R_p = \left(\frac{1}{3.00} + \frac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$

$\mathcal{P} = I^2 R$: $\quad \mathcal{P}_2 = (2.67\ \text{A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\ \text{A})^2(4.00\ \Omega) = \boxed{28.4\ \text{W}}$ in $4.00\ \Omega$

$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V}, \quad \Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V} \quad (= \Delta V_3 = \Delta V_1)$

$\mathcal{P}_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \frac{(\Delta V_1)^2}{R_1} = \frac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in $1.00\ \Omega$

28.16 Denoting the two resistors as x and y,

$x + y = 690$, and $\frac{1}{150} = \frac{1}{x} + \frac{1}{y}$

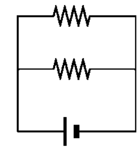
$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)}$

$x^2 - 690x + 103{,}500 = 0$

$x = \frac{690 \pm \sqrt{(690)^2 - 414{,}000}}{2}$

$x = \boxed{470\ \Omega} \qquad y = \boxed{220\ \Omega}$

28.17 (a) $\Delta V = IR$: $33.0\text{ V} = I_1(11.0\ \Omega)$ $33.0\text{ V} = I_2(22.0\ \Omega)$



$I_1 = 3.00\text{ A}$ $I_2 = 1.50\text{ A}$

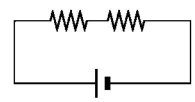
$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (3.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.50\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 99.0\text{ W}$ $\mathcal{P}_2 = 49.5\text{ W}$

The 11.0-Ω resistor uses more power.

(b) $\mathcal{P}_1 + \mathcal{P}_2 =$ 148 W $\mathcal{P} = I(\Delta V) = (4.50)(33.0) =$ 148 W

(c) $R_s = R_1 + R_2 = 11.0\ \Omega + 22.0\ \Omega = 33.0\ \Omega$



$\Delta V = IR$: $33.0\text{ V} = I(33.0\ \Omega)$, so $I = 1.00\text{ A}$

$\mathcal{P} = I^2R$: $\mathcal{P}_1 = (1.00\text{ A})^2(11.0\ \Omega)$ $\mathcal{P}_2 = (1.00\text{ A})^2(22.0\ \Omega)$

$\mathcal{P}_1 = 11.0\text{ W}$ $\mathcal{P}_2 = 22.0\text{ W}$

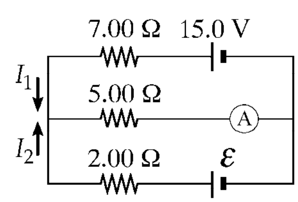
The 22.0-Ω resistor uses more power.

(d) $\mathcal{P}_1 + \mathcal{P}_2 = I^2(R_1 + R_2) = (1.00\text{ A})^2(33.0\ \Omega) =$ 33.0 W

$\mathcal{P} = I(\Delta V) = (1.00\text{ A})(33.0\text{ V}) =$ 33.0 W

(e) The parallel configuration uses more power.

28.18 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$



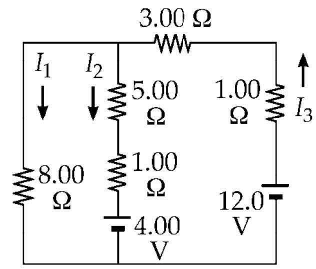
$5.00 = 7.00I_1$ so $I_1 = 0.714\text{ A}$

$I_3 = I_1 + I_2 = 2.00\text{ A}$

$0.714 + I_2 = 2.00$ so $I_2 = 1.29\text{ A}$

$+\mathcal{E} - 2.00(1.29) - (5.00)(2.00) = 0$ $\mathcal{E} = 12.6\text{ V}$

28.19 We name the currents I_1, I_2, and I_3 as shown.



From Kirchhoff's current rule, $I_3 = I_1 + I_2$

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$12.0 \text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00 \text{ V} = 0$

$8.00 = (4.00)I_3 + (6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

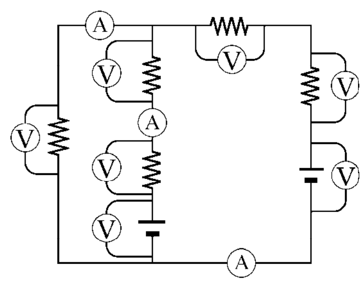
$-(6.00)I_2 - 4.00 \text{ V} + (8.00)I_1 = 0$

$(8.00)I_1 = 4.00 + (6.00)I_2$

Solving the above linear systems, $\boxed{I_1 = 846 \text{ mA}, \quad I_2 = 462 \text{ mA}, \quad I_3 = 1.31 \text{ A}}$

All currents flow in the directions indicated by the arrows in the circuit diagram.

***28.20** The solution figure is shown to the right.



***28.21** We use the results of Problem 19.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)It = 4.00 \text{ V}(-0.462 \text{ A})120 \text{ s} = \boxed{-222 \text{ J}}$

By the 12.0-V battery: $12.0 \text{ V } (1.31 \text{ A}) \ 120 \text{ s} = \boxed{1.88 \text{ kJ}}$

(b) By the 8.00 Ω resistor: $I^2 Rt = (0.846 \text{ A})^2(8.00 \ \Omega) \ 120 \text{ s} = \boxed{687 \text{ J}}$

By the 5.00 Ω resistor: $(0.462 \text{ A})^2(5.00 \ \Omega) \ 120 \text{ s} = \boxed{128 \text{ J}}$

By the 1.00 Ω resistor: $(0.462 \text{ A})^2(1.00 \ \Omega) \ 120 \text{ s} = \boxed{25.6 \text{ J}}$

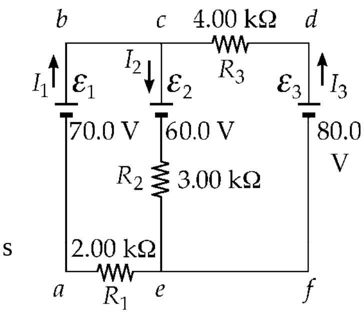
By the 3.00 Ω resistor: $(1.31 \text{ A})^2(3.00 \ \Omega) \ 120 \text{ s} = \boxed{616 \text{ J}}$

By the 1.00 Ω resistor: $(1.31 \text{ A})^2(1.00 \ \Omega) \ 120 \text{ s} = \boxed{205 \text{ J}}$

(c) $-222 \text{ J} + 1.88 \text{ kJ} = \boxed{1.66 \text{ kJ}}$ from chemical to electrical.

$687 \text{ J} + 128 \text{ J} + 25.6 \text{ J} + 616 \text{ J} + 205 \text{ J} = 1.66 \text{ kJ}$ from electrical to heat.

28.22 We name the currents I_1, I_2, and I_3 as shown.



[1] $70.0 - 60.0 - I_2(3.00\text{ k}\Omega) - I_1(2.00\text{ k}\Omega) = 0$

[2] $80.0 - I_3(4.00\text{ k}\Omega) - 60.0 - I_2(3.00\text{ k}\Omega) = 0$

[3] $I_2 = I_1 + I_3$

(a) Substituting for I_2 and solving the resulting simultaneous equations yields

$I_1 = \boxed{0.385\text{ mA}}$ (through R_1)

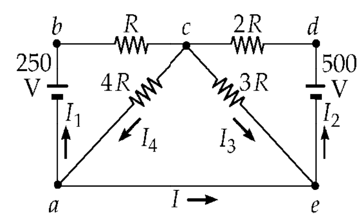
$I_3 = \boxed{2.69\text{ mA}}$ (through R_3)

$I_2 = \boxed{3.08\text{ mA}}$ (through R_2)

(b) $\Delta V_{cf} = -60.0\text{ V} - (3.08\text{ mA})(3.00\text{ k}\Omega) = \boxed{-69.2\text{ V}}$

$\boxed{\text{Point } c \text{ is at higher potential.}}$

28.23 Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

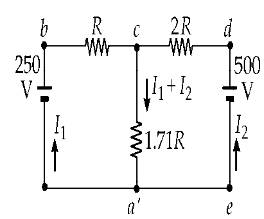


(a)

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250 \quad \text{and} \quad (1.71R)I_1 + (3.71R)I_2 = 500$$

With $R = 1000\ \Omega$, simultaneous solution of these equations yields:



(b)

$I_1 = 10.0\text{ mA}$ and $I_2 = 130.0\text{ mA}$

From Figure (b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$

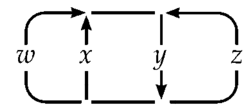
Thus, from Figure (a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\text{ V}}{4000\ \Omega} = 60.0\text{ mA}$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$,

or $I = \boxed{50.0\text{ mA flowing from point } a \text{ to point } e}$.

28.24 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are



$$-200w - 40.0 + 80.0x = 0$$

$$-80.0x + 40.0 + 360 - 20.0y = 0$$

$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution.
$$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x:
$$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $430 - 70.0w - 1575 + 1215w = 0$

$w = 70.0/70.0 =$ $\boxed{\text{1.00 A upward in 200 }\Omega}$

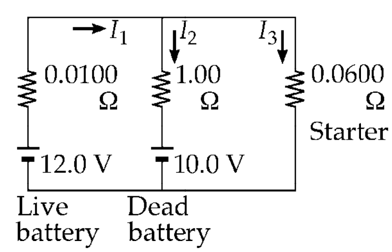
Now $z =$ $\boxed{\text{4.00 A upward in 70.0 }\Omega}$

$x =$ $\boxed{\text{3.00 A upward in 80.0 }\Omega}$

$y =$ $\boxed{\text{8.00 A downward in 20.0 }\Omega}$

and for the 200 Ω, $\Delta V = IR = (1.00\text{ A})(200\ \Omega) =$ $\boxed{200\text{ V}}$

28.25 Using Kirchhoff's rules,



$$12.0 - (0.0100)I_1 - (0.0600)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

and $I_1 = I_2 + I_3$

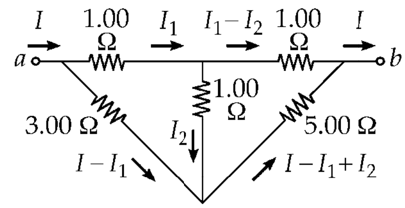
$$12.0 - (0.0100)I_2 - (0.0700)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

Solving simultaneously, $I_2 =$ $\boxed{\text{0.283 A downward}}$ in the dead battery,

and $I_3 =$ $\boxed{\text{171 A downward}}$ in the starter.

28.26 $V_{ab} = (1.00)I_1 + (1.00)(I_1 - I_2)$



$V_{ab} = (1.00)I_1 + (1.00)I_2 + (5.00)(I - I_1 + I_2)$

$V_{ab} = (3.00)(I - I_1) + (5.00)(I - I_1 + I_2)$

Let $I = 1.00$ A, $I_1 = x$, and $I_2 = y$

Then, the three equations become:

$V_{ab} = 2.00x - y$, or $y = 2.00x - V_{ab}$

$V_{ab} = -4.00x + 6.00y + 5.00$

and $V_{ab} = 8.00 - 8.00x + 5.00y$

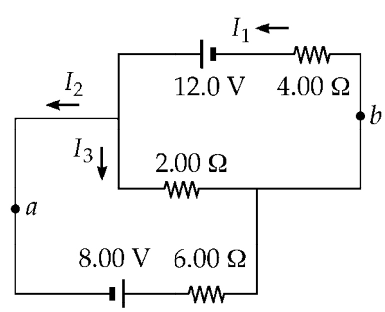
Substituting the first into the last two gives:

$7.00V_{ab} = 8.00x + 5.00$ and $6.00V_{ab} = 2.00x + 8.00$

Solving these simultaneously yields $V_{ab} = \frac{27}{17}$ V

Then, $R_{ab} = \frac{V_{ab}}{I} = \frac{27/17 \text{ V}}{1.00 \text{ A}}$ or $\boxed{R_{ab} = \frac{27}{17}\ \Omega}$

28.27 We name the currents I_1, I_2, and I_3 as shown.



(a) $I_1 = I_2 + I_3$

Counterclockwise around the top loop,

$12.0 \text{ V} - (2.00\ \Omega)I_3 - (4.00\ \Omega)I_1 = 0$

Traversing the bottom loop,

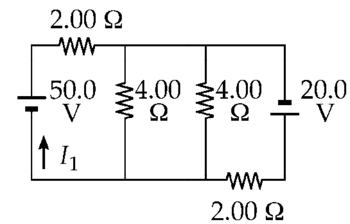
$8.00 \text{ V} - (6.00\ \Omega)I_2 + (2.00\ \Omega)I_3 = 0$

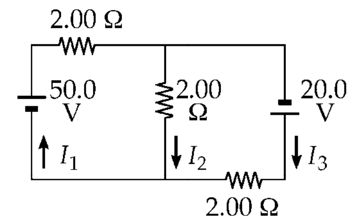
$I_1 = 3.00 - \frac{1}{2}I_3$ $I_2 = \frac{4}{3} + \frac{1}{3}I_3$ and $\boxed{I_3 = 909 \text{ mA}}$

(b) $V_a - (0.909 \text{ A})(2.00\ \Omega) = V_b$

$V_b - V_a = \boxed{-1.82 \text{ V}}$

28.28 We apply Kirchhoff's rules to the second diagram.





$50.0 - 2.00I_1 - 2.00I_2 = 0$ (1)

$20.0 - 2.00I_3 + 2.00I_2 = 0$ (2)

$I_1 = I_2 + I_3$ (3)

Substitute (3) into (1), and solve for I_1, I_2, and I_3

$I_1 = 20.0$ A; $I_2 = 5.00$ A; $I_3 = 15.0$ A

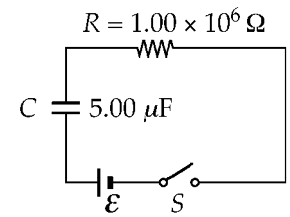
Then apply $P = I^2R$ to each resistor:

$(2.00\ \Omega)_1$: $P = I_1^2(2.00\ \Omega) = (20.0\text{ A})^2(2.00\ \Omega) = \boxed{800\text{ W}}$

$(4.00\ \Omega)$: $P = \left(\frac{5.00}{2}\text{ A}\right)^2(4.00\ \Omega) = \boxed{25.0\text{ W}}$
(Half of I_2 goes through each)

$(2.00\ \Omega)_3$: $P = I_3^2(2.00\ \Omega) = (15.0\text{ A})^2(2.00\ \Omega) = \boxed{450\text{ W}}$

28.29 (a) $RC = (1.00\times10^6\ \Omega)(5.00\times10^{-6}\text{ F}) = \boxed{5.00\text{ s}}$



(b) $Q = C\mathcal{E} = (5.00\times10^{-6}\text{ C})(30.0\text{ V}) = \boxed{150\ \mu\text{C}}$

(c) $I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = \frac{30.0}{1.00\times10^6}\exp\left[\frac{-10.0}{(1.00\times10^6)(5.00\times10^{-6})}\right] = \boxed{4.06\ \mu\text{A}}$

28.30 (a) $I(t) = -I_0e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\text{ C}}{(1300\ \Omega)(2.00\times10^{-9}\text{ F})} = 1.96\text{ A}$$

$$I(t) = -(1.96\text{ A})\exp\left[\frac{-9.00\times10^{-6}\text{ s}}{(1300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{-61.6\text{ mA}}$$

(b) $q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00\times10^{-6}\text{ s}}{(1300\ \Omega)(2.00\times10^{-9}\text{ F})}\right] = \boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the current is $\boxed{I_0 = 1.96\text{ A}}$

28.31 $U = \frac{1}{2}C(\Delta V)^2$ and $\Delta V = Q/C$

Therefore, $U = Q^2/2C$ and when the charge decreases to half its original value, the stored energy is one-quarter its original value: $\boxed{U_f = \frac{1}{4}U_0}$

28.32 (a) $\tau = RC = (1.50 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.50\ \text{s}}$

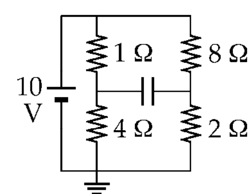
(b) $\tau = (1.00 \times 10^5\ \Omega)(10.0 \times 10^{-6}\ \text{F}) = \boxed{1.00\ \text{s}}$

(c) The battery carries current $\dfrac{10.0\ \text{V}}{50.0 \times 10^3\ \Omega} = 200\ \mu\text{A}$

The 100 kΩ carries current of magnitude $I = I_0 e^{-t/RC} = \left(\dfrac{10.0\ \text{V}}{100 \times 10^3\ \Omega}\right) e^{-t/\ 1.00\ \text{s}}$

So the switch carries downward current $\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\ \text{s}}}$

28.33 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



V_L = 8.00 V because of voltage divider: $I_L = \dfrac{10.0\ \text{V}}{5.00\ \Omega} = 2.00\ \text{A}$

$V_L = 10.0\ \text{V} - (2.00\ \text{A})(1.00\ \Omega) = 8.00\ \text{V}$

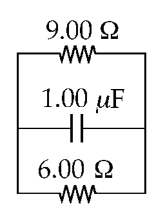
Likewise, $V_R = \left(\dfrac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right) 10.0\ \text{V} = 2.00\ \text{V}$

or $I_R = \dfrac{10.0\ \text{V}}{10.0\ \Omega} = 1.00\ \text{A}$

$V_R = (10.0\ \text{V}) - (8.00\ \Omega)(1.00\ \text{A}) = 2.00\ \text{V}$

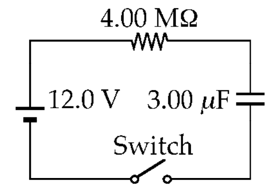
Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\ \text{V}}$

(b) Redraw the circuit $R = \dfrac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$



$RC = 3.60 \times 10^{-6}\ \text{s}$

and $e^{-t/RC} = \dfrac{1}{10}$ so $t = RC \ln 10 = \boxed{8.29\ \mu\text{s}}$



28.34 (a) $\tau = RC = \left(4.00\times10^{6}\ \Omega\right)\left(3.00\times10^{-6}\ \text{F}\right) = \boxed{12.0\ \text{s}}$

(b) $I = \frac{\mathcal{E}}{R}e^{-t/RC} = \frac{12.0}{4.00\times10^{6}}\,e^{-t/12.0\ \text{s}}$

$q = C\mathcal{E}\left[1 - e^{-t/RC}\right] = 3.00\times10^{-6}(12.0)\left[1 - e^{-t/12.0}\right)$

$\boxed{q = 36.0\ \mu\text{C}\left[1 - e^{-t/12.0}\right]}$ $\boxed{I = 3.00\ \mu\text{A}e^{-t/12.0}}$

28.35 $\Delta V_0 = \frac{Q}{C}$

Then, if $q(t) = Qe^{-t/RC}$ $\Delta V(t) = \Delta V_0 e^{-t/RC}$

$$\frac{\Delta V(t)}{\Delta V_0} = e^{-t/RC}$$

Therefore $\frac{1}{2} = \exp\left(-\frac{4.00}{R\left(3.60\times10^{-6}\right)}\right)$

$$\ln\left(\frac{1}{2}\right) = -\frac{4.00}{R\left(3.60\times10^{-6}\right)}$$

$$R = \boxed{1.60\ \text{M}\Omega}$$

28.36 $\Delta V_0 = \frac{Q}{C}$

Then, if $q(t) = Q\,e^{-t/RC}$ $\Delta V(t) = \left(\Delta V_0\right)e^{-t/RC}$

and $\frac{\Delta V(t)}{\left(\Delta V_0\right)} = e^{-t/RC}$

When $\Delta V(t) = \frac{1}{2}\left(\Delta V_0\right)$, then $e^{-t/RC} = \frac{1}{2}$

$$-\frac{t}{RC} = \ln\left(\frac{1}{2}\right) = -\ln 2$$

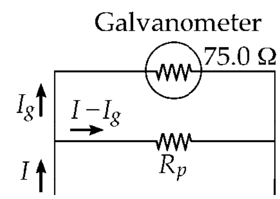
Thus, $\boxed{R = \frac{t}{C(\ln 2)}}$

28.37 $q(t) = Q\left[1 - e^{-t/RC}\right]$ so $\dfrac{q(t)}{Q} = 1 - e^{-t/RC}$

$0.600 = 1 - e^{-0.900/RC}$ or $e^{-0.900/RC} = 1 - 0.600 = 0.400$

$\dfrac{-0.900}{RC} = \ln(0.400)$ thus $RC = \dfrac{-0.900}{\ln(0.400)} = \boxed{0.982\ \text{s}}$

28.38 Applying Kirchhoff's loop rule, $-I_g(75.0\ \Omega) + \left(I - I_g\right)R_p = 0$



Therefore, if $I = 1.00$ A when $I_g = 1.50$ mA,

$$R_p = \frac{I_g(75.0\ \Omega)}{\left(I - I_g\right)} = \frac{\left(1.50 \times 10^{-3}\ \text{A}\right)(75.0\ \Omega)}{1.00\ \text{A} - 1.50 \times 10^{-3}\ \text{A}} = \boxed{0.113\ \Omega}$$

28.39 Series Resistor $\rightarrow$ Voltmeter

$\Delta V = IR$: $25.0 = 1.50 \times 10^{-3}(R_s + 75.0)$

Solving, $\boxed{R_s = 16.6\ \text{k}\Omega}$

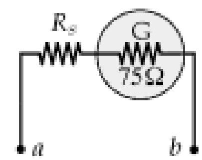


Figure for Goal Solution

<u>Goal Solution</u>

The galvanometer described in the preceding problem can be used to measure voltages. In this case a large resistor is wired in series with the galvanometer in a way similar to that shown in Figure P28.24b This arrangement, in effect, limits the current that flows through the galvanometer when large voltages are applied. Most of the potential drop occurs across the resistor placed in series. Calculate the value of the resistor that enables the galvanometer to measure an applied voltage of 25.0 V at full-scale deflection.

G: The problem states that the value of the resistor must be "large" in order to limit the current through the galvanometer, so we should expect a resistance of kΩ to MΩ.

O: The unknown resistance can be found by applying the definition of resistance to the portion of the circuit shown in Figure 28.24b.

A: $\Delta V_{ab} = 25.0$ V; From Problem 38, $I = 1.50$ mA and $R_g = 75.0\ \Omega$. For the two resistors in series, $R_{eq} = R_s + R_g$ so the definition of resistance gives us: $\Delta V_{ab} = I(R_s + R_g)$

Therefore, $R_s = \dfrac{\Delta V_{ab}}{I} - R_g = \dfrac{25.0\ \text{V}}{1.50 \times 10^{-3}\ \text{A}} - 75.0\ \Omega = 16.6\ \text{k}\Omega$

L: The resistance is relatively large, as expected. It is important to note that some caution would be necessary if this arrangement were used to measure the voltage across a circuit with a comparable resistance. For example, if the circuit resistance was 17 kΩ, the voltmeter in this problem would cause a measurement inaccuracy of about 50%, because the meter would divert about half the current that normally would go through the resistor being measured. Problems 46 and 59 address a similar concern about measurement error when using electrical meters.

28.40 We will use the values required for the 1.00-V voltmeter to obtain the internal resistance of the galvanometer. $\Delta V = I_g(R + r_g)$

Solve for r_g: $$r_g = \frac{\Delta V}{I_g} - R = \frac{1.00\text{ V}}{1.00 \times 10^{-3}\text{ A}} - 900\ \Omega = 100\ \Omega$$

We then obtain the series resistance required for the 50.0-V voltmeter:

$$R = \frac{V}{I_g} - r_g = \frac{50.0\text{ V}}{1.00 \times 10^{-3}\text{ A}} - 100\ \Omega = \boxed{49.9\text{ k}\Omega}$$

28.41 $$\Delta V = I_g r_g = \left(I - I_g\right)R_p, \text{ or } R_p = \frac{I_g r_g}{\left(I - I_g\right)} = \frac{I_g(60.0\ \Omega)}{\left(I - I_g\right)}$$

Therefore, to have $I = 0.100\text{ A} = 100\text{ mA}$ when $I_g = 0.500\text{ mA}$:

$$R_p = \frac{(0.500\text{ mA})(60.0\ \Omega)}{99.5\text{ mA}} = \boxed{0.302\ \Omega}$$

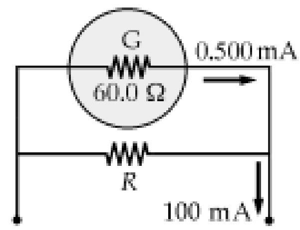


Figure for Goal Solution

Goal Solution

Assume that a galvanometer has an internal resistance of $60.0\ \Omega$ and requires a current of 0.500 mA to produce full-scale deflection. What resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter that has a full-scale deflection for a current of 0.100 A?

G: An ammeter reads the flow of current in a portion of a circuit; therefore it must have a low resistance so that it does not significantly alter the current that would exist without the meter. Therefore, the resistance required is probably less than $1\ \Omega$.

O: From the values given for a full-scale reading, we can find the voltage across and the current through the shunt (parallel) resistor, and the resistance value can then be found from the definition of resistance.

A: The voltage across the galvanometer must be the same as the voltage across the shunt resistor in parallel, so when the ammeter reads full scale,

$$\Delta V = (0.500\text{ mA})(60.0\ \Omega) = 30.0\text{ mV}$$

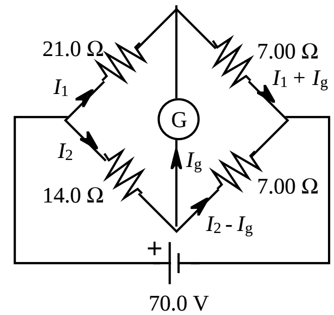
Through the shunt resistor, $$I = 100\text{ mA} - 0.500\text{ mA} = 99.5\text{ mA}$$

Therefore, $$R = \frac{\Delta V}{I} = \frac{30.0\text{ mV}}{99.5\text{ mA}} = 0.302\ \Omega$$

L: The shunt resistance is less than $1\ \Omega$ as expected. It is important to note that some caution would be necessary if this meter were used in a circuit that had a low resistance. For example, if the circuit resistance was $3\ \Omega$, adding the ammeter to the circuit would reduce the current by about 10%, so the current displayed by the meter would be lower than without the meter. Problems 46 and 59 address a similar concern about measurement error when using electrical meters.

28.42 $$R_x = \frac{R_2 R_3}{R_1} = \frac{R_2 R_3}{2.50 R_2} = \frac{1000\ \Omega}{2.50} = \boxed{400\ \Omega}$$

28.43 Using Kirchhoff's rules with $R_g << 1$,



$$-(21.0\ \Omega)I_1 + (14.0\ \Omega)I_2 = 0, \text{ so } I_1 = \frac{2}{3}I_2$$

$$70.0 - 21.0I_1 - 7.00\left(I_1 + I_g\right) = 0, \text{ and}$$

$$70.0 - 14.0I_2 - 7.00\left(I_2 - I_g\right) = 0$$

The last two equations simplify to

$$10.0 - 4.00\left(\tfrac{2}{3}I_2\right) = I_g, \quad \text{and} \quad 10.0 - 3.00I_2 = -I_g$$

Solving simultaneously yields: $I_g = \boxed{0.588\ \text{A}}$

28.44 $$R = \frac{\rho L}{A} \quad \text{and} \quad R_i = \frac{\rho L_i}{A_i}$$

But, $V = AL = A_i L_i$, so $R = \frac{\rho L^2}{V}$ and $R_i = \frac{\rho L_i^2}{V}$

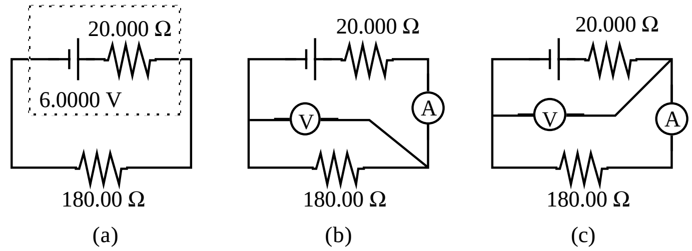
Therefore, $R = \frac{\rho\left(L_i + \Delta L\right)^2}{V} = \frac{\rho L_i\left[1 + \left(\Delta L / L_i\right)\right]^2}{V} = R_i[1+\alpha]^2$ where $\alpha \equiv \frac{\Delta L}{L}$

This may be written as: $\boxed{R = R_i(1 + 2\alpha + \alpha^2)}$

28.45 $$\frac{\mathcal{E}_x}{R_s} = \frac{\mathcal{E}_s}{R_s}; \quad \mathcal{E}_x = \frac{\mathcal{E}_s R_x}{R_s} = \left(\frac{48.0\ \Omega}{36.0\ \Omega}\right)(1.0186\ \text{V}) = \boxed{1.36\ \text{V}}$$

***28.46** (a) In Figure (a), the emf sees an equivalent resistance of 200.00 Ω.

$$I = \frac{6.000\ 0\ \text{V}}{200.00\ \Omega} = \boxed{0.030\ 000\ \text{A}}$$



The terminal potential difference is $\Delta V = IR = (0.030\ 000\ \text{A})(180.00\ \Omega) = \boxed{5.400\ 0\ \text{V}}$

(b) In Figure (b), $R_{eq} = \left(\frac{1}{180.00\ \Omega} + \frac{1}{20\ 000\ \Omega}\right)^{-1} = 178.39\ \Omega$

The equivalent resistance across the emf is $178.39\ \Omega + 0.500\ 00\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The ammeter reads $I = \frac{\mathcal{E}}{R} = \frac{6.000\ 0\ \text{V}}{198.89\ \Omega} = \boxed{0.030\ 167\ \text{A}}$

and the voltmeter reads $\Delta V = IR = (0.030\ 167\ \text{A})(178.39\ \Omega) = \boxed{5.381\ 6\ \text{V}}$

(c) In Figure (c), $\left(\frac{1}{180.50\ \Omega} + \frac{1}{20\ 000\ \Omega}\right)^{-1} = 178.89\ \Omega$

Therefore, the emf sends current through $R_{tot} = 178.89\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The current through the battery is $I = \frac{6.000\ 0\ \text{V}}{198.89\ \Omega} = 0.030\ 168\ \text{A}$

but not all of this goes through the ammeter.

The voltmeter reads $\Delta V = IR = (0.030\ 168\ \text{A})(178.89\ \Omega) = \boxed{5.396\ 6\ \text{V}}$

The ammeter measures current $I = \frac{\Delta V}{R} = \frac{5.396\ 6\ \text{V}}{180.50\ \Omega} = \boxed{0.029\ 898\ \text{A}}$

The connection shown in Figure (c) is better than that shown in Figure (b) for accurate readings.

28.47 (a) $\mathcal{P} = I(\Delta V)$ So for the Heater, $I = \frac{\mathcal{P}}{\Delta V} = \frac{1500\ \text{W}}{120\ \text{V}} = \boxed{12.5\ \text{A}}$

For the Toaster, $I = \frac{750\ \text{W}}{120\ \text{W}} = \boxed{6.25\ \text{A}}$

And for the Grill, $I = \frac{1000\ \text{W}}{120\ \text{V}} = \boxed{8.33\ \text{A}}$ (Grill)

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1\ \text{A}}$ The current draw is greater than 25.0 amps, so this would not be sufficient.

28.48 (a) $\mathcal{P} = I^2R = I^2\left(\frac{\rho\ell}{A}\right) = \frac{(1.00\text{ A})^2(1.70\times10^{-8}\ \Omega\cdot\text{m})(16.0\text{ ft})(0.3048\text{ m / ft})}{\pi(0.512\times10^{-3}\text{ m})^2} = \boxed{0.101\text{ W}}$

(b) $\mathcal{P} = I^2R = 100(0.101\ \Omega) = \boxed{10.1\text{ W}}$

28.49 $I_{\text{Al}}^2 R_{\text{Al}} = I_{\text{Cu}}^2 R_{\text{Cu}}$ so $I_{\text{Al}} = \sqrt{\frac{R_{\text{Cu}}}{R_{\text{Al}}}}I_{\text{Cu}} = \sqrt{\frac{\rho_{\text{Cu}}}{\rho_{\text{Al}}}}I_{\text{Cu}} = \sqrt{\frac{1.70}{2.82}}(20.0) = 0.776(20.0) = \boxed{15.5\text{ A}}$

***28.50** (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho\ell}{A} \cong \frac{\left(10^{13}\ \Omega\cdot\text{m}\right)\left(10^{-3}\text{ m}\right)}{4\times10^{-6}\text{ m}^2} \cong 2\times10^{15}\ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2\times10^{15}\ \Omega + 10^4\ \Omega + 2\times10^{15}\ \Omega \cong 5\times10^{15}\ \Omega$$

It is: $I = \frac{\Delta V}{R} \sim \frac{120\text{ V}}{5\times10^{15}\ \Omega}$ $\boxed{\sim 10^{-14}\text{ A}}$

(b) The resistors form a voltage divider, with the center of your hand at potential $V_h/2$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is $\Delta V = IR \sim \left(10^{-14}\text{ A}\right)\left(10^4\ \Omega\right) \sim 10^{-10}\text{ V}$. So the points where the rubber meets your fingers are at potentials of

$\boxed{\sim \frac{V_h}{2} + 10^{-10}\text{ V}}$ and $\boxed{\sim \frac{V_h}{2} - 10^{-10}\text{ V}}$

***28.51** The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4\times 1.50\text{ V} = 6.00\text{ V}$. The chemical energy they store is

$$\Delta U = q\Delta V = (240\text{ C})(6.00\text{ J/C}) = 1440\text{ J}$$

The radio draws current $I = \frac{\Delta V}{R} = \frac{6.00\text{ V}}{200\ \Omega} = 0.0300\text{ A}$

So, its power is $\mathcal{P} = (\Delta V)I = (6.00\text{ V})(0.0300\text{ A}) = 0.180\text{ W} = 0.180\text{ J/s}$

Then for the time the energy lasts, we have $\mathcal{P} = E/t$: $t = \frac{E}{\mathcal{P}} = \frac{1440\text{ J}}{0.180\text{ J / s}} = 8.00\times10^3\text{ s}$

We could also compute this from $I = Q/t$: $t = \frac{Q}{I} = \frac{240\text{ C}}{0.0300\text{ A}} = 8.00\times10^3\text{ s} = \boxed{2.22\text{ h}}$

***28.52** $I = \dfrac{\mathcal{E}}{R+r}$, so $\mathcal{P} = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$ or $(R+r)^2 = \left(\dfrac{\mathcal{E}^2}{\mathcal{P}}\right) R$

Let $x \equiv \dfrac{\mathcal{E}^2}{\mathcal{P}}$, then $(R+r)^2 = xR$ or $R^2 + (2r - x)R - r^2 = 0$

With $r = 1.20\ \Omega$, this becomes $R^2 + (2.40 - x)R - 1.44 = 0$,

which has solutions of $R = \dfrac{-(2.40 - x) \pm \sqrt{(2.40 - x)^2 - 5.76}}{2}$

(a) With $\mathcal{E} = 9.20$ V and $\mathcal{P} = 12.8$ W, $x = 6.61$: $R = \dfrac{+4.21 \pm \sqrt{(4.21)^2 - 5.76}}{2} =$ $\boxed{3.84\ \Omega}$ or $\boxed{0.375\ \Omega}$

(b) For $\mathcal{E} = 9.20$ V and $\mathcal{P} = 21.2$ W, $x \equiv \dfrac{\mathcal{E}^2}{\mathcal{P}} = 3.99$ $R = \dfrac{+1.59 \pm \sqrt{(1.59)^2 - 5.76}}{2} = \dfrac{1.59 \pm \sqrt{-3.22}}{2}$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R = r = 1.20\ \Omega$, and that maximum is: $\mathcal{P}_{max} = \mathcal{E}^2 / 4r = 17.6$ W

28.53 Using Kirchhoff's loop rule for the closed loop, $+12.0 - 2.00I - 4.00I = 0$, so $I = 2.00$ A

$V_b - V_a = +4.00\text{ V} - (2.00\text{ A})(4.00\ \Omega) - (0)(10.0\ \Omega) = -4.00\text{ V}$

Thus, $|\Delta V_{ab}| =$ $\boxed{4.00\text{ V}}$ and $\boxed{\text{point } a \text{ is at the higher potential}}$.

28.54 The potential difference across the capacitor $\Delta V(t) = \Delta V_{max}\left[1 - e^{-t/RC}\right]$

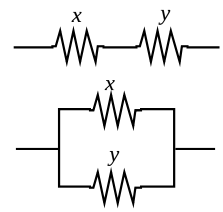
Using $1\text{ Farad} = 1\text{ s}/\Omega$, $4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/R(10.0\times10^{-6}\text{ s}/\Omega)}\right]$

Therefore, $0.400 = 1.00 - e^{-(3.00\times10^5\ \Omega)/R}$ or $e^{-(3.00\times10^5\ \Omega)/R} = 0.600$

Taking the natural logarithm of both sides, $-\dfrac{3.00\times10^5\ \Omega}{R} = \ln(0.600)$

and $R = -\dfrac{3.00\times10^5\ \Omega}{\ln(0.600)} = +5.87\times10^5\ \Omega =$ $\boxed{587\text{ k}\Omega}$

28.55 Let the two resistances be x and y.



Then, $R_s = x + y = \frac{P_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega$ $\qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{P_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega$ $\qquad x^2 - 9.00x + 18.0 = 0$

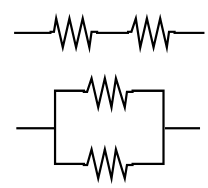
Factoring the second equation, $\qquad (x - 6.00)(x - 3.00) = 0$

so $\qquad x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives $\qquad y = 3.00\ \Omega$ or $y = 6.00\ \Omega$

The two resistances are found to be $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

28.56 Let the two resistances be x and y.



Then, $R_s = x + y = \frac{P_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{P_p}{I^2}$.

From the first equation, $y = \frac{P_s}{I^2} - x$, and the second

becomes $\frac{x\left(P_s/I^2 - x\right)}{x + \left(P_s/I^2 - x\right)} = \frac{P_p}{I^2}$ or $x^2 - \left(\frac{P_s}{I^2}\right)x + \frac{P_s P_p}{I^4} = 0$.

Using the quadratic formula, $x = \frac{P_s \pm \sqrt{P_s^2 - 4P_s P_p}}{2I^2}$.

Then, $y = \frac{P_s}{I^2} - x$ gives $y = \frac{P_s > \sqrt{P_s^2 - 4P_s P_p}}{2I^2}$.

The two resistances are $\boxed{\frac{P_s + \sqrt{P_s^2 - 4P_s P_p}}{2I^2}}$ and $\boxed{\frac{P_s - \sqrt{P_s^2 - 4P_s P_p}}{2I^2}}$

28.57 The current in the simple loop circuit will be $I = \dfrac{\mathcal{E}}{R+r}$

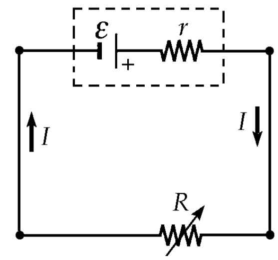


Figure for Goal Solution

(a) $\Delta V_{\text{ter}} = \mathcal{E} - Ir = \dfrac{\mathcal{E}R}{R+r}$ and $\Delta V_{\text{ter}} \to \mathcal{E}$ as $\boxed{R \to \infty}$

(b) $I = \dfrac{\mathcal{E}}{R+r}$ and $I \to \dfrac{\mathcal{E}}{r}$ as $\boxed{R \to 0}$

(c) $\mathcal{P} = I^2R = \mathcal{E}^2 \dfrac{R}{(R+r)^2}$

$$\frac{d\mathcal{P}}{dR} = \mathcal{E}^2 R(-2)(R+r)^{-3} + \mathcal{E}^2(R+r)^{-2} = \frac{-2\mathcal{E}^2 R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} = 0$$

Then $2R = R + r$ and $\boxed{R = r}$

Goal Solution

A battery has an emf $\mathcal{E}$ and internal resistance r. A variable resistor R is connected across the terminals of the battery. Determine the value of R such that (a) the potential difference across the terminals is a maximum, (b) the current in the circuit is a maximum, (c) the power delivered to the resistor is a maximum.

G: If we consider the limiting cases, we can imagine that the **potential** across the battery will be a maximum when $R = \infty$ (open circuit), the **current** will be a maximum when $R = 0$ (short circuit), and the **power** will be a maximum when R is somewhere between these two extremes, perhaps when $R = r$.

O: We can use the definition of resistance to find the voltage and current as functions of R, and the power equation can be differentiated with respect to R.

A: (a) The battery has a voltage $\Delta V_{\text{terminal}} = \mathcal{E} - Ir = \dfrac{\mathcal{E}R}{R+r}$ or as $R \to \infty$, $\Delta V_{\text{terminal}} \to \mathcal{E}$

(b) The circuit's current is $I = \dfrac{\mathcal{E}}{R+r}$ or as $R \to 0$, $I \to \dfrac{\mathcal{E}}{r}$

(c) The power delivered is $\mathcal{P} = I^2R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$

To maximize the power $\mathcal{P}$ as a function of R, we differentiate with respect to R, and require that $d\mathcal{P}/dR = 0$

$$\frac{d\mathcal{P}}{dR} = \mathcal{E}^2 R(-2)(R+r)^{-3} + \mathcal{E}^2(R+r)^{-2} = \frac{-2\mathcal{E}^2 R}{(R+r)^3} + \frac{\mathcal{E}^2}{(R+r)^2} = 0$$

Then $2R = R + r$ and $R = r$

L: The results agree with our predictions. Making load resistance equal to the source resistance to maximize power transfer is called impedance matching.

28.58 (a) $\mathcal{E} - I(\Sigma R) - (\mathcal{E}_1 + \mathcal{E}_2) = 0$

$40.0 \text{ V} - (4.00 \text{ A})[(2.00 + 0.300 + 0.300 + R)\Omega] - (6.00 + 6.00) \text{ V} = 0;$ so $R = \boxed{4.40\ \Omega}$

(b) Inside the supply, $\mathcal{P} = I^2 R = (4.00 \text{ A})^2 (2.00\ \Omega) = \boxed{32.0 \text{ W}}$

Inside both batteries together, $\mathcal{P} = I^2 R = (4.00 \text{ A})^2 (0.600\ \Omega) = \boxed{9.60 \text{ W}}$

For the limiting resistor, $\mathcal{P} = (4.00 \text{ A})^2 (4.40\ \Omega) = \boxed{70.4 \text{ W}}$

(c) $\mathcal{P} = I(\mathcal{E}_1 + \mathcal{E}_2) = (4.00 \text{ A})[(6.00 + 6.00)\text{V}] = \boxed{48.0 \text{ W}}$

28.59 Let R_m = measured value, R = actual value,

I_R = current through the resistor R

I = current measured by the ammeter.

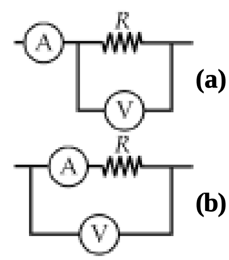


Figure for Goal solution

(a) When using circuit (a), $I_R R = \Delta V = 20\,000(I - I_R)$ or $R = 20\,000\left[\dfrac{I}{I_R} - 1\right]$

But since $I = \dfrac{\Delta V}{R_m}$ and $I_R = \dfrac{\Delta V}{R}$, we have $\dfrac{I}{I_R} = \dfrac{R}{R_m}$

and $R = 20\,000\,\dfrac{(R - R_m)}{R_m}$ (1)

When $R > R_m$, we require $\dfrac{(R - R_m)}{R} \le 0.0500$

Therefore, $R_m \ge R(1 - 0.0500)$ and from (1) we find $\boxed{R \le 1050\ \Omega}$

(b) When using circuit (b), $I_R R = \Delta V - I_R(0.5\ \Omega)$.

But since $I_R = \dfrac{\Delta V}{R_m}$, $R_m = (0.500 + R)$ (2)

When $R_m > R$, we require $\dfrac{(R_m - R)}{R} \le 0.0500$

From (2) we find $\boxed{R \ge 10.0\ \Omega}$

Goal Solution

The value of a resistor R is to be determined using the ammeter-voltmeter setup shown in Figure P28.59. The ammeter has a resistance of $0.500\ \Omega$, and the voltmeter has a resistance of $20\,000\ \Omega$. Within what range of actual values of R will the measured values be correct to within 5.00% if the measurement is made using (a) the circuit shown in Figure P28.59a (b) the circuit shown in Figure P28.59b?

G: An ideal ammeter has zero resistance, and an ideal voltmeter has infinite resistance, so that adding the meter does not alter the current or voltage of the existing circuit. For the non-ideal meters in this problem, a low values of R will give a large voltage measurement error in circuit (b), while a large value of R will give significant current measurement error in circuit (a). We could hope that these meters yield accurate measurements in either circuit for typical resistance values of $1\ \Omega$ to $1\ \text{M}\Omega$.

O: The definition of resistance can be applied to each circuit to find the minimum and maximum current and voltage allowed within the 5.00% tolerance range.

A: (a) In Figure P28.59a, at least a little current goes through the voltmeter, so less current flows through the resistor than the ammeter reports, and the resistance computed by dividing the voltage by the inflated ammeter reading will be too small. Thus, we require that $\Delta V / I = 0.950R$ where I is the current through the ammeter. Call I_R the current through the resistor; then $I - I_R$ is the current in the voltmeter. Since the resistor and the voltmeter are in parallel, the voltage across the meter equals the voltage across the resistor. Applying the definition of resistance:

$$\Delta V = I_R R = (I - I_R)(20\,000\ \Omega) \quad \text{so} \quad I = \frac{I_R(R + 20\,000\ \Omega)}{20\,000\ \Omega}$$

Our requirement is $$\frac{I_R R}{\left(\dfrac{I_R(R + 20\,000\ \Omega)}{20\,000\ \Omega}\right)} \geq 0.95R$$

Solving, $$20\,000\ \Omega \geq 0.95(R + 20\,000\ \Omega) = 0.95R + 19\,000\ \Omega$$

and $$R \leq \frac{1000\ \Omega}{0.95} \quad \text{or} \quad R \leq 1.05\ \text{k}\Omega$$

(b) If R is too small, the resistance of an ammeter in series will significantly reduce the current that would otherwise flow through R. In Figure 28.59b, the voltmeter reading is $I(0.500\ \Omega) + IR$, at least a little larger than the voltage across the resistor. So the resistance computed by dividing the inflated voltmeter reading by the ammeter reading will be too large.

We require $$\frac{V}{I} \leq 1.05R \quad \text{so that} \quad \frac{I\,(0.500\ \Omega) + IR}{I} \leq 1.05R$$

Thus, $$0.500\ \Omega \leq 0.0500R \quad \text{and} \quad R \geq 10.0\ \Omega$$

L: The range of R values seems correct since the ammeter's resistance should be less than 5% of the smallest R value ($0.500\ \Omega \leq 0.05R$ means that R should be greater than $10\ \Omega$), and R should be less than 5% of the voltmeter's internal resistance ($R \leq 0.05 \times 20\ \text{k}\Omega = 1\ \text{k}\Omega$). Only for the restricted range between 10 ohms and 1000 ohms can we indifferently use either of the connections (a) and (b) for a reasonably accurate resistance measurement. For low values of the resistance R, circuit (a) must be used. Only circuit (b) can accurately measure a large value of R.

28.60 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-1/RC}\right)$

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R}\exp\left(-\frac{t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C\exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0-1] = \mathcal{E}^2 C$$

The heating power of the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total heat is $\int dE = \int_0^{\infty}\frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty}\exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy is conserved: $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

28.61 (a) $q = C(\Delta V)\left[1 - e^{-t/RC}\right]$

$$q = \left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1 - e^{-\frac{10.0}{\left(2.00\times10^6\right)\left(1.00\times10^{-6}\right)}}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$

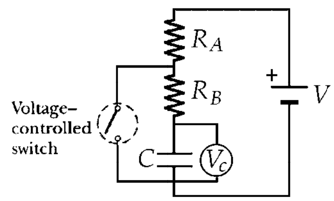
$$I = \left(\frac{10.0\text{ V}}{2.00\times10^6\ \Omega}\right)e^{-5.00} = 3.37\times10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

(c) $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \frac{q}{C}\frac{dq}{dt} = \left(\frac{q}{C}\right)I$

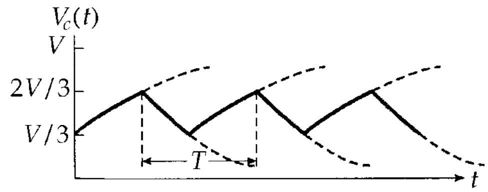
$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right) = 3.34\times10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

(d) $\mathcal{P}_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V}) = 3.37\times10^{-7}\text{ W} = \boxed{337\text{ nW}}$

28.62 Start at the point when the voltage has just reached $\frac{2}{3}V$ and the switch has just closed. The voltage is $\frac{2}{3}V$ and is decaying towards 0 V with a time constant R_BC.



$$V_C(t) = \left[\frac{2}{3}V\right]e^{-t/R_BC}$$



We want to know when $V_C(t)$ will reach $\frac{1}{3}V$.

Therefore, $\left(\frac{1}{3}\right)V = \left[\frac{2}{3}V\right]e^{-t/R_BC}$ or $e^{-t/R_BC} = \frac{1}{2}$

or $t_1 = R_BC \ln 2$

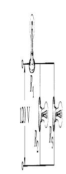
After the switch opens, the voltage is $\frac{1}{3}V$, increasing toward V with time constant $(R_A + R_B)C$:

$$V_C(t) = V - \left[\frac{2}{3}V\right]e^{-t/(R_A+R_B)C}$$

When $V_C(t) = \frac{2}{3}V$, $\frac{2}{3}V = V - \frac{2}{3}Ve^{-t/(R_A+R_B)C}$ or $e^{-t/(R_A+R_B)C} = \frac{1}{2}$

so $t_2 = (R_A + R_B)C \ln 2$ and $T = t_1 + t_2 = \boxed{(R_A + 2R_B)C \ln 2}$

28.63 (a) First determine the resistance of each light bulb: $\mathcal{P} = (\Delta V)^2/R$



$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by applying Equations 28.6 and 28.7:

$$R_{eq} = R_1 + \frac{1}{(1/R_2) + (1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega$$

The total power dissipated in the 360 Ω is $\mathcal{P} = \frac{(\Delta V)^2}{R_{eq}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$

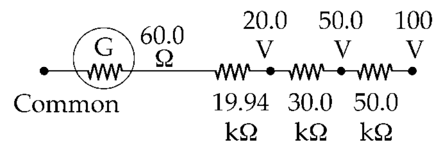
(b) The current through the network is given by $\mathcal{P} = I^2R_{eq}$: $I = \sqrt{\frac{\mathcal{P}}{R_{eq}}} = \sqrt{\frac{40.0\text{ W}}{360\ \Omega}} = \frac{1}{3}\text{ A}$

The potential difference across R_1 is $\Delta V_1 = IR_1 = \left(\frac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\text{ A}\right)\left(\frac{1}{(1/240\ \Omega) + (1/240\ \Omega)}\right) = \boxed{40.0\text{ V}}$$

28.64 $\Delta V = IR$



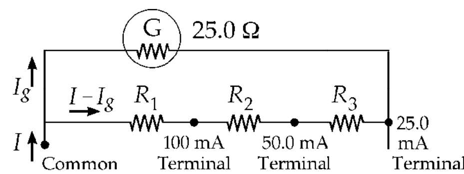
(a) $20.0\text{ V} = (1.00 \times 10^{-3}\text{ A})(R_1 + 60.0\ \Omega)$

$R_1 = 1.994 \times 10^4\ \Omega = \boxed{19.94\text{ k}\Omega}$

(b) $50.0\text{ V} = (1.00 \times 10^{-3}\text{ A})(R_2 + R_1 + 60.0\ \Omega)$ $\qquad R_2 = \boxed{30.0\text{ k}\Omega}$

(c) $100\text{ V} = (1.00 \times 10^{-3}\text{ A})(R_3 + R_1 + 60.0\ \Omega)$ $\qquad R_3 = \boxed{50.0\text{ k}\Omega}$

28.65 Consider the circuit diagram shown, realizing that $I_g = 1.00$ mA. For the 25.0 mA scale:



$$(24.0\text{ mA})(R_1 + R_2 + R_3) = (1.00\text{ mA})(25.0\ \Omega)$$

or $$R_1 + R_2 + R_3 = \left(\frac{25.0}{24.0}\right)\Omega \qquad (1)$$

For the 50.0 mA scale: $$(49.0\text{ mA})(R_1 + R_2) = (1.00\text{ mA})(25.0\ \Omega + R_3)$$

or $$49.0(R_1 + R_2) = 25.0\ \Omega + R_3 \qquad (2)$$

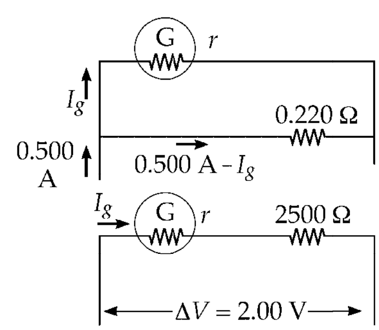
For the 100 mA scale: $$(99.0\text{ mA})R_1 = (1.00\text{ mA})(25.0\ \Omega + R_2 + R_3)$$

or $$99.0R_1 = 25.0\ \Omega + R_2 + R_3 \qquad (3)$$

Solving (1), (2), and (3) simultaneously yields

$$\boxed{R_1 = 0.260\ \Omega, \quad R_2 = 0.261\ \Omega, \quad R_3 = 0.521\ \Omega}$$

28.66 <u>Ammeter:</u> $I_g r = (0.500\text{ A} - I_g)(0.220\ \Omega)$



or $I_g(r + 0.220\ \Omega) = 0.110\text{ V}$ $\qquad (1)$

<u>Voltmeter:</u> $2.00\text{ V} = I_g(r + 2500\ \Omega)$ $\qquad (2)$

Solve (1) and (2) simultaneously to find:

$$I_g = \boxed{0.756\text{ mA}} \quad \text{and} \quad r = \boxed{145\ \Omega}$$

28.67 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0 \quad \text{(steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2: $$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1 + R_2} = \frac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)} = \boxed{333\ \mu\text{A (steady-state)}}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

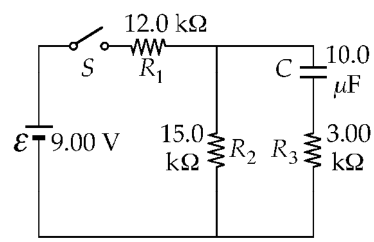
$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\text{ k}\Omega) = \boxed{50.0\ \mu\text{C}}$$

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of $(R_2 + R_3)C$ = (15.0 kΩ + 3.00 kΩ)(10.0 μF) = 0.180 s. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

$$I_i = \frac{(\Delta V)_C}{(R_2 + R_3)} = \frac{IR_2}{(R_2 + R_3)} = \frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega + 3.00\text{ k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})} \quad (\text{for } t > 0)}$$



(a)

(d) The charge q on the capacitor decays from Q_i to $Q_i/5$ according to

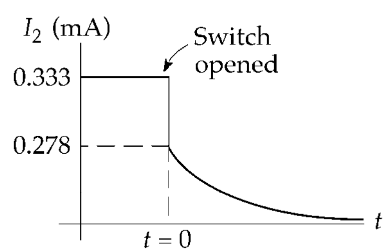
$$q = Q_i e^{-t/(R_2+R_3)C}$$

$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\text{ s})}$$

$$5 = e^{t/0.180\text{ s}}$$

$$\ln 5 = \frac{t}{180\text{ ms}}$$

$$t = (0.180\text{ s})(\ln 5) = \boxed{290\text{ ms}}$$



(b)

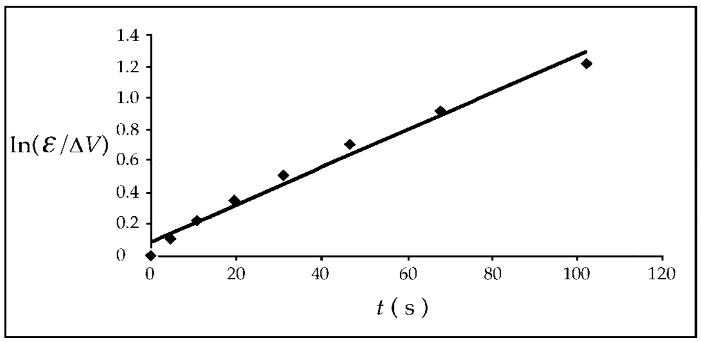
28.68 $\Delta V = \mathcal{E} e^{-t/RC}$ so $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = \left(\frac{1}{RC}\right)t$

A plot of $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope $= \frac{1}{RC}$.

Using the given data values:

t (s)	ΔV (V)	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

(a) A least-square fit to this data yields the graph to the right.



$\Sigma x_i = 282$, $\Sigma x_i^2 = 1.86 \times 10^4$, $\Sigma x_i y_i = 244$, $\Sigma y_i = 4.03$, $N = 8$

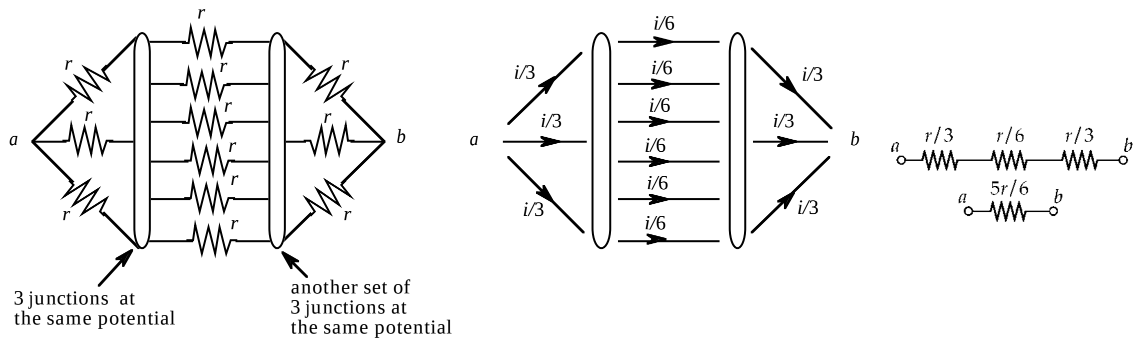
$$\text{Slope} = \frac{N(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0118 \qquad \text{Intercept} = \frac{(\Sigma x_i^2)(\Sigma y_i) - (\Sigma x_i)(\Sigma x_i y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0882$$

The equation of the best fit line is: $\boxed{\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = (0.0118)t + 0.0882}$

(b) Thus, the time constant is $\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.0118} = \boxed{84.7 \text{ s}}$

and the capacitance is $C = \frac{\tau}{R} = \frac{84.7 \text{ s}}{10.0 \times 10^6 \ \Omega} = \boxed{8.47 \ \mu\text{F}}$

28.69



28.70 (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

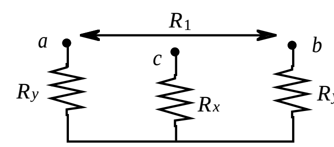


Figure 1

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $$R_y = \frac{1}{2}R_1 \qquad (1)$$

For the second measurement, the equivalent circuit is shown in Figure 2.

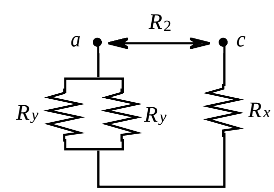


Figure 2

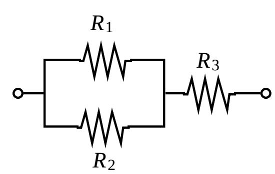
Thus, $$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad (2)$$

Substitute (1) into (2) to obtain: $R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x$, or $\boxed{R_x = R_2 - \frac{1}{4}R_1}$

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$

The antenna is inadequately grounded since this exceeds the limit of $2.00\ \Omega$.

28.71 Since the total current passes through R_3, that resistor will dissipate the most power. When that resistor is operating at its power limit of 32.0 W, the current through it is



$R_1 = R_2 = R_3 = 2.00\ \Omega$

$$I_{\text{total}}^2 = \frac{\mathcal{P}}{R} = \frac{32.0\ \text{W}}{2.00\ \Omega} = 16.0\ \text{A}^2, \text{ or } I_{\text{total}} = 4.00\ \text{A}$$

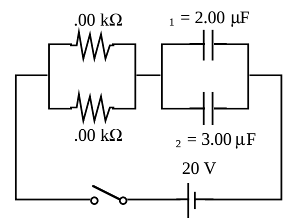
Half of this total current (2.00 A) flows through each of the other two resistors, so the power dissipated in each of them is:

$$\mathcal{P} = \left(\tfrac{1}{2}I_{\text{total}}\right)^2 R = (2.00\ \text{A})^2(2.00\ \Omega) = 8.00\ \text{W}$$

Thus, the total power dissipated in the entire circuit is:

$P_{total} = 32.0\text{ W} + 8.00\text{ W} + 8.00\text{ W} = \boxed{48.0\text{ W}}$

28.72 The total resistance between points b and c is:



$$R = \frac{(2.00\text{ k}\Omega)(3.00\text{ k}\Omega)}{2.00\text{ k}\Omega + 3.00\text{ k}\Omega} = 1.20\text{ k}\Omega$$

The total capacitance between points d and e is:

$$C = 2.00\ \mu\text{F} + 3.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

The potential difference between point d and e in this series RC circuit at any time is:

$$\Delta V = \mathcal{E}\left[1 - e^{-t/RC}\right] = (120.0\text{ V})\left[1 - e^{-1000t/6}\right]$$

Therefore, the charge on each capacitor between points d and e is:

$$q_1 = C_1(\Delta V) = (2.00\ \mu\text{F})(120.0\text{ V})\left[1 - e^{-1000t/6}\right] = \boxed{(240\ \mu\text{C})\left[1 - e^{-1000t/6}\right]}$$

and $q_2 = C_2(\Delta V) = (3.00\ \mu\text{F})(120.0\text{ V})\left[1 - e^{-1000t/6}\right] = \boxed{(360\ \mu\text{C})\left[1 - e^{-1000t/6}\right]}$

***28.73** (a) $R_{eq} = 3R$ $\quad I = \dfrac{\mathcal{E}}{3R}$ $\quad P_{series} = \mathcal{E}I = \boxed{\mathcal{E}^2/3R}$

(b) $R_{eq} = \dfrac{1}{(1/R) + (1/R) + (1/R)} = \dfrac{R}{3}$ $\quad I = \dfrac{3\mathcal{E}}{R}$ $\quad P_{parallel} = \mathcal{E}I = \boxed{\dfrac{3\mathcal{E}^2}{R}}$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

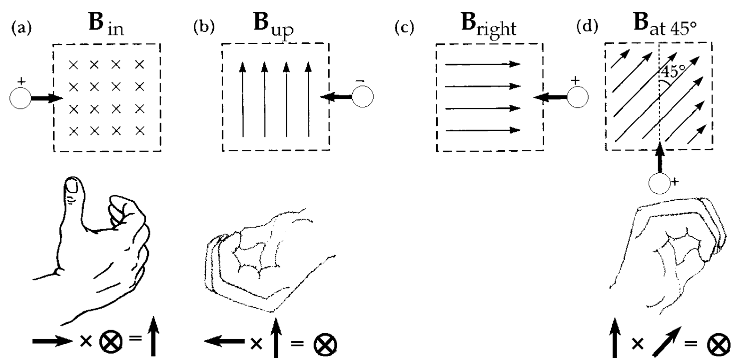
Chapter 29 Solutions

29.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page



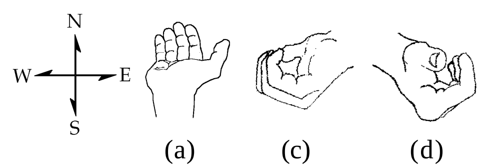
29.2 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ is opposite in direction to $\mathbf{v} \times \mathbf{B}$. Figures are drawn looking down.

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$

(b) North × North = sin 0° = 0: $\boxed{\text{Zero deflection}}$

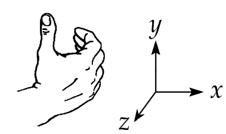
(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$

(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$



29.3 $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$; $\quad |\mathbf{F}_B|(-\mathbf{j}) = -e\,|\mathbf{v}|\,\mathbf{i} \times \mathbf{B}$

Therefore, $B = |\mathbf{B}|(-\mathbf{k})$ which indicates the $\boxed{\text{negative } z \text{ direction}}$



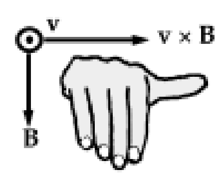
***29.4** (a) $F_B = qvB\sin\theta = (1.60 \times 10^{-19}\text{ C})(3.00 \times 10^{6}\text{ m/s})(3.00 \times 10^{-1}\text{ T})\sin 37.0°$

$$F_B = \boxed{8.67 \times 10^{-14}\text{ N}}$$

(b) $$a = \frac{F}{m} = \frac{8.67 \times 10^{-14}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = \boxed{5.19 \times 10^{13}\text{ m/s}^2}$$

29.5 $F = ma = (1.67 \times 10^{-27}\text{ kg})(2.00 \times 10^{13}\text{ m/s}^2) = 3.34 \times 10^{-14}\text{ N} = qvB\sin 90°$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14}\text{ N}}{(1.60 \times 10^{-19}\text{ C})(1.00 \times 10^{7}\text{ m/s})} = \boxed{2.09 \times 10^{-2}\text{ T}}$$



The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

***29.6** First find the speed of the electron: $\Delta K = \frac{1}{2} mv^2 = e(\Delta V) = \Delta U$

$$v = \sqrt{\frac{2e(\Delta V)}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{C}\right)\left(2400\ \text{J/C}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)}} = 2.90\times10^{7}\ \text{m/s}$$

(a) $F_{B,\,\max} = qvB = (1.60\times10^{-19}\ \text{C})(2.90\times10^{7}\ \text{m/s})(1.70\ \text{T}) = \boxed{7.90\times10^{-12}\ \text{N}}$

(b) $F_{B,\,\min} = \boxed{0}$ occurs when v is either parallel to or anti-parallel to B

29.7 Gravitational force: $F_g = mg = (9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{8.93\times10^{-30}\ \text{N down}}$

Electric force: $F_e = qE = (-1.60\times10^{-19}\ \text{C})100\ \text{N/C down} = \boxed{1.60\times10^{-17}\ \text{N up}}$

Magnetic force: $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\ \text{C}\right)\left(6.00\times10^{6}\ \frac{\text{m}}{\text{s}}\ \mathbf{E}\right)\times\left(50.0\times10^{-6}\ \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\ \mathbf{N}\right)$

$\mathbf{F}_B = -\,4.80\times10^{-17}\ \text{N up} = \boxed{4.80\times10^{-17}\ \text{N down}}$

29.8 We suppose the magnetic force is small compared to gravity. Then its horizontal velocity component stays nearly constant. We call it $v\,\mathbf{i}$.

From $v_y{}^2 = v_{yi}{}^2 + 2a_y(y - y_i)$, the vertical component at impact is $-\sqrt{2gh}\,\mathbf{j}$. Then,

$$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = Q\left(v\mathbf{i} - \sqrt{2gh}\ \mathbf{j}\right)\times B\mathbf{k} = QvB(-\mathbf{j}) - Q\sqrt{2gh}\ B\mathbf{i}$$

$\mathbf{F}_B = QvB$ vertical $+\ Q\sqrt{2gh}\ B$ horizontal

$\mathbf{F}_B = 5.00\times10^{-6}\ \text{C}(20.0\ \text{m/s})(0.0100\ \text{T})\ \mathbf{j} + 5.00\times10^{-6}\ \text{C}\sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m})}\ (0.0100\ \text{T})\ \mathbf{i}$

$\mathbf{F}_B = \boxed{(1.00\times10^{-6}\ \text{N})\ \text{vertical} + (0.990\times10^{-6}\ \text{N})\ \text{horizontal}}$

29.9 $F_B = qvB\sin\theta$ so $8.20\times10^{-13}\ \text{N} = (1.60\times10^{-19}\ \text{C})(4.00\times10^{6}\ \text{m/s})(1.70\ \text{T})\sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9° \text{ or } 131°}$

29.10 $q\mathbf{E} = \left(-1.60\times10^{-19}\text{ C}\right)\left(20.0\text{ N / C}\right)\mathbf{k} = \left(-3.20\times10^{-18}\text{ N}\right)\mathbf{k}$

$\Sigma\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = m\mathbf{a}$

$\left(-3.20\times10^{-18}\text{ N}\right)\mathbf{k} - 1.60\times10^{-19}\text{ C}(1.20\times10^{4}\text{ m / s }\mathbf{i})\times\mathbf{B} = (9.11\times10^{-31})(2.00\times10^{12}\text{ m / s}^2)\mathbf{k}$

$-(3.20\times10^{-18}\text{ N})\mathbf{k} - (1.92\times10^{-15}\text{ C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = (1.82\times10^{-18}\text{ N})\mathbf{k}$

$(1.92\times10^{-15}\text{ C}\cdot\text{m / s})\mathbf{i}\times\mathbf{B} = -(5.02\times10^{-18}\text{ N})\mathbf{k}$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z = \boxed{0}$ and $B_y = \boxed{-2.62\text{ mT}}$

29.11 $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\mathbf{i} + (1+6)\mathbf{j} + (4+4)\mathbf{k} = 10\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$$

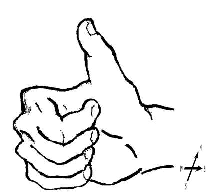
$|\mathbf{v}\times\mathbf{B}| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\text{ T}\cdot\text{m / s}$

$|\mathbf{F}_B| = q|\mathbf{v}\times\mathbf{B}| = \left(1.60\times10^{-19}\text{ C}\right)\left(14.6\text{ T}\cdot\text{m/s}\right) = \boxed{2.34\times10^{-18}\text{ N}}$

29.12 $$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\right)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3.70\times10^{5} & 0 \\ 1.40 & 2.10 & 0 \end{vmatrix}$$

$$\mathbf{F}_B = \left(-1.60\times10^{-19}\text{ C}\right)\left[(0-0)\mathbf{i} + (0-0)\mathbf{j} + \left(0 - (1.40\text{ T})\left(3.70\times10^{5}\text{ m/s}\right)\right)\mathbf{k}\right] = \boxed{\left(8.29\times10^{-14}\;\mathbf{k}\right)\text{N}}$$

29.13 $F_B = ILB\sin\theta$ with $F_B = F_g = mg$



$mg = ILB\sin\theta$ so $\dfrac{m}{L}g = IB\sin\theta$

$I = 2.00\text{ A}$ and $\dfrac{m}{L} = \left(0.500\text{ g / cm}\right)\left(\dfrac{100\text{ cm / m}}{1000\text{ g / kg}}\right) = 5.00\times10^{-2}\text{ kg / m}$

Thus $(5.00\times10^{-2})(9.80) = (2.00)B\sin 90.0°$

$B = \boxed{0.245\text{ Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$

Goal Solution

A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

G: Since $\mathbf{I} = 2.00$ A south, **B** must be to the east to make **F** upward according to the right-hand rule for currents in a magnetic field.

The magnitude of **B** should be significantly greater than the earth's magnetic field (~ 50 μT), since we do not typically see wires levitating when current flows through them.

O: The force on a current-carrying wire in a magnetic field is $\mathbf{F}_B = I\mathbf{l}\times\mathbf{B}$, from which we can find **B**.

A: With I to the south and **B** to the east, the force on the wire is simply $F_B = IlB\sin 90°$, which must oppose the weight of the wire, mg. So,

$$B = \frac{F_B}{Il} = \frac{mg}{Il} = \frac{g}{I}\left(\frac{m}{l}\right) = \left(\frac{9.80\text{ m/s}^2}{2.00\text{ A}}\right)\left(0.500\ \frac{\text{g}}{\text{cm}}\right)\left(\frac{10^2\text{ cm/m}}{10^3\text{ g/kg}}\right) = 0.245\text{ T}$$

L: The required magnetic field is about 5000 times stronger than the earth's magnetic field. Thus it was reasonable to ignore the earth's magnetic field in this problem. In other situations the earth's field can have a significant effect.

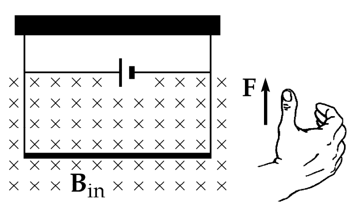
29.14 $\mathbf{F}_B = I\mathbf{L}\times\mathbf{B} = (2.40\text{ A})(0.750\text{ m})\mathbf{i}\times(1.60\text{ T})\mathbf{k} = \boxed{(-2.88\ \mathbf{j})\text{ N}}$

29.15 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

29.16 $$\frac{|\mathbf{F}_B|}{L} = \frac{mg}{L} = \frac{I|\mathbf{L}\times\mathbf{B}|}{L}$$



$$I = \frac{mg}{BL} = \frac{(0.0400\text{ kg/m})(9.80\text{ m/s}^2)}{3.60\text{ T}} = \boxed{0.109\text{ A}}$$

The direction of I in the bar is $\boxed{\text{to the right}}$.

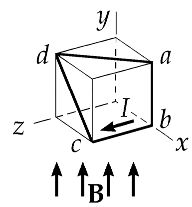
29.17 The magnetic and gravitational forces must balance. Therefore, it is necessary to have $F_B = BIL = mg$, or $I = (mg/BL) = (\lambda g/B)$ [λ is the mass per unit length of the wire].

Thus, $I = \dfrac{(1.00\times10^{-3}\text{ kg/m})(9.80\text{ m/s}^2)}{(5.00\times10^{-5}\text{ T})} = \boxed{196\text{ A}}$ (if $B = 50.0\ \mu$T)

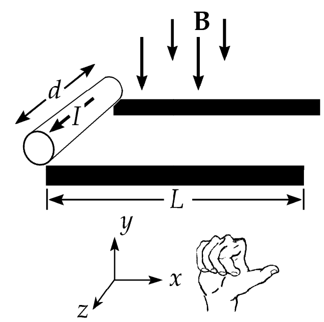
The required direction of the current is $\boxed{\text{eastward}}$, since East × North = Up.

29.18 For each segment, $I = 5.00$ A and $\mathbf{B} = 0.0200\text{ N/A}\cdot\text{m }\mathbf{j}$

Segment	$\mathbf{L}$	$\mathbf{F}_B = I(\mathbf{L}\times\mathbf{B})$
ab	$-0.400\text{ m }\mathbf{j}$	$\boxed{0}$
bc	$0.400\text{ m }\mathbf{k}$	$\boxed{(40.0\text{ mN})(-\mathbf{i})}$
cd	$-0.400\text{ m }\mathbf{i} + 0.400\text{ m }\mathbf{j}$	$\boxed{(40.0\text{ mN})(-\mathbf{k})}$
da	$0.400\text{ m }\mathbf{i} - 0.400\text{ m }\mathbf{k}$	$\boxed{(40.0\text{ mN})(\mathbf{k}+\mathbf{i})}$



29.19 The rod feels force $\mathbf{F}_B = I(\mathbf{d}\times\mathbf{B}) = Id(\mathbf{k})\times B(-\mathbf{j}) = IdB(\mathbf{i})$



The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad IdBL = \tfrac{3}{4}mv^2$$

$$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\text{ A})(0.120\text{ m})(0.240\text{ T})(0.450\text{ m})}{3(0.720\text{ kg})}} = \boxed{1.07\text{ m/s}}$$

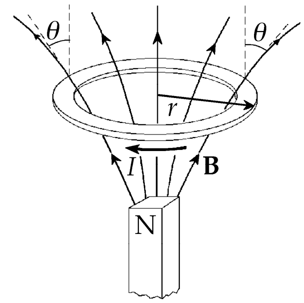
29.20 The rod feels force $\mathbf{F}_B = I(\mathbf{d}\times\mathbf{B}) = Id(\mathbf{k})\times B(-\mathbf{j}) = IdB(\mathbf{i})$

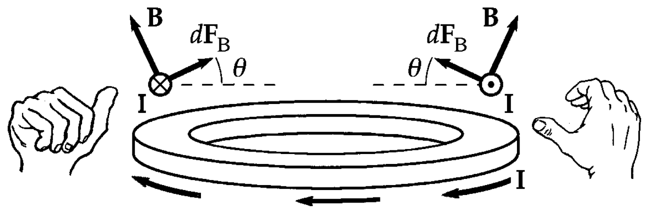
The work-energy theorem is $(K_{\text{trans}} + K_{\text{rot}})_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}})_f$

$$0 + 0 + Fs\cos\theta = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$IdBL\cos 0^\circ = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \quad \text{and} \quad v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

29.21 The magnetic force on each bit of ring is $I\,d\mathbf{s} \times \mathbf{B} = I\,ds\,B$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $I\,ds\,B \sin\theta$ all add to $\boxed{I\,2\pi rB \sin\theta \text{ up}}$.

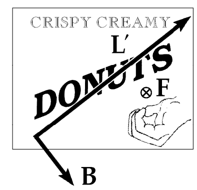




***29.22** Take the x-axis east, the y-axis up, and the z-axis south. The field is

$$\mathbf{B} = (52.0\ \mu\text{T})\cos 60.0^\circ\,(-\mathbf{k}) + (52.0\ \mu\text{T})\sin 60.0^\circ\,(-\mathbf{j})$$

The current then has equivalent length: $\mathbf{L}' = 1.40\ \text{m}(-\mathbf{k}) + 0.850\ \text{m}(\mathbf{j})$



$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B} = (0.0350\ \text{A})(0.850\,\mathbf{j} - 1.40\,\mathbf{k})\text{m} \times (-45.0\,\mathbf{j} - 26.0\,\mathbf{k})10^{-6}\ \text{T}$$

$$\mathbf{F}_B = 3.50 \times 10^{-8}\ \text{N}(-22.1\,\mathbf{i} - 63.0\,\mathbf{i}) = 2.98 \times 10^{-6}\ \text{N}(-\mathbf{i}) = \boxed{2.98\ \mu\text{N west}}$$

29.23 (a) $2\pi r = 2.00$ m so $r = 0.318$ m

$$\mu = IA = (17.0 \times 10^{-3}\ \text{A})\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA} \cdot \text{m}^2}$$

(b) $\tau = \boldsymbol{\mu} \times \mathbf{B}$ so $\tau = (5.41 \times 10^{-3}\ \text{A} \cdot \text{m}^2)(0.800\ \text{T}) = \boxed{4.33\ \text{mN} \cdot \text{m}}$

***29.24** $\tau = \mu B \sin\theta$ so $4.60 \times 10^{-3}\ \text{N} \cdot \text{m} = \mu(0.250)\sin 90.0^\circ$

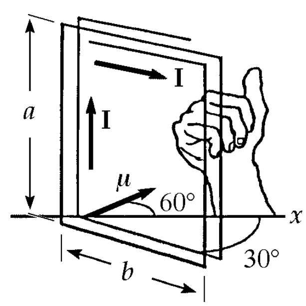
$$\mu = 1.84 \times 10^{-2}\ \text{A} \cdot \text{m}^2 = \boxed{18.4\ \text{mA} \cdot \text{m}^2}$$

29.25 $\tau = NBAI \sin\theta$

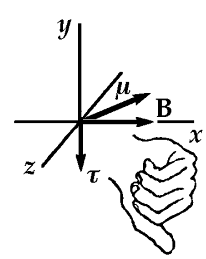
$$\tau = 100(0.800\ \text{T})(0.400 \times 0.300\ \text{m}^2)(1.20\ \text{A})\sin 60^\circ$$

$$\tau = \boxed{9.98\ \text{N} \cdot \text{m}}$$

Note that θ is the angle between the magnetic moment and the **B** field. The loop will rotate so as to align the magnetic moment with the **B** field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.



(a)



(b)

29.26 (a) Let θ represent the unknown angle; L, the total length of the wire; and d, the length of one side of the square coil. Then, use the right-hand rule to find

$$\mu = NAI = \left(\frac{L}{4d}\right)d^2 I \quad \text{at angle } \theta \text{ with the horizontal.}$$

At equilibrium, $\Sigma\tau = (\boldsymbol{\mu} \times \mathbf{B}) - (\mathbf{r} \times m\mathbf{g}) = 0$

$$\left(\frac{ILBd}{4}\right)\sin(90.0° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0 \quad \text{and} \quad \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{ILBd}{4}\right)\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{ILB}{2mg}\right) = \tan^{-1}\left(\frac{(3.40 \text{ A})(4.00 \text{ m})(0.0100 \text{ T})}{2(0.100 \text{ kg})(9.80 \text{ m/s}^2)}\right) = \boxed{3.97°}$$

(b) $$\tau_m = \left(\frac{ILBd}{4}\right)\cos\theta = \frac{1}{4}(3.40 \text{ A})(4.00 \text{ m})(0.0100 \text{ T})(0.100 \text{ m})\cos 3.97° = \boxed{3.39 \text{ mN} \cdot \text{m}}$$

29.27 From $\tau = \boldsymbol{\mu} \times \mathbf{B} = I\mathbf{A} \times \mathbf{B}$, the magnitude of the torque is $IAB \sin 90.0°$

(a) Each side of the triangle is 40.0 cm / 3.

Its altitude is $\sqrt{13.3^2 - 6.67^2}$ cm = 11.5 cm and its area is

$$A = \frac{1}{2}(11.5 \text{ cm})(13.3 \text{ cm}) = 7.70 \times 10^{-3} \text{ m}^2$$

Then $\tau = (20.0 \text{ A})(7.70 \times 10^{-3} \text{ m}^2)(0.520 \text{ N} \cdot \text{s/C} \cdot \text{m}) = \boxed{80.1 \text{ mN} \cdot \text{m}}$

(b) Each side of the square is 10.0 cm and its area is $100 \text{ cm}^2 = 10^{-2} \text{ m}^2$.

$$\tau = (20.0 \text{ A})(10^{-2} \text{ m}^2)(0.520 \text{ T}) = \boxed{0.104 \text{ N} \cdot \text{m}}$$

(c) $r = 0.400 \text{ m}/2\pi = 0.0637 \text{ m}$

$$A = \pi r^2 = 1.27 \times 10^{-2} \text{ m}^2$$

$$\tau = (20.0 \text{ A})(1.27 \times 10^{-2} \text{ m}^2)(0.520) = \boxed{0.132 \text{ N} \cdot \text{m}}$$

(d) The circular loop experiences the largest torque.

***29.28** Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B \sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. Its energy is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B \sin\theta \, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \quad \text{or} \quad \boxed{U = -\boldsymbol{\mu} \cdot \mathbf{B}}$$

***29.29** (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\min} = -\mu B\cos 0^\circ = -\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = -5.34\times10^{-7}\ \text{J}$

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\max} = -\mu B\cos 180^\circ = +\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) = +5.34\times10^{-7}\ \text{J}$

(b) $U_{\min} + W = U_{\max}$: $W = U_{\max} - U_{\min} = +5.34\times10^{-7}\ \text{J} - \left(-5.34\times10^{-7}\ \text{J}\right) = \boxed{1.07\ \mu\text{J}}$

29.30 (a) $\tau = \boldsymbol{\mu}\times\mathbf{B}$, so $\tau = |\boldsymbol{\mu}\times\mathbf{B}| = \mu B\sin\theta = NIAB\sin\theta$

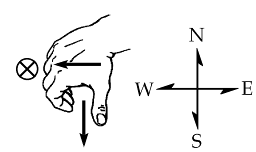
$\tau_{\max} = NIAB\sin 90.0^\circ = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = \boxed{118\ \mu\text{N}\cdot\text{m}}$

(b) $U = -\boldsymbol{\mu}\cdot\mathbf{B}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\ \text{A})\left[\pi(0.0500\ \text{m})^2\right]\left(3.00\times10^{-3}\ \text{T}\right) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$

29.31 (a) $B = 50.0\times10^{-6}$ T; $v = 6.20\times10^{6}$ m/s



Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$F_B = qvB\sin\theta$

$F_B = (1.60\times10^{-19}\ \text{C})(6.20\times10^{6}\ \text{m/s})(50.0\times10^{-6}\ \text{T})\sin 90.0^\circ = \boxed{4.96\times10^{-17}\ \text{N}}$

(b) $F = \dfrac{mv^2}{r}$ so $r = \dfrac{mv^2}{F} = \dfrac{(1.67\times10^{-27}\ \text{kg})(6.20\times10^{6}\ \text{m/s})^2}{4.96\times10^{-17}\ \text{N}} = \boxed{1.29\ \text{km}}$

29.32 (a) $\frac{1}{2}mv^2 = q(\Delta V)$ $\frac{1}{2}(3.20\times10^{-26}\ \text{kg})\,v^2 = (1.60\times10^{-19}\ \text{C})(833\ \text{V})$ $v = 91.3$ km/s

The magnetic force provides the centripetal force: $qvB\sin\theta = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB\sin 90.0°} = \frac{(3.20 \times 10^{-26}\ \text{kg})(9.13 \times 10^{4}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.920\ \text{N} \cdot \text{s/C} \cdot \text{m})} = \boxed{1.98\ \text{cm}}$$

29.33 For each electron, $|q|vB\sin 90.0° = \dfrac{mv^2}{r}$ and $v = \dfrac{eBr}{m}$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \tfrac{1}{2}mv_{1i}^2 + 0 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2$$

$$K = \tfrac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \tfrac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right) = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{e(1.60\times10^{-19}\text{ C})(0.0440\text{ N}\cdot\text{s / C}\cdot\text{m})^2}{2(9.11\times10^{-31}\text{ kg})}\left[(0.0100\text{ m})^2 + (0.0240\text{ m})^2\right] = \boxed{115\text{ keV}}$$

29.34 We begin with $qvB = \dfrac{mv^2}{R}$, so $v = \dfrac{qRB}{m}$

The time to complete one revolution is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{\left(\dfrac{qRB}{m}\right)} = \dfrac{2\pi m}{qB}$

Solving for B, $B = \dfrac{2\pi m}{qT} = \boxed{6.56\times10^{-2}\text{ T}}$

29.35 $q(\Delta V) = \tfrac{1}{2}mv^2$ or $v = \sqrt{\dfrac{2q(\Delta V)}{m}}$

Also, $qvB = \dfrac{mv^2}{r}$ so $r = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2q(\Delta V)}{m}} = \sqrt{\dfrac{2m(\Delta V)}{qB^2}}$

Therefore, $r_p^2 = \dfrac{2m_p(\Delta V)}{eB^2}$

$$r_d^2 = \frac{2m_d(\Delta V)}{q_dB^2} = \frac{2(2m_p)(\Delta V)}{eB^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

and $$r_\alpha{}^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p{}^2$$

The conclusion is: $\boxed{r_\alpha = r_d = \sqrt{2}\,r_p}$

Goal Solution

29.35 A proton (charge $+e$, mass m_p), a deuteron (charge $+e$, mass $2m_p$), and an alpha particle, (charge $+2e$, mass $4m_p$) are accelerated through a common potential difference ΔV. The particles enter a uniform magnetic field **B** with a velocity in a direction perpendicular to **B**. The proton moves in a circular path of radius r_p. Determine the values of the radii of the circular orbits for the deuteron r_d and the alpha particle r_α in terms of r_p.

G: In general, particles with greater speed, more mass, and less charge will have larger radii as they move in a circular path due to a constant magnetic force. Since the effects of mass and charge have opposite influences on the path radius, it is somewhat difficult to predict which particle will have the larger radius. However, since the mass and charge ratios of the three particles are all similar in magnitude within a factor of four, we should expect that the radii also fall within a similar range.

O: The radius of each particle's path can be found by applying Newton's second law, where the force causing the centripetal acceleration is the magnetic force: $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$. The speed of the particles can be found from the kinetic energy resulting from the change in electric potential given.

A: An electric field changes the speed of each particle according to $(K+U)_i = (K+U)_f$. Therefore, assuming that the particles start from rest, we can write $q\Delta V = \frac{1}{2}mv^2$.

The magnetic field changes their direction as described by $\Sigma\mathbf{F} = m\mathbf{a}$: $qvB\sin 90° = \dfrac{mv^2}{r}$

thus $$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

For the protons, $$r_p = \frac{1}{B}\sqrt{\frac{2m_p\Delta V}{e}}$$

For the deuterons, $$r_d = \frac{1}{B}\sqrt{\frac{2(2m_p)\Delta V}{e}} = \sqrt{2}r_p$$

For the alpha particles, $$r_\alpha = \frac{1}{B}\sqrt{\frac{2(4m_p)\Delta V}{2e}} = \sqrt{2}r_p$$

L: Somewhat surprisingly, the radii of the deuterons and alpha particles are the same and are only 41% greater than for the protons.

29.36 (a) We begin with $qvB = \dfrac{mv^2}{R}$, or $qRB = mv$. But, $L = mvR = qR^2B$.

Therefore, $$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00\times10^{-25}\ \text{J}\cdot\text{s}}{(1.60\times10^{-19}\ \text{C})(1.00\times10^{-3}\ \text{T})}} = 0.0500\ \text{m} = \boxed{5.00\ \text{cm}}$$

(b) Thus, $$v = \frac{L}{mR} = \frac{4.00\times10^{-25}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.0500\ \text{m})} = \boxed{8.78\times10^{6}\ \text{m/s}}$$

29.37 $$\omega = \frac{qB}{m} = \frac{(1.60\times10^{-19}\text{ C})(5.20\text{ T})}{1.67\times10^{-27}\text{ kg}} = \boxed{4.98\times10^{8}\text{ rad/s}}$$

29.38 $$\frac{1}{2}mv^2 = q(\Delta V) \quad\text{so}\quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

$$r = \frac{mv}{qB} \quad\text{so}\quad r = \frac{m\sqrt{2q(\Delta V)/m}}{qB}$$

$$r^2 = \frac{m}{q}\cdot\frac{2(\Delta V)}{B^2} \quad\text{and}\quad (r')^2 = \frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$$

$$m = \frac{qB^2r^2}{2(\Delta V)} \quad\text{and}\quad (m') = \frac{(q')B^2(r')^2}{2(\Delta V)} \quad\text{so}\quad \frac{m'}{m} = \frac{q'}{q}\cdot\frac{(r')^2}{r^2} = \left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2 = \boxed{8}$$

29.39 $$E = \frac{1}{2}mv^2 = e(\Delta V) \quad\text{and}\quad evB\sin 90° = mv^2/R$$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e(\Delta V)}{m}} = \frac{1}{R}\sqrt{\frac{2m(\Delta V)}{e}}$$

$$B = \frac{1}{5.80\times10^{10}\text{ m}}\sqrt{\frac{2(1.67\times10^{-27}\text{ kg})(10.0\times10^{6}\text{ V})}{1.60\times10^{-19}\text{ C}}} = \boxed{7.88\times10^{-12}\text{ T}}$$

29.40 $$r = \frac{mv}{qB} \quad\text{so}\quad m = \frac{rqB}{v} = \frac{(7.94\times10^{-3}\text{ m})(1.60\times10^{-19}\text{ C})(1.80\text{ T})}{4.60\times10^{5}\text{ m/s}}$$

$$m = 4.97\times10^{-27}\text{ kg}\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = \boxed{2.99\text{ u}}$$

The particle is singly ionized: either a tritium ion, $\boxed{{}^{3}_{1}\text{H}^{+}}$, or a helium ion, $\boxed{{}^{3}_{2}\text{He}^{+}}$.

29.41 $F_B = F_e$ so $qvB = qE$ where $v = \sqrt{2K/m}$. K is kinetic energy of the electrons.

$$E = vB = \sqrt{\frac{2K}{m}}B = \left(\frac{2(750)(1.60\times10^{-19})}{9.11\times10^{-31}}\right)^{1/2}(0.0150) = \boxed{244\text{ kV/m}}$$

29.42 $K = \frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$$|\mathbf{F}_B| = |q\mathbf{v}\times\mathbf{B}| = \frac{mv^2}{r} \quad r = \frac{mv}{qB} = \frac{m}{q}\frac{\sqrt{2q(\Delta V)/m}}{B} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $r_{238} = \sqrt{\frac{2(238\times 1.66\times 10^{-27})2000}{1.60\times 10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28\times 10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$

(b) $r_{235} = \boxed{8.23\text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.0064$$

The ratios of the orbit radius for different ions are independent of ΔV and B.

29.43 In the velocity selector: $v = \frac{E}{B} = \frac{2500\text{ V/m}}{0.0350\text{ T}} = 7.14\times 10^4\text{ m/s}$

In the deflection chamber: $r = \frac{mv}{qB} = \frac{(2.18\times 10^{-26}\text{ kg})(7.14\times 10^4\text{ m/s})}{(1.60\times 10^{-19}\text{ C})(0.0350\text{ T})} = \boxed{0.278\text{ m}}$

29.44 $K = \frac{1}{2}mv^2$: $(34.0\times 10^6\text{ eV})(1.60\times 10^{-19}\text{ J/eV}) = \frac{1}{2}(1.67\times 10^{-27}\text{ kg})v^2$

$v = 8.07\times 10^7\text{ m/s}$ $r = \frac{mv}{qB} = \frac{(1.67\times 10^{-27}\text{ kg})(8.07\times 10^7\text{ m/s})}{(1.60\times 10^{-19}\text{ C})(5.20\text{ T})} = \boxed{0.162\text{ m}}$

29.45 (a) $F_B = qvB = \frac{mv^2}{R}$

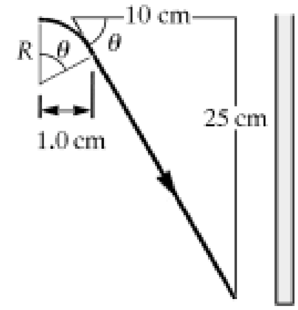
$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{(1.60\times 10^{-19}\text{ C})(0.450\text{ T})}{1.67\times 10^{-27}\text{ kg}} = \boxed{4.31\times 10^7\text{ rad/s}}$$

(b) $v = \frac{qBR}{m} = \frac{(1.60\times 10^{-19}\text{ C})(0.450\text{ T})(1.20\text{ m})}{1.67\times 10^{-27}\text{ kg}} = \boxed{5.17\times 10^7\text{ m/s}}$

29.46 $F_B = qvB = \frac{mv^2}{r}$

$$B = \frac{mv}{qr} = \frac{4.80\times 10^{-16}\text{ kg}\cdot\text{m/s}}{(1.60\times 10^{-19}\text{ C})(1000\text{ m})} = \boxed{3.00\text{ T}}$$

29.47 $\theta = \tan^{-1}\dfrac{25.0}{10.0} = 68.2°$ and $R = \dfrac{1.00\text{ cm}}{\sin 68.2°} = 1.08\text{ cm}$



Ignoring relativistic correction, the kinetic energy of the electrons is

$\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\dfrac{2q(\Delta V)}{m}} = 1.33\times10^8\text{ m / s}$

From the centripetal force $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11\times10^{-31}\text{ kg})(1.33\times10^8\text{ m / s})}{(1.60\times10^{-19}\text{ C})(1.08\times10^{-2}\text{ m})} = \boxed{70.1\text{ mT}}$$

29.48 (a) $R_H \equiv \dfrac{1}{nq}$ so $n = \dfrac{1}{qR_H} = \dfrac{1}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.840\times10^{-10}\text{ m}^3/\text{C}\right)} = \boxed{7.44\times10^{28}\text{ m}^{-3}}$

(b) $\Delta V_H = \dfrac{IB}{nqt}$

$$B = \frac{nqt(\Delta V_H)}{I} = \frac{\left(7.44\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(0.200\times10^{-3}\text{ m}\right)\left(15.0\times10^{-6}\text{ V}\right)}{20.0\text{ A}} = \boxed{1.79\text{ T}}$$

29.49 $$\frac{1}{nq} = \frac{t(\Delta V_H)}{IB} = \frac{(35.0\times10^{-6}\text{ V})(0.400\times10^{-2}\text{ m})}{(21.0\text{ A})(1.80\text{ T})} = \boxed{3.70\times10^{-9}\text{ m}^3/\text{C}}$$

29.50 Since $\Delta V_H = \dfrac{IB}{nqt}$, and given that $I = 50.0$ A, $B = 1.30$ T, and $t = 0.330$ mm, the number of charge carriers per unit volume is

$$n = \frac{IB}{e(\Delta V_H)t} = \boxed{1.28\times10^{29}\text{ m}^{-3}}$$

The number density of atoms we compute from the density:

$$n_0 = \frac{8.92\text{ g}}{\text{cm}^3}\left(\frac{1\text{ mole}}{63.5\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{\text{mole}}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right) = 8.46\times10^{28}\text{ atom / m}^3$$

So the number of conduction electrons per atom is

$$\frac{n}{n_0} = \frac{1.28\times10^{29}}{8.46\times10^{28}} = \boxed{1.52}$$

29.51 $B = \dfrac{nqt(\Delta V_H)}{I} = \dfrac{(8.48\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.00\times10^{-3}\ \text{m})(5.10\times10^{-12}\ \text{V})}{8.00\ \text{A}}$

$B = 4.32\times10^{-5}\ \text{T} = \boxed{43.2\ \mu\text{T}}$

Goal Solution

In an experiment designed to measure the Earth's magnetic field using the Hall effect, a copper bar 0.500 cm thick is positioned along an east-west direction. If a current of 8.00 A in the conductor results in a Hall voltage of 5.10 pV, what is the magnitude of the Earth's magnetic field? (Assume that $n = 8.48\times10^{28}$ electrons/m^3 and that the plane of the bar is rotated to be perpendicular to the direction of **B**.)

G: The Earth's magnetic field is about 50 μT (see Table 29.1), so we should expect a result of that order of magnitude.

O: The magnetic field can be found from the Hall effect voltage:

$$\Delta V_H = \frac{IB}{nqt} \quad \text{or} \quad B = \frac{nqt\Delta V_H}{I}$$

A: From the Hall voltage,

$$B = \frac{(8.48\times10^{28}\ \text{e}^-/\text{m}^3)(1.60\times10^{-19}\ \text{C/e}^-)(0.00500\ \text{m})(5.10\times10^{-12}\ \text{V})}{8.00\ \text{A}} = 4.32\times10^{-5}\ \text{T} = 43.2\ \mu\text{T}$$

L: The calculated magnetic field is slightly less than we expected but is reasonable considering that the Earth's local magnetic field varies in both magnitude and direction.

29.52 (a) $\Delta V_H = \dfrac{IB}{nqt}$ so $\dfrac{nqt}{I} = \dfrac{B}{\Delta V_H} = \dfrac{0.0800\ \text{T}}{0.700\times10^{-6}\ \text{V}} = 1.14\times10^5\ \dfrac{\text{T}}{\text{V}}$

Then, the unknown field is $B = \left(\dfrac{nqt}{I}\right)(\Delta V_H)$

$B = (1.14\times10^5\ \text{T/V})(0.330\times10^{-6}\ \text{V}) = 0.0377\ \text{T} = \boxed{37.7\ \text{mT}}$

(b) $\dfrac{nqt}{I} = 1.14\times10^5\ \dfrac{\text{T}}{\text{V}}$ so $n = \left(1.14\times10^5\ \dfrac{\text{T}}{\text{V}}\right)\dfrac{I}{qt}$

$n = \left(1.14\times10^5\ \dfrac{\text{T}}{\text{V}}\right)\dfrac{0.120\ \text{A}}{(1.60\times10^{-19}\ \text{C})(2.00\times10^{-3}\ \text{m})} = \boxed{4.29\times10^{25}\ \text{m}^{-3}}$

29.53 $|q|\,vB\sin 90° = \frac{mv^2}{r}$ $\quad\therefore\ \omega = \frac{v}{r} = \frac{eB}{m} = \frac{\theta}{t}$

(a) The time it takes the electron to complete π radians is

$$t = \frac{\theta}{\omega} = \frac{\theta m}{eB} = \frac{(\pi\text{ rad})(9.11\times10^{-31}\text{ kg})}{(1.60\times10^{-19}\text{ C})(0.100\text{ N}\cdot\text{s/C}\cdot\text{m})} = \boxed{1.79\times10^{-10}\text{ s}}$$

(b) Since $v = \frac{|q|Br}{m}$,

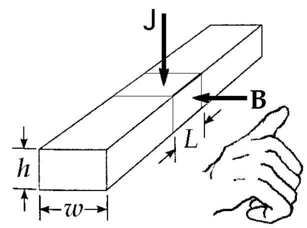
$$K_e = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m} = \frac{e(1.60\times10^{-19}\text{ C})(0.100\text{ N}\cdot\text{s / Cm})^2(2.00\times10^{-2}\text{ m})^2}{2(9.11\times10^{-31}\text{ kg})} = \boxed{351\text{ keV}}$$

29.54 $\sum F_y = 0$: $\quad +n - mg = 0$

$\sum F_x = 0$: $\quad -\mu_k n + IBd\sin 90.0° = 0$

$$B = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\text{ kg})(9.80\text{ m/s}^2)}{(10.0\text{ A})(0.500\text{ m})} = \boxed{39.2\text{ mT}}$$

29.55 (a) The $\boxed{\text{electric current experiences a magnetic force}}$.



$I(\mathbf{h}\times\mathbf{B})$ in the direction of $\mathbf{L}$.

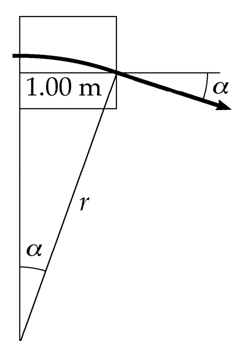
(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $\mathbf{J}\times(\text{area}) = \mathbf{J}Lw$.

The current then feels a magnetic force $\quad I|\mathbf{h}\times\mathbf{B}| = JLwhB\sin 90°$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}$$

29.56 The magnetic force on each proton, $\quad \mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = qvB\sin 90°$



downward perpendicular to velocity, supplies centripetal force, guiding it into a circular path of radius r, with

$$qvB = \frac{mv^2}{r} \qquad \text{and} \qquad r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed: $\quad K = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.00\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{1.67\times10^{-27}\text{ kg}}} = 3.10\times10^7\text{ m/s}$$

Now, $r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(3.10\times10^{7}\text{ m/s})(\text{C}\cdot\text{m})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ N}\cdot\text{s})} = 6.46\text{ m}$

(b) From the figure, observe that $\sin\propto = \frac{1.00\text{ m}}{r} = \frac{1\text{ m}}{6.46\text{ m}}$ $\boxed{\alpha = 8.90°}$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$-(1.67\times10^{-27}\text{ kg})(3.10\times10^{7}\text{ m/s})\sin(8.90°) = \boxed{-8.00\times10^{-21}\text{ kg}\cdot\text{m/s}}$

***29.57** (a) If $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$, $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}) = 0 + ev_iB_y\,\mathbf{k} - ev_iB_z\,\mathbf{j}$

Since the force actually experienced is $\mathbf{F}_B = F_i\,\mathbf{j}$, observe that

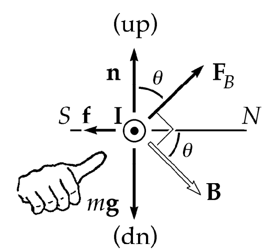
$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -F_i/ev_i}$

(b) If $\mathbf{v} = -v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = e(-v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

(c) If $q = -e$ and $\mathbf{v} = v_i\,\mathbf{i}$, then $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = -e(v_i\,\mathbf{i})\times(B_x\,\mathbf{i} + 0\,\mathbf{j} - F_i/ev_i\,\mathbf{k}) = \boxed{-F_i\,\mathbf{j}}$

Reversing either the velocity or the sign of the charge reverses the force.

29.58 A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B = BIL$ or $B = F_B/IL$



When the wire is just able to move, $\Sigma F_y = n + F_B\cos\theta - mg = 0$

so $n = mg - F_B\cos\theta$

and $f = \mu(mg - F_B\cos\theta)$

Also, $\Sigma F_x = F_B\sin\theta - f = 0$

so $F_B\sin\theta = f$: $F_B\sin\theta = \mu(mg - F_B\cos\theta)$ and $F_B = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$

We minimize B by minimizing F_B: $\frac{dF_B}{d\theta} = (\mu mg)\frac{\cos\theta - \mu\sin\theta}{(\sin\theta + \mu\cos\theta)^2} = 0 \Rightarrow \mu\sin\theta = \cos\theta$

Thus, $\theta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}(5.00) = 78.7°$ for the smallest field, and

$$B = \frac{F_B}{IL} = \left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta + \mu\cos\theta}$$

$$B_{\min} = \left[\frac{(0.200)(9.80\text{ m/s}^2)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7° + (0.200)\cos 78.7°} = 0.128\text{ T}$$

$\boxed{B_{\min} = 0.128\text{ T pointing north at an angle of 78.7° below the horizontal}}$

29.59 (a) The net force is the Lorentz force given by $\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

$$\mathbf{F} = \left(3.20\times10^{-19}\right)\left[\left(4\,\mathbf{i} - 1\mathbf{j} - 2\,\mathbf{k}\right) + \left(2\,\mathbf{i} + 3\,\mathbf{j} - 1\,\mathbf{k}\right)\times\left(2\,\mathbf{i} + 4\,\mathbf{j} + 1\,\mathbf{k}\right)\right]\text{N}$$

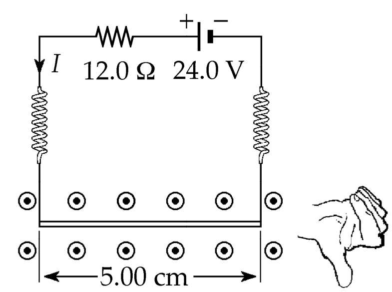
Carrying out the indicated operations, we find: $\mathbf{F} = \boxed{\left(3.52\,\mathbf{i} - 1.60\,\mathbf{j}\right)\times10^{-18}\ \text{N}}$

(b) $$\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$$

29.60 $$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27})(1.50\times10^{8})}{(1.60\times10^{-19})(5.00\times10^{-5})}\ \text{m} = \boxed{3.13\times10^{4}\ \text{m}} = 31.3\ \text{km}$$

No, $\boxed{\text{the proton will not hit the Earth}}$.

29.61 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\,\Delta x_2$ where k is the force constant of the spring and can be determined from $k = mg/2\Delta x_1$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find



$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\,\Delta x_2}{\Delta x_1};\quad \text{but}\quad \left|\mathbf{F}_B\right| = I\left|\mathbf{L}\times\mathbf{B}\right| = ILB$$

Therefore, where $I = \dfrac{24.0\ \text{V}}{12.0\ \Omega} = 2.00\ \text{A}$, $B = \dfrac{mg\,\Delta x_2}{IL\,\Delta x_1} = \dfrac{(0.0100)(9.80)(3.00\times10^{-3})}{(2.00)(0.0500)(5.00\times10^{-3})} = \boxed{0.588\ \text{T}}$

***29.62** Suppose the input power is $120\ \text{W} = (120\ \text{V})I$: $\boxed{I \sim 1\ \text{A} = 10^{0}\ \text{A}}$

Suppose $\omega = 2000\ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) \sim 200\ \dfrac{\text{rad}}{\text{s}}$

and the output power is $20\ \text{W} = \tau\omega = \tau\left(200\ \dfrac{\text{rad}}{\text{s}}\right)$ $\boxed{\tau \sim 10^{-1}\ \text{N}\cdot\text{m}}$

Suppose the area is about $(3\ \text{cm})\times(4\ \text{cm})$, or $\boxed{A \sim 10^{-3}\ \text{m}^2}$

From Table 29.1, suppose that the field is $\boxed{B \sim 10^{-1}\ \text{T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$0.1\ \text{N}\cdot\text{m} \sim N\left(1\ \dfrac{\text{C}}{\text{s}}\right)\left(10^{-3}\ \text{m}^2\right)\left(10^{-1}\ \dfrac{\text{N}\cdot\text{s}}{\text{Cm}}\right)$ giving $\boxed{N \sim 10^{3}}$

29.63 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \qquad \text{or} \qquad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \qquad \text{or} \qquad T\cos\theta = mg$$

Therefore, $\tan\theta = \dfrac{ILB}{mg} = \dfrac{IB}{(m/L)g}$ or $B = \dfrac{(m/L)g}{I}\tan\theta$

$$B = \frac{(0.0100\ \text{kg/m})(9.80\ \text{m/s}^2)}{5.00\ \text{A}}\tan(45.0^\circ) = \boxed{19.6\ \text{mT}}$$

29.64 Call the length of the rod L and the tension in each wire alone $T/2$. Then, at equilibrium:

$$\Sigma F_x = T\sin\theta - ILB\sin 90.0^0 = 0 \qquad \text{or} \qquad T\sin\theta = ILB$$
$$\Sigma F_y = T\cos\theta - mg = 0, \qquad \text{or} \qquad T\cos\theta = mg$$

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g} \qquad \text{or} \qquad B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\mu g}{I}\tan\theta}$$

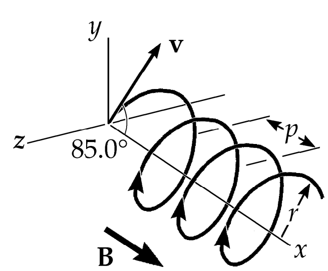
29.65 $\Sigma F = ma$ or $qvB\sin 90.0° = \dfrac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\dfrac{v}{r} = \omega = \dfrac{qB}{m} = 2\pi f$ and

$$\Delta f = f_{12} - f_{14} = \frac{qB}{2\pi}\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{(1.60\times10^{-19}\ \text{C})(2.40\ \text{T})}{2\pi(1.66\times10^{-27}\ \text{kg / u})}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$$\Delta f = f_{12} - f_{14} = 4.38\times10^5\ \text{s}^{-1} = \boxed{438\ \text{kHz}}$$

29.66 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.



(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (see Equation 29.15).

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times10^6)(\cos 85.0°)(2\pi)(9.11\times10^{-31})}{(0.150)(1.60\times10^{-19})} = \boxed{1.04\times10^{-4}\ \text{m}}$$

(b) From Equation 29.13, $r = \dfrac{mv_\perp}{Bq} = \dfrac{mv\sin 85.0°}{Bq}$

$$r = \frac{(9.11\times10^{-31})(5.00\times10^6)(\sin\ 85.0°)}{(0.150)(1.60\times10^{-19})} = \boxed{1.89\times10^{-4}\ \text{m}}$$

29.67 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \dfrac{\Delta q}{\Delta t} = \dfrac{q}{T}$

and the period of the motion is $T = \dfrac{2\pi R}{v}$

The electron's speed in its orbit is found by requiring $\dfrac{k_e q^2}{R^2} = \dfrac{mv^2}{R}$ or $v = q\sqrt{\dfrac{k_e}{mR}}$

Substituting this expression for v into the equation for T, we find $T = 2\pi\sqrt{\dfrac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{(9.11\times10^{-31})(5.29\times10^{-11})^3}{(1.60\times10^{-19})^2(8.99\times10^9)}} = 1.52\times10^{-16}\ \text{s}$$

Therefore, $|\tau| = \left(\dfrac{q}{T}\right)AB = \dfrac{1.60\times10^{-19}}{1.52\times10^{-16}}\pi(5.29\times10^{-11})^2(0.400) = \boxed{3.70\times10^{-24}\ \text{N}\cdot\text{m}}$

Goal Solution

Consider an electron orbiting a proton and maintained in a fixed circular path of radius $R = 5.29 \times 10^{-11}$ m by the Coulomb force. Treating the orbiting charge as a current loop, calculate the resulting torque when the system is in a magnetic field of 0.400 T directed perpendicular to the magnetic moment of the electron.

G: Since the mass of the electron is very small (~10^{-30} kg), we should expect that the torque on the orbiting charge will be very small as well, perhaps ~10^{-30} N·m.

O: The torque on a current loop that is perpendicular to a magnetic field can be found from $|\tau| = IAB\sin\theta$. The magnetic field is given, $\theta = 90°$, the area of the loop can be found from the radius of the circular path, and the current can be found from the centripetal acceleration that results from the Coulomb force that attracts the electron to proton.

A: The area of the loop is $A = \pi r^2 = \pi\left(5.29\times10^{-11}\ \text{m}\right)^2 = 8.79\times10^{-21}\ \text{m}^2$.

If v is the speed of the electron, then the period of its circular motion will be $T = 2\pi R/v$, and the effective current due to the orbiting electron is $I = \Delta Q / \Delta t = e/T$. Applying Newton's second law with the Coulomb force acting as the central force gives

$$\Sigma F = \frac{k_e q^2}{R^2} = \frac{mv^2}{R} \quad \text{so that} \quad v = q\sqrt{\frac{k_e}{mR}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$$

$$T = 2\pi\sqrt{\frac{(9.10\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})^3}{\left(1.60\times10^{-19}\ \text{C}\right)^2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}} = 1.52\times10^{-16}\ \text{s}$$

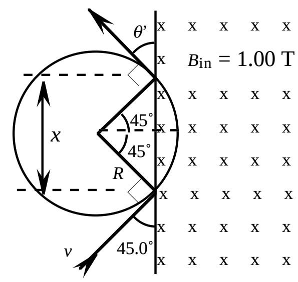
The torque is $|\tau| = \left(\dfrac{q}{T}\right)AB$: $|\tau| = \dfrac{1.60\times10^{19}\ \text{C}}{1.52\times10^{-16}\ \text{s}}(\pi)(5.29\times10^{-11}\ \text{m})^2(0.400\ \text{T}) = 3.70\times10^{-24}\ \text{N}\cdot\text{m}$

L: The torque is certainly small, but a million times larger than we guessed. This torque will cause the atom to precess with a frequency proportional to the applied magnetic field. A similar process on the nuclear, rather than the atomic, level leads to nuclear magnetic resonance (NMR), which is used for magnetic resonance imaging (MRI) scans employed for medical diagnostic testing (see Section 44.2).

29.68 Use the equation for cyclotron frequency $\omega = \frac{qB}{m}$ or $m = \frac{qB}{\omega} = \frac{qB}{2\pi f}$

$$m = \frac{(1.60\times10^{-19}\text{ C})(5.00\times10^{-2}\text{ T})}{(2\pi)(5.00\text{ rev }/\ 1.50\times10^{-3}\text{ s})} = \boxed{3.82\times10^{-25}\text{ kg}}$$

29.69 (a) $K = \frac{1}{2}mv^2 = 6.00\text{ MeV} = \left(6.00\times10^6\text{ eV}\right)\left(1.60\times10^{-19}\ \frac{\text{J}}{\text{eV}}\right)$



$K = 9.60\times10^{-13}$ J

$$v = \sqrt{\frac{2\left(9.60\times10^{-13}\text{ J}\right)}{1.67\times10^{-27}\text{ kg}}} = 3.39\times10^7\text{ m s}$$

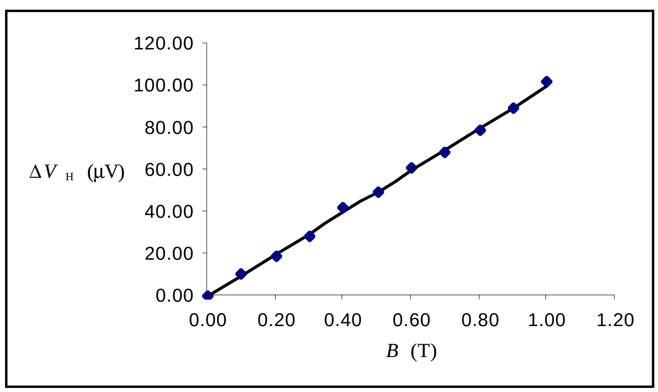
$F_B = qvB = \frac{mv^2}{R}$ so $R = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.39\times10^7\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)(1.00\text{ T})} = 0.354\text{ m}$

Then, from the diagram, $x = 2R\sin 45.0° = 2(0.354\text{ m})\sin 45.0° = \boxed{0.501\text{ m}}$

(b) From the diagram, observe that $\theta' = \boxed{45.0°}$.

29.70 (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_H = \left(1.00\times10^{-4}\text{ V/T}\right)B}$$



(b) Comparing the equation of the line which fits the data best to

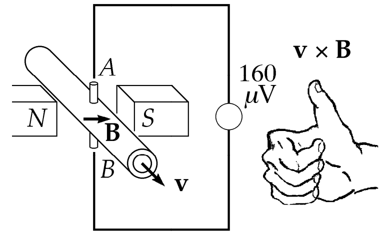
$$\Delta V_H = \left(\frac{I}{nqt}\right)B$$

observe that: $\frac{I}{nqt} = 1.00\times10^{-4}\text{ V/T}$, or $t = \frac{I}{nq\left(1.00\times10^{-4}\text{ V/T}\right)}$

Then, if $I = 0.200$ A, $q = 1.60\times10^{-19}$ C, and $n = 1.00\times10^{26}\text{ m}^{-3}$, the thickness of the sample is

$$t = \frac{0.200\text{ A}}{\left(1.00\times10^{26}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-4}\text{ V/T}\right)} = 1.25\times10^{-4}\text{ m} = \boxed{0.125\text{ mm}}$$

***29.71** (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force,

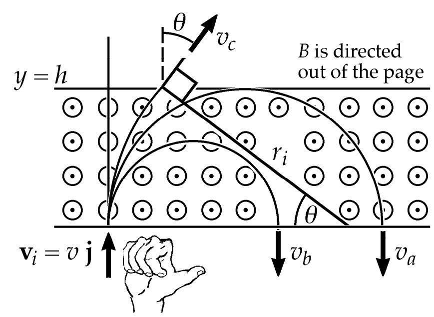


so $$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{160\times10^{-6}\ \text{V}}{(0.040\,0\ \text{T})(3.00\times10^{-3}\ \text{m})} = \boxed{1.33\ \text{m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

***29.72** When in the field, the particles follow a circular path according to $qvB = mv^2/r$, so the radius of the path is: $r = mv/qB$



(a) When $r = h = \frac{mv}{qB}$, that is, when $\boxed{v = \frac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

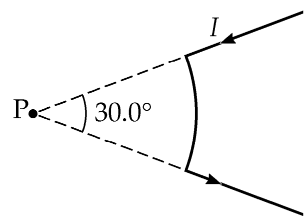
(b) When $v < \frac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r = \frac{mv}{qB} < h$. It will leave the field at $(2r, 0, 0)$ with velocity $\boxed{\mathbf{v}_f = -v\mathbf{j}}$.

(c) When $v > \frac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r = \frac{mv}{qB} > h$, centered at $(r, 0, 0)$. The arc subtends an angle given by $\theta = \sin^{-1}(h/r)$. It will leave the field at the point with coordinates $[r(1-\cos\theta), h, 0]$ with velocity $\boxed{\mathbf{v}_f = v\sin\theta\,\mathbf{i} + v\cos\theta\,\mathbf{j}}$.

Chapter 30 Solutions

30.1 $B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q(v/2\pi R)}{2R} = \boxed{12.5\ \text{T}}$

***30.2** We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\mathbf{s}$ is at 0° or 180° to ~, so $d\mathbf{s} \times \sim = 0$. Thus, only the curved section of wire contributes to $\mathbf{B}$ at P. Hence, $d\mathbf{s}$ is tangent to the arc and ~ is radially inward; so $d\mathbf{s} \times \sim = |ds|\ \ell \sin 90° = |ds|\ \otimes$. All points along the curve are the same distance $r = 0.600$ m from the field point, so



$$B = \int_{\text{all current}} |d\mathbf{B}| = \int \frac{\mu_0}{4\pi} \frac{I\,|d\mathbf{s} \times \sim|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int |ds| = \frac{\mu_0}{4\pi} \frac{I}{r^2}\, s$$

where s is the arclength of the curved wire,

$$s = r\theta = (0.600\ \text{m})30.0°\left(\frac{2\pi}{360°}\right) = 0.314\ \text{m}$$

Then, $B = \left(10^{-7}\,\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{(3.00\ \text{A})}{(0.600\ \text{m})^2}(0.314\ \text{m})$

$B = \boxed{261\ \text{nT into the page}}$

30.3 (a) $B = \frac{4\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right)$ where $a = \frac{1}{2}$

is the distance from any side to the center.

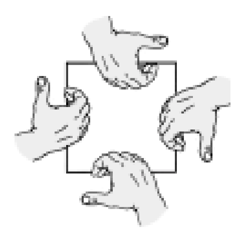


Figure for Goal Solution

$$B = \frac{4.00 \times 10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 2\sqrt{2}\ \times 10^{-5}\ \text{T} = \boxed{28.3\ \mu\text{T into the paper}}$$

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi^2 \times 10^{-7})(10.0)}{4(0.400)} = \boxed{24.7\ \mu\text{T into the paper}}$$

Goal Solution

(a) A conductor in the shape of a square of edge length $\ell = 0.400$ m carries a current I = 10.0 A (Fig. P30.3). Calculate the magnitude and direction of the magnetic field at the center of the square. (b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

G: As shown in the diagram above, the magnetic field at the center is directed into the page from the clockwise current. If we consider the sides of the square to be sections of four infinite wires, then we could expect the magnetic field at the center of the square to be a little less than four times the strength of the field at a point $\ell/2$ away from an infinite wire with current I.

$$B < 4\frac{\mu_0 I}{2\pi a} = 4\left(\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m / A}\right)(10.0\ \text{A})}{2\pi(0.200\ \text{m})}\right) = 40.0\ \mu\text{T}$$

Forming the wire into a circle should not significantly change the magnetic field at the center since the average distance of the wire from the center will not be much different.

O: Each side of the square is simply a section of a thin, straight conductor, so the solution derived from the Biot-Savart law in Example 30.1 can be applied to part (a) of this problem. For part (b), the Biot-Savart law can also be used to derive the equation for the magnetic field at the center of a circular current loop as shown in Example 30.3.

A: (a) We use Equation 30.4 for the field created by each side of the square. Each side contributes a field away from you at the center, so together they produce a magnetic field:

$$B = \frac{4\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right) = \frac{4\left(4\pi\times10^{-6}\ \text{T}\cdot\text{m / A}\right)(10.0\ \text{A})}{4\pi(0.200\ \text{m})}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)$$

so at the center of the square, $\mathbf{B} = 2.00\sqrt{2}\times10^{-5}\ \text{T} = 28.3\ \mu\text{T}$ perpendicularly into the page

(b) As in the first part of the problem, the direction of the magnetic field will be into the page. The new radius is found from the length of wire: $4\ell = 2\pi R$, so $R = 2\ell/\pi = 0.255$ m. Equation 30.8 gives the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m / A})(10.0\ \text{A})}{2(0.255\ \text{m})} = 2.47\times10^{-5}\ \text{T} = 24.7\ \mu\text{T}$$

Caution! If you use your calculator, it may not understand the keystrokes: To get the right answer, you may need to use .

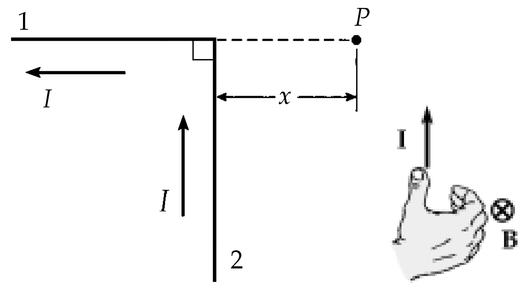
L: The magnetic field in part (a) is less than 40μT as we predicted. Also, the magnetic fields from the square and circular loops are similar in magnitude, with the field from the circular loop being about 15% less than from the square loop.

Quick tip: A simple way to use your right hand to find the magnetic field due to a current loop is to curl the fingers of your right hand in the direction of the current. Your extended thumb will then point in the direction of the magnetic field within the loop or solenoid.

30.4 $$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\,(1.00\text{ A})}{2\pi(1.00\text{ m})} = \boxed{2.00 \times 10^{-7}\text{ T}}$$

30.5 For leg 1, $d\mathbf{s} \times \sim = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x}\text{ into the paper}}$$



30.6 $$B = \frac{\mu_0 I}{2R} \qquad R = \frac{\mu_0 I}{2B} = \frac{20.0\pi \times 10^{-7}}{2.00 \times 10^{-5}} = \boxed{31.4\text{ cm}}$$

30.7 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$B = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} = \left(1 + \frac{1}{\pi}\right)\frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A}\right)\left(7.00\text{ A}\right)}{2\left(0.100\text{ m}\right)} = 5.80 \times 10^{-5}\text{ T}$$

or $\boxed{\mathbf{B} = 58.0\ \mu\text{T} \quad (\text{directed into the page})}$

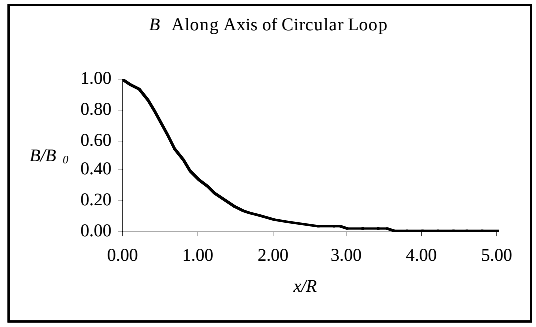
30.8 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$\boxed{\mathbf{B} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} \quad (\text{directed into the page})}$$

30.9 For the straight sections $d\mathbf{s} \times \sim = 0$. The quarter circle makes one-fourth the field of a full loop:

$$\mathbf{B} = \frac{1}{4}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8R}\text{ into the paper} \qquad \mathbf{B} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A})(5.00\text{ A})}{8(0.0300\text{ m})} = \boxed{26.2\ \mu\text{T into the paper}}$$

30.10 Along the axis of a circular loop of radius R,



$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{\left(x/R\right)^2 + 1}\right]^{3/2}$$

where $B_0 \equiv \mu_0 I/2R$.

x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

30.11 $$dB = \frac{\mu_0 I}{4\pi}\frac{\left|d\mathbf{l}\times\hat{\mathbf{r}}\right|}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{\frac{1}{6}2\pi a}{a^2} - \frac{\frac{1}{6}2\pi b}{b^2}\right)$$

$$\boxed{\mathbf{B} = \frac{\mu_0 I}{12}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ directed out of the paper}}$$

30.12 Apply Equation 30.4 three times:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2 + a^2}}\right) \text{ toward you}$$

$$+ \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2 + a^2}} + \frac{a}{\sqrt{d^2 + a^2}}\right) \text{ away from you}$$

$$+ \frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2 + a^2}} - \cos 180°\right) \text{ toward you}$$

$$\mathbf{B} = \boxed{\frac{\mu_0 I\left(a^2 + d^2 - d\sqrt{a^2 + d^2}\right)}{2\pi a d\sqrt{a^2 + d^2}} \text{ away from you}}$$

30.13 The picture requires $L = 2R$

$$\mathbf{B} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) + \frac{\mu_0 I}{4\pi R}(\cos 90.0° - \cos 135°) + \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 135°)$$

$$+ \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 90.0°) \text{ into the page}$$

$$\mathbf{B} = \frac{\mu_0 I}{R}\left(\frac{1}{4} + \frac{1}{\pi\sqrt{2}}\right) = \boxed{0.475\left(\frac{\mu_0 I}{R}\right) \text{ (into the page)}}$$

30.14 Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be B_1, B_2, and B_3 respectively.

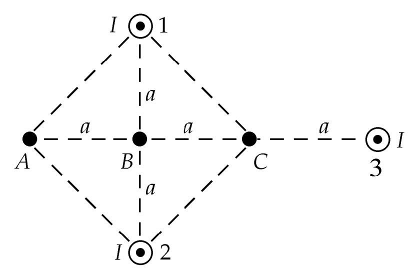


Figure (a)

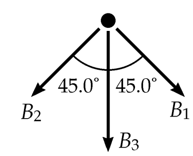


Figure (b)

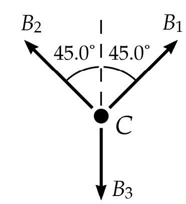


Figure (c)

(a) At Point A: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.

The directions of these fields are shown in Figure (b). Observe that the horizontal components of B_1 and B_2 cancel while their vertical components both add to B_3.

Therefore, the net field at point A is:

$$B_A = B_1 \cos 45.0° + B_2 \cos 45.0° + B_3 = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi\left(1.00\times 10^{-2}\ \text{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right] = \boxed{53.3\ \mu\text{T}}$$

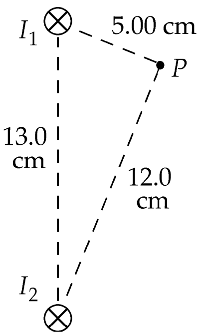
(b) At point B: B_1 and B_2 cancel, leaving $B_B = B_3 = \dfrac{\mu_0 I}{2\pi(2a)}$.

$$B_B = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(2)\left(1.00\times 10^{-2}\ \text{m}\right)} = \boxed{20.0\ \mu\text{T}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of B_1 and B_2 cancel. The vertical components both oppose B_3 giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2\cos 45.0°}{\sqrt{2}} - 1\right] = \boxed{0}$$

30.15 Take the x-direction to the right and the y-direction up in the plane of the paper. Current 1 creates at P a field



$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{(2.00\times10^{-7}\ \mathrm{T\cdot m})(3.00\ \mathrm{A})}{\mathrm{A}(0.0500\ \mathrm{m})}$$

$\mathbf{B}_1 = 12.0\ \mu\mathrm{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.

Current 2 contributes

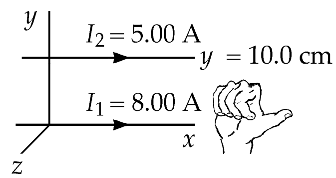
$$B_2 = \frac{(2.00\times10^{-7}\ \mathrm{T\cdot m})(3.00\ \mathrm{A})}{\mathrm{A}(0.120\ \mathrm{m})}$$ clockwise perpendicular to 12.0 cm

$\mathbf{B}_2 = 5.00\ \mu\mathrm{T}$ to the right and down, at angle –22.6°

Then, $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (12.0\ \mu\mathrm{T})(-\mathbf{i}\cos 67.4° - \mathbf{j}\sin 67.4°) + (5.00\ \mu\mathrm{T})(\mathbf{i}\cos 22.6° - \mathbf{j}\sin 22.6°)$

$$\mathbf{B} = (-11.1\ \mu\mathrm{T})\mathbf{j} - (1.92\ \mu\mathrm{T})\mathbf{j} = \boxed{(-13.0\ \mu\mathrm{T})\mathbf{j}}$$

***30.16** Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.



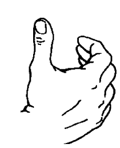
(a) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{k} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\mathbf{k}$$

$\mathbf{B} = \boxed{1.00\times10^{-5}\ \mathrm{T}\ \text{out of the page}}$

(b) $\mathbf{F}_B = I_2\mathbf{L}\times\mathbf{B} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\mathbf{i}\times(1.00\times10^{-5}\ \mathrm{T})\mathbf{k}\right] = (8.00\times10^{-5}\ \mathrm{N})(-\mathbf{j})$

$\boxed{\mathbf{F}_B = 8.00\times10^{-5}\ \mathrm{N}\ \text{toward the first wire}}$

(c) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}(-\mathbf{k}) = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}(-\mathbf{k}) = (1.60\times10^{-5}\ \mathrm{T})(-\mathbf{k})$$

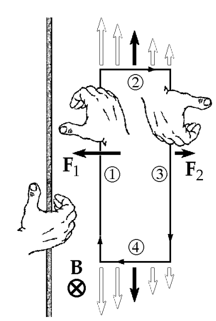


$\mathbf{B} = \boxed{1.60\times10^{-5}\ \mathrm{T}\ \text{into the page}}$

(d) $\mathbf{F}_B = I_1\mathbf{L}\times\mathbf{B} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\mathbf{i}\times(1.60\times10^{-5}\ \mathrm{T})(-\mathbf{k})\right] = (8.00\times10^{-5}\ \mathrm{N})(+\mathbf{j})$

$\mathbf{F}_B = \boxed{8.00\times10^{-5}\ \mathrm{N}\ \text{toward the second wire}}$

30.17 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 30.12)



$$\mathbf{F}_B = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i}$$

Substituting given values $\mathbf{F}_B = -2.70\times10^{-5}\ \mathbf{i}\ \mathrm{N} = \boxed{-27.0\ \mu\mathrm{N}\ \mathbf{i}}$

Goal Solution

In Figure P30.17, the current in the long, straight wire is I_1 = 5.00 A and the wire lies in the plane of the rectangular loop, which carries 10.0 A. The dimensions are c = 0.100 m, a = 0.150 m, and ℓ = 0.450 m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

G: Even though there are forces in opposite directions on the loop, we must remember that the magnetic field is stronger near the wire than it is farther away. By symmetry the forces exerted on sides 2 and 4 (the horizontal segments of length a) are equal and opposite, and therefore cancel. The magnetic field in the plane of the loop is directed into the page to the right of I_1. By the right-hand rule, $\mathbf{F} = I\boldsymbol{\ell}\times\mathbf{B}$ is directed toward the **left** for side 1 of the loop and a smaller force is directed toward the **right** for side 3. Therefore, we should expect the net force to be to the left, possibly in the μN range for the currents and distances given.

O: The magnetic force between two parallel wires can be found from Equation 30.11, which can be applied to sides 1 and 3 of the loop to find the net force resulting from these opposing force vectors.

A: $$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\mathbf{i}$$

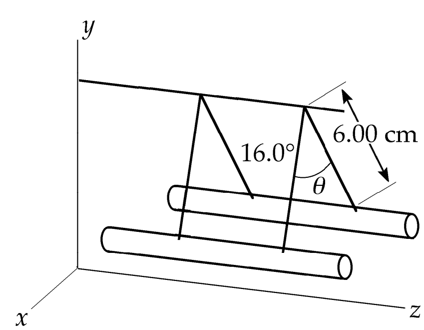
$$\mathbf{F} = \frac{\left(4\pi\times10^{-7}\ \text{N}/\text{A}^2\right)(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi}\left(\frac{-0.150\ \text{m}}{(0.100\ \text{m})(0.250\ \text{m})}\right)\mathbf{i}$$

$$\mathbf{F} = (-2.70\times10^{-5}\,\mathbf{i})\ \text{N} \qquad \text{or} \qquad \mathbf{F} = 2.70\times10^{-5}\ \text{N} \quad \text{toward the left}$$

L: The net force is to the left and in the μN range as we expected. The symbolic representation of the net force on the loop shows that the net force would be zero if either current disappeared, if either dimension of the loop became very small ($a \to 0$ or $\ell \to 0$), or if the magnetic field were uniform ($c \to \infty$).

30.18 The separation between the wires is

a = 2(6.00 cm) sin 8.00° = 1.67 cm.



(a) Because the wires repel, the currents are in

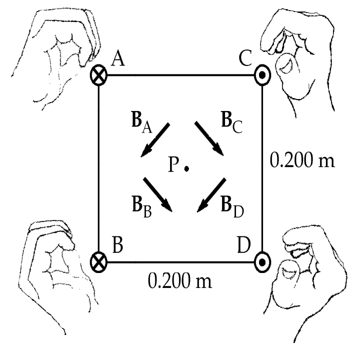
opposite directions .

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a\ mg} = \tan 8.00°$$

$$I^2 = \frac{mg\ 2\pi a}{\ell\,\mu_0}\tan 8.00° \qquad \text{so} \qquad I = \boxed{67.8\ \text{A}}$$

30.19 Each wire is distant from P by (0.200 m) cos 45.0° = 0.141 m



Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00 \times 10^{-7}\ \text{T} \cdot \text{m})(5.00\ \text{A})}{\text{A}(0.141\ \text{m})} = 7.07\ \mu\text{T}$$

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is downward and to the left. The total field is then

$$4\ (7.07\ \mu\text{T}) \sin 45.0° = \boxed{20.0\ \mu\text{T}}$$ toward the page's bottom

30.20 Let the current I flow to the right. It creates a field $B = \mu_0 I / 2\pi d$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$$mg(-\,\mathbf{j}) + qv(-\,\mathbf{i}) \times \frac{\mu_0 I}{2\pi d}(\mathbf{k}) = 0$$ at a distance d from the wire

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{(1.60 \times 10^{-19}\ \text{C})(2.30 \times 10^{4}\ \text{m / s})(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A})(1.20 \times 10^{-6}\ \text{A})}{2\pi(1.67 \times 10^{-27}\ \text{kg})\ (9.80\ \text{m / s}^2)} = \boxed{5.40\ \text{cm}}$$

30.21 From Ampère's law, the magnetic field at point a is given by $B_a = \mu_0 I_a / 2\pi r_a$, where I_a is the net current flowing through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A}\right)(1.00\ \text{A})}{2\pi(1.00 \times 10^{-3}\ \text{m})} = \boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current flowing through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00\ \text{A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A})(2.00\ \text{A})}{2\pi(3.00 \times 10^{-3}\ \text{m})} = \boxed{133\ \mu\text{T toward bottom of page}}$$

***30.22** (a) In $B = \dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\text{ cm}}$

(b) $\mathbf{B} = \dfrac{\mu_0 I}{2\pi r_1}\,\mathbf{k} + \dfrac{\mu_0 I}{2\pi r_2}(-\mathbf{k})$ so $B = \dfrac{4\pi \times 10^{-7}\text{ T}\cdot\text{m }(2.00\text{ A})}{2\pi\text{ A}}\left(\dfrac{1}{0.3985\text{ m}} - \dfrac{1}{0.4015\text{ m}}\right) = \boxed{7.50\text{ nT}}$

(c) Call r the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\text{ T} = \left(2.00 \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2.00\text{ A})\,\frac{(3.00 \times 10^{-3}\text{ m})}{r^2 - 2.25 \times 10^{-6}\text{ m}^2} \quad \text{so} \quad r = \boxed{1.26\text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

30.23 (a) $B_{\text{inner}} = \dfrac{\mu_0 N I}{2\pi r} = \boxed{3.60\text{ T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 N I}{2\pi r} = \boxed{1.94\text{ T}}$

***30.24** (a) $B = \dfrac{\mu_0 I}{2\pi a^2}\,r$ for $r \le a$ so $B = \dfrac{\mu_0(2.50\text{ A})}{2\pi(0.0250\text{ m})^2}(0.0125\text{ m}) = \boxed{10.0\ \mu\text{T}}$

(b) $r = \dfrac{\mu_0 I}{2\pi B} = \dfrac{\mu_0(2.50\text{ A})}{2\pi(10.0 \times 10^{-6}\text{ T})} = 0.0500\text{ m} = \boxed{\text{2.50 cm beyond the conductor's surface}}$

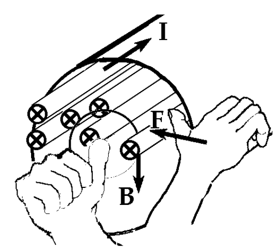
30.25 (a) One wire feels force due to the field of the other ninety-nine.

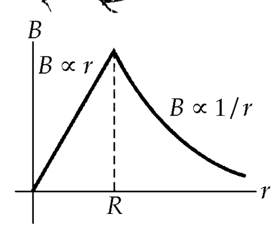
Within the bundle, $B = \left(\dfrac{\mu_0 I}{2\pi R^2}\right) r = 3.17 \times 10^{-3}\text{ T}.$

The force, *acting inward*, is $F_B = I\ell B$, and the force per unit length is

$$\frac{F_B}{\ell} = \boxed{6.34 \times 10^{-3}\text{ N/m inward}}$$

(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.





Figures for Goal Solution

Goal Solution

A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius R = 0.500 cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) Would a wire on the outer edge of the bundle experience a force greater or less than the value calculated in part (a)?

G: The force **on** one wire comes from its interaction with the magnetic field created **by** the other ninety-nine wires. According to Ampere's law, at a distance r from the center, only the wires enclosed within a radius r contribute to this net magnetic field; the other wires outside the radius produce magnetic field vectors in opposite directions that cancel out at r. Therefore, the magnetic field (and also the force on a given wire at radius r) will be greater for larger radii within the bundle, and will decrease for distances beyond the radius of the bundle, as shown in the graph to the right. Applying $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$, the magnetic force on a single wire will be directed toward the center of the bundle, so that all the wires tend to attract each other.

O: Using Ampere's law, we can find the magnetic field at any radius, so that the magnetic force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ on a single wire can then be calculated.

A: (a) Ampere's law is used to derive Equation 30.15, which we can use to find the magnetic field at r = 0.200 cm from the center of the cable:

$$B = \frac{\mu_o I_o r}{2\pi R^2} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(99)(2.00\ \mathrm{A})\left(0.200\times10^{-2}\ \mathrm{m}\right)}{2\pi\,(0.500\times10^{-2}\ \mathrm{m})^2} = 3.17\times10^{-3}\ \mathrm{T}$$

This field points tangent to a circle of radius 0.200 cm and exerts a force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ toward the center of the bundle, on the single hundredth wire:

$$F/l = IB\sin\theta = (2.00\ \mathrm{A})\left(3.17\times10^{-3}\ \mathrm{T}\right)(\sin 90°) = 6.34\ \mathrm{mN/m}$$

(b) As is shown above in Figure 30.12 from the text, the magnetic field increases linearly as a function of r until it reaches a maximum at the outer surface of the cable. Therefore, the force on a single wire at the outer radius $r = 5.00$ cm would be greater than at $r = 2.00$ cm by a factor of $5/2$.

L: We did not estimate the expected magnitude of the force, but 200 amperes is a lot of current. It would be interesting to see if the magnetic force that pulls together the individual wires in the bundle is enough to hold them against their own weight: If we assume that the insulation accounts for about half the volume of the bundle, then a single copper wire in this bundle would have a cross sectional area of about

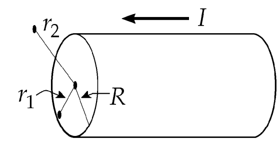
$$(1/2)(0.01)\pi(0.500\ \mathrm{cm})^2 = 4\times10^{-7}\ \mathrm{m}^2$$

with a weight per unit length of $\rho gA = \left(8920\ \mathrm{kg/m^3}\right)(9.8\ \mathrm{N/kg})\left(4\times10^{-7}\ \mathrm{m^2}\right) = 0.03\ \mathrm{N/m}$

Therefore, the outer wires experience an inward magnetic force that is about half the magnitude of their own weight. If placed on a table, this bundle of wires would form a loosely held mound without the outer sheathing to hold them together.

30.26 From $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$, $I = \dfrac{2\pi rB}{\mu_0} = \dfrac{(2\pi)(1.00\times10^{-3})(0.100)}{4\pi\times10^{-7}} = \boxed{500\ \mathrm{A}}$

30.27 Use Ampère's law, $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$. For current density $\mathbf{J}$, this becomes



$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$$

(a) For $r_1 < R$, this gives

$$2\pi r_1 = \mu_0 \int_0^{r_1} (br)(2\pi r\, dr) \quad \text{and}$$

$$B = \boxed{\dfrac{\mu_0 b r_1^2}{3} \quad (\text{for } r_1 < R \text{ or inside the cylinder})}$$

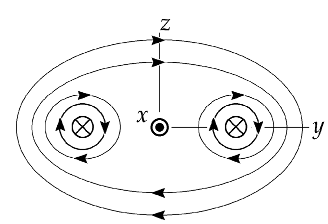
(b) When $r_2 > R$, Ampère's law yields

$$(2\pi r_2)B = \mu_0 \int_0^{R} (br)(2\pi r\, dr) = 2\pi \mu_0 b R^3/3,$$

or $$B = \boxed{\dfrac{\mu_0 b R^3}{3 r_2} \quad (\text{for } r_2 > R \text{ or outside the cylinder})}$$

30.28 (a) See Figure (a) to the right.

(b) At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding



(Currents are into the paper)

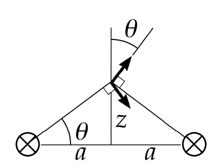
Figure (a)

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi(a^2+z^2)}$$

The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi(a^2+z^2)^2} + \frac{\mu_0 I}{\pi(a^2+z^2)} = 0, \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{(a^2-z^2)}{(a^2+z^2)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a}$.



At a distance z above the plane of the conductors

Figure (b)

30.29 $B = \mu_0 \dfrac{N}{\ell} I$ so $I = \dfrac{B}{\mu_0 n} = \boxed{31.8\text{ mA}}$

30.30 (a) $I = \dfrac{10.0}{(4\pi \times 10^{-7})(2000)} = \boxed{3.98\text{ kA}}$

(b) $\dfrac{F_B}{\ell} = IB = \boxed{39.8\text{ kN/m radially outward}}$

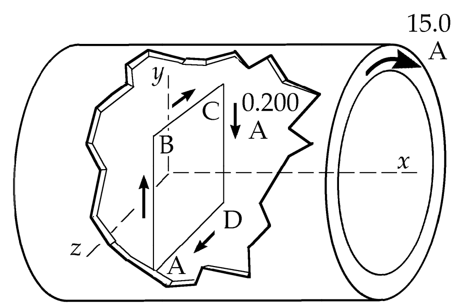
This is the force the windings will have to resist when the magnetic field in the solenoid is 10.0 T.

30.31 The resistance of the wire is $R_e = \dfrac{\rho\ell}{\pi r^2}$, so it carries current $I = \dfrac{\mathcal{E}}{R_e} = \dfrac{\mathcal{E}\pi r^2}{\rho\ell}$.

If there is a single layer of windings, the number of turns per length is the reciprocal of the wire diameter: $n = 1/2r$.

So, $B = n\mu_0 I = \dfrac{\mu_0 \mathcal{E}\pi r^2}{\rho\ell 2r} = \dfrac{\mu_0 \mathcal{E}\pi r}{2\rho\ell} = \dfrac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(20.0\text{ V})\pi(2.00 \times 10^{-3}\text{ m})}{2(1.70 \times 10^{-8}\ \Omega\cdot\text{m})(10.0\text{ m})} = \boxed{464\text{ mT}}$

***30.32** The field produced by the solenoid in its interior is given by



$$\mathbf{B} = \mu_0 n I(-\mathbf{i}) = \left(4\pi \times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\frac{30.0}{10^{-2}\text{ m}}\right)(15.0\text{ A})(-\mathbf{i})$$

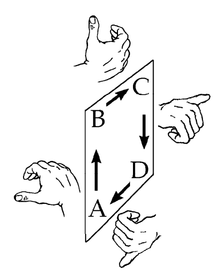
$$\mathbf{B} = -\left(5.65 \times 10^{-2}\text{ T}\right)\mathbf{i}$$

The force exerted on side *AB* of the square current loop is

$$(\mathbf{F}_B)_{AB} = I\mathbf{L} \times \mathbf{B} = (0.200\text{ A})\left[\left(2.00 \times 10^{-2}\text{ m}\right)\mathbf{j} \times \left(5.65 \times 10^{-2}\text{ T}\right)(-\mathbf{i})\right]$$

$$(\mathbf{F}_B)_{AB} = \left(2.26 \times 10^{-4}\text{ N}\right)\mathbf{k}$$

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of $\boxed{226\ \mu\text{N directed away from the center}}$. From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by



$$\mu = I\mathbf{A} = (0.200\text{ A})\left(2.00 \times 10^{-2}\text{ m}\right)^2(-\mathbf{i}) = -80.0\ \mu\text{A}\cdot\text{m}^2\,\mathbf{i}$$

The torque exerted on the loop is then $\tau = \mu \times \mathbf{B} = \left(-80.0\ \mu\text{A}\cdot\text{m}^2\,\mathbf{i}\right) \times \left(-5.65 \times 10^{-2}\text{ T}\mathbf{i}\right) = \boxed{0}$

30.33 (a) $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \mathbf{B} \cdot \mathbf{A} = (5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})\text{T} \cdot (2.50 \times 10^{-2} \text{ m})^2 \mathbf{i}$

$$\Phi_B = 3.13 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.13 \times 10^{-3} \text{ Wb} = \boxed{3.13 \text{ mWb}}$$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B} \cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

30.34 (a) $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA$ where A is the cross-sectional area of the solenoid.

$$\Phi_B = \left(\frac{\mu_0 NI}{l}\right)(\pi r^2) = \boxed{7.40 \ \mu\text{Wb}}$$

(b) $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA = \left(\frac{\mu_0 NI}{l}\right)\left[\pi(r_2^2 - r_1^2)\right]$

$$\Phi_B = \left[\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(300)(12.0 \text{ A})}{(0.300 \text{ m})}\right]\pi\left[(8.00)^2 - (4.00)^2\right](10^{-3} \text{ m})^2 = \boxed{2.27 \ \mu\text{Wb}}$$

30.35 (a) $(\Phi_B)_{\text{flat}} = \mathbf{B} \cdot \mathbf{A} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$

(b) The net flux out of the closed surface is zero: $(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$

$$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

30.36 $\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$

(a) $\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \boxed{7.19 \times 10^{11} \text{ V/m} \cdot \text{s}}$

(b) $\oint B \cdot ds = \epsilon_0 \mu_0 \frac{\Phi_E}{dt}$ so $2\pi r B = \epsilon_0 \mu_0 \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A} \cdot \pi r^2\right]$

$$B = \frac{\mu_0 I r}{2A} = \frac{\mu_0 (0.200)(5.00 \times 10^{-2})}{2\pi (0.100)^2} = \boxed{2.00 \times 10^{-7} \text{ T}}$$

30.37 (a) $\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{(0.100 \text{ A})}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{11.3 \times 10^9 \text{ V} \cdot \text{m/s}}$

(b) $I_d = \epsilon_0 \frac{d\Phi_E}{dt} = I = \boxed{0.100 \text{ A}}$

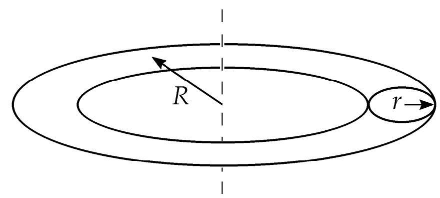
30.38 (a) $I = \frac{ev}{2\pi r}$

$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27 \times 10^{-24}\ \text{A} \cdot \text{m}^2}$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

30.39 Assuming a uniform B inside the toroid is equivalent to assuming $r << R$, then $B_0 \cong \mu_0 \frac{NI}{2\pi R}$ and a *tightly* wound solenoid.



$$B_0 = \mu_0 \frac{(630)(3.00)}{2\pi(0.200)} = 0.00189 \text{ T}$$

With the steel, $B = \kappa_m B_0 = (1 + \chi)B_0 = (101)(0.00189 \text{ T})$ $\boxed{B = 0.191 \text{ T}}$

30.40 $B = \mu n I = \mu\left(\frac{N}{2\pi r}\right)I$ so $I = \frac{(2\pi r)B}{\mu N} = \frac{2\pi(0.100 \text{ m})(1.30 \text{ T})}{5000(4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m})(470)} = \boxed{277 \text{ mA}}$

30.41 $\Phi_B = \mu n I A$

$$B = \mu n I = (750 \times 4\pi \times 10^{-7})\left(\frac{500}{2\pi(0.200)}\right)(0.500) = 0.188 \text{ T}$$

$A = 8.00 \times 10^{-4} \text{ m}^2$ and $\Phi_B = (0.188 \text{ T})(8.00 \times 10^{-4} \text{ m}^2) = 1.50 \times 10^{-4} \text{ T} \cdot \text{m}^2) = \boxed{150\ \mu\text{T} \cdot \text{m}^2}$

30.42 The period is $T = 2\pi/\omega$. The spinning constitutes a current $I = \frac{Q}{T} = \frac{Q\omega}{2\pi}$.

$\mu = IA = \frac{Q\omega}{2\pi}\pi R^2 = \frac{Q\omega R^2}{2}$ in the direction of ω

$$\mu = \frac{(6.00 \times 10^{-6} \text{ C})(4.00 / \text{s})(0.0200 \text{ m})^2}{2} = \boxed{4.80 \times 10^{-9} \text{ A} \cdot \text{m}^2}$$

30.43 $B = \mu_0(H + M)$ so $H = \dfrac{B}{\mu_0} - M = \boxed{2.62 \times 10^6 \text{ A/m}}$

30.44 $B = \mu_0(H + M)$

If $\mu_0 M = 2.00$ T, then the magnetization of the iron is $M = \dfrac{2.00 \text{ T}}{\mu_0}$.

But $M = xn\mu_B$ where μ_B is the Bohr magneton, n is the number of atoms per unit volume, and x is the number of electrons that contribute per atom. Thus,

$$x = \frac{M}{n\mu_B} = \frac{2.00 \text{ T}}{n\mu_B\mu_0} = \frac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N} \cdot \text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \boxed{2.02}$$

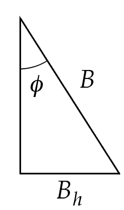
***30.45** (a) Comparing Equations 30.29 and 30.30, we see that the applied field is described by $\mathbf{B}_0 = \mu_0 \mathbf{H}$. Then Eq. 30.35 becomes $M = C\dfrac{B_0}{T} = \dfrac{C}{T}\mu_0 H$, and the definition of susceptibility (Eq. 30.32) is

$$\boxed{\chi = \frac{M}{H} = \frac{C}{T}\mu_0}$$

(b) $$C = \frac{\chi T}{\mu_0} = \frac{\left(2.70 \times 10^{-4}\right)(300 \text{ K})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = \boxed{6.45 \times 10^4 \ \frac{\text{K} \cdot \text{A}}{\text{T} \cdot \text{m}}}$$

30.46 (a) $$B_h = B_{coil} = \frac{\mu_0 NI}{2R} = \frac{(4\pi \times 10^{-7})(5.00)(0.600)}{0.300} = \boxed{12.6 \ \mu\text{T}}$$

(b) $$B_h = B\sin\phi \rightarrow B = \frac{B_h}{\sin\phi} = \frac{12.6 \ \mu T}{\sin 13.0°} = \boxed{56.0 \ \mu\text{T}}$$



30.47 (a) Number of unpaired electrons = $$\frac{8.00 \times 10^{22} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} = \boxed{8.63 \times 10^{45}}$$

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63 \times 10^{45}\right)$.

(b) Mass = $$\frac{(4.31 \times 10^{45} \text{ atoms})(7900 \text{ kg/m}^3)}{8.50 \times 10^{28} \text{ atoms/m}^3} = \boxed{4.01 \times 10^{20} \text{ kg}}$$

Goal Solution

The magnetic moment of the Earth is approximately 8.00×10^{22} A·m². (a) If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? (b) At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? (Iron has a density of 7900 kg/m³, and approximately 8.50×10^{28} atoms/m³.)

G: We know that most of the Earth is not iron, so if the situation described provides an accurate model, then the iron deposit must certainly be less than the mass of the Earth ($M_{\text{Earth}} = 5.98 \times 10^{24}$ kg). One mole of iron has a mass of 55.8 g and contributes $2(6.02 \times 10^{23})$ unpaired electrons, so we should expect the total unpaired electrons to be less than 10^{50}.

O: The Bohr magneton μ_B is the measured value for the magnetic moment of a single unpaired electron. Therefore, we can find the number of unpaired electrons by dividing the magnetic moment of the Earth by μ_B. We can then use the density of iron to find the mass of the iron atoms that each contribute two electrons.

A: (a) $\mu_B = \left(9.27 \times 10^{-24} \frac{\text{J}}{\text{T}}\right)\left(1 \frac{\text{N} \cdot \text{m}}{\text{J}}\right)\left(\frac{1 \text{ T}}{\text{N} \cdot \text{s/C} \cdot \text{m}}\right)\left(\frac{1 \text{ A}}{\text{C / s}}\right) = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$

The number of unpaired electrons is $$N = \frac{8.00 \times 10^{22} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} = 8.63 \times 10^{45} \text{ e}^-$$

(b) Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}N = \frac{1}{2}(8.63 \times 10^{45}) = 4.31 \times 10^{45}$ iron atoms.

Thus, $$M_{Fe} = \frac{(4.31 \times 10^{45} \text{ atoms})(7900 \text{ kg / m}^3)}{8.50 \times 10^{28} \text{ atoms / m}^3} = 4.01 \times 10^{20} \text{ kg}$$

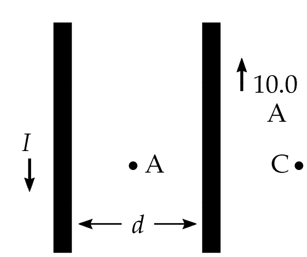
L: The calculated answers seem reasonable based on the limits we expected. From the data in this problem, the iron deposit required to produce the magnetic moment would only be about 1/15 000 the mass of the Earth and would form a sphere 500 km in diameter. Although this is certainly a large amount of iron, it is much smaller than the inner core of the Earth, which is estimated to have a diameter of about 3000 km.

30.48 $B = \frac{\mu_0 I}{2\pi R} = 2.00 \times 10^{-5} \text{ T} = \boxed{20.0 \ \mu\text{T}}$

30.49 $B = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}}$ so $\boxed{I = 2.00 \times 10^9 \text{ A}}$ flowing west

30.50 (a) $B_C = \frac{\mu_0 I}{2\pi(0.270)} - \frac{\mu_0(10.0)}{2\pi(0.0900)} = 0$ so $\boxed{I = 30.0 \text{ A}}$

(b) $B_A = \frac{4\mu_0(10.0)}{2\pi(0.0900)} = \boxed{88.9 \ \mu\text{T}}$ out of paper



***30.51** Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\frac{200\text{ W}}{12\text{ V}} = 17$ A current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \cong \mu_0$. Model the current as straight. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})17}{2\pi(0.6)} \boxed{\sim 10^{-5}\text{ T}}$$

If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim 10^{-1}\text{ times as large,}}$ enough to affect the compass noticeably.

30.52 A ring of radius r and width dr has area $dA = 2\pi r\, dr$. The current inside radius r is

$$I = \int_0^r 2\pi J r\, dr = 2\pi J_0 \int_0^r r\, dr - 2\pi\left(J_0/R^2\right)\int_0^r r^3\, dr = 2\pi J_0 r^2/2 - 2\pi\left(J_0/R^2\right)\left(r^4/4\right)$$

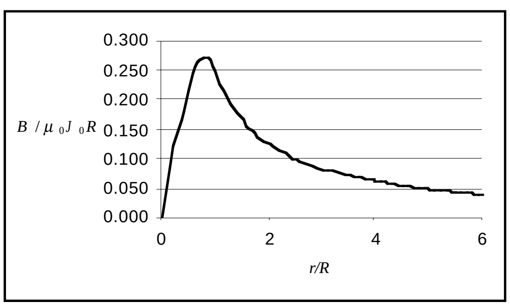
(a) Ampère's law says $B(2\pi r) = \mu_0 I = \mu_0 \pi J_0\left(r^2 - r^4/2R^2\right)$,

or $$\boxed{B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{r}{R}\right) - \frac{1}{4}\left(\frac{r}{R}\right)^3\right] \text{ for } r \le R}$$

and $$B(2\pi r) = \mu_0 I_{\text{total}} = \mu_0\left[\pi J_0 R^2 - \pi J_0 R^2/2\right] = \mu_0 \pi J_0 R^2/2$$

or $$\boxed{B = \frac{\mu_0 J_0 R^2}{4r} = \frac{\mu_0 J_0 R}{4(r/R)} \text{ for } r \ge R}$$

(b)

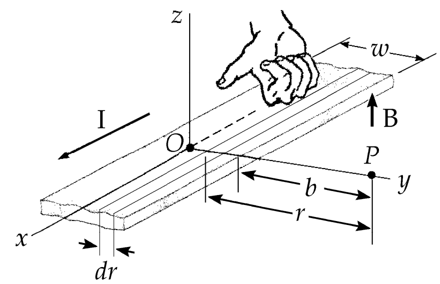


(c) To locate the maximum in the region $r \le R$, require that $\frac{dB}{dr} = \frac{\mu_0 J_0}{2} - 3\frac{\mu_0 J_0 r^2}{4R^2} = 0$

This gives the position of the maximum as $\boxed{r = \sqrt{2/3}\,R}$.

Here $B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{2}{3}\right)^{1/2} - \frac{1}{4}\left(\frac{2}{3}\right)^{3/2}\right] = \boxed{0.272\,\mu_0 J_0 R}$

30.53 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is



$$dB = \frac{\mu_0 dI}{2\pi r} \quad \text{where} \quad dI = I(dr/w)$$

$$\mathbf{B} = \int d\mathbf{B} = \int_b^{b+w} \frac{\mu_0 I\, dr}{2\pi w r}\,\mathbf{k} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1 + \frac{w}{b}\right)\mathbf{k}}$$

30.54 We find the total number of turns: $B = \frac{\mu_0 N I}{l}$

$$N = \frac{Bl}{\mu_0 I} = \frac{(0.0300\text{ T})(0.100\text{ m})\text{A}}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m})(1.00\text{ A})} = 2.39 \times 10^3$$

Each layer contains $(10.0\text{ cm}/0.0500\text{ cm}) = 200$ closely wound turns

so she needs $(2.39 \times 10^3/200) = \boxed{12\text{ layers}}$.

The inner diameter of the innermost layer is 10.0 mm. The outer diameter of the outermost layer is $10.0\text{ mm} + 2 \times 12 \times 0.500\text{ mm} = 22.0\text{ mm}$. The average diameter is 16.0 mm, so the total length of wire is

$$(2.39 \times 10^3)\pi(16.0 \times 10^{-3}\text{ m}) = \boxed{120\text{ m}}$$

30.55 On the axis of a current loop, the magnetic field is given by $B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$

where in this case $I = \frac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a strength of

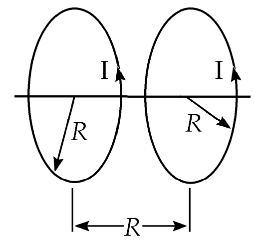
$$B = \frac{\mu_0 \omega R^2 q}{4\pi(x^2 + R^2)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2(10.0 \times 10^{-6})}{4\pi\left[(0.0500)^2 + (0.100)^2\right]^{3/2}} = \boxed{1.43 \times 10^{-10}\text{ T}}$$

30.56 On the axis of a current loop, the magnetic field is given by $B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$

where in this case $I = \frac{q}{(2\pi/\omega)}$. Therefore, $B = \frac{\mu_0 \omega R^2 q}{4\pi(x^2 + R^2)^{3/2}}$

when $x = \frac{R}{2}$, then $B = \frac{\mu_0 \omega q R^2}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\frac{\mu_0 q \omega}{2.5\sqrt{5}\,\pi R}}$

30.57 (a) Use Equation 30.7 twice: $B_x = \dfrac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$



$$B = B_{x1} + B_{x2} = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{\left((R-x)^2 + R^2\right)^{3/2}}\right]$$

$$\boxed{B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2xR)^{3/2}}\right]}$$

(b) $$\frac{dB}{dx} = \frac{\mu_0 I R^2}{2}\left[-\frac{3}{2}(2x)\left(x^2 + R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2 + x^2 - 2xR\right)^{-5/2}(2x - 2R)\right]$$

Substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{dB}{dx} = 0}$

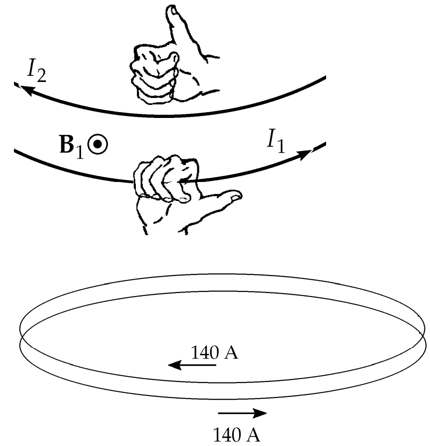
$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 I R^2}{2}\left[(x^2 + R^2)^{-5/2} - 5x^2(x^2 + R^2)^{-7/2} + (2R^2 + x^2 - 2xR)^{-5/2} - 5(x-R)^2(2R^2 + x^2 - 2xR)^{-7/2}\right]$$

Again substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{d^2B}{dx^2} = 0}$

30.58 "Helmholtz pair" $\rightarrow$ separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[(R/2)^2 + R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4} + 1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \quad \text{for 1 turn}$$

For N turns in each coil, $B = \dfrac{\mu_0 N I}{1.40R} = \dfrac{\left(4\pi \times 10^{-7}\right)100(10.0)}{1.40(0.500)} = \boxed{1.80 \times 10^{-3}\ \text{T}}$



30.59 Model the two wires as straight parallel wires (!)

(a) $F_B = \dfrac{\mu_0 I^2 L}{2\pi a}$ (Equation 30.12)

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2 2\pi(0.100)}{2\pi(1.00 \times 10^{-3})} = \boxed{2.46 \text{ N}} \quad \text{upward}$$

(b) $$a_{\text{loop}} = \frac{2.46 \text{ N} - m_{\text{loop}}\, g}{m_{\text{loop}}} = \boxed{107 \text{ m/s}^2} \quad \text{upward}$$

***30.60** (a) In $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} I d\mathbf{s} \times \sim$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\mathbf{s}$ is the direction of $\mathbf{v}$, so $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\mathbf{v} \times \sim$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi r^2} q\mathbf{v} \times \sim}$$

(b) $$B = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(2.00 \times 10^{7} \text{ m/s}\right)}{4\pi\left(1.00 \times 10^{-3}\right)^2} \sin 90.0° = \boxed{3.20 \times 10^{-13} \text{ T}}$$

(c) $$F_B = q\left|\mathbf{v} \times \mathbf{B}\right| = \left(1.60 \times 10^{-19} \text{ C}\right)\left(2.00 \times 10^{7} \text{ m/s}\right)\left(3.20 \times 10^{-13} \text{ T}\right) \sin 90.0°$$

$$F_B = \boxed{1.02 \times 10^{-24} \text{ N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^{9} \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(1.00 \times 10^{-3}\right)^2}$$

$$F_e = \boxed{2.30 \times 10^{-22} \text{ N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

***30.61** (a) $$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(24.0 \text{ A})}{2\pi(0.0175 \text{ m})} = \boxed{2.74 \times 10^{-4} \text{ T}}$$

(b) At point C, conductor AB produces a field $\frac{1}{2}\left(2.74 \times 10^{-4} \text{ T}\right)(-\mathbf{j})$,  conductor DE produces a field of $\frac{1}{2}\left(2.74 \times 10^{-4} \text{ T}\right)(-\mathbf{j})$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74 \times 10^{-4} \text{ T}(-\mathbf{j})}$.

(c) $\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} = (24.0\ \text{A})(0.0350\ \text{m}\,\mathbf{k}) \times \left[5\left(2.74 \times 10^{-4}\ \text{T}\right)(-\mathbf{j})\right] = \boxed{\left(1.15 \times 10^{-3}\ \text{N}\right)\mathbf{i}}$

(d) $\mathbf{a} = \dfrac{\Sigma \mathbf{F}}{m} = \dfrac{\left(1.15 \times 10^{-3}\ \text{N}\right)\mathbf{i}}{3.00 \times 10^{-3}\ \text{kg}} = \boxed{\left(0.384\ \dfrac{\text{m}}{\text{s}^2}\right)\mathbf{i}}$

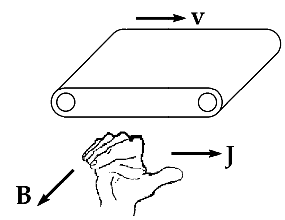
(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2\left(0.384\ \text{m/s}^2\right)(1.30\ \text{m})$, so $\mathbf{v}_f = \boxed{(0.999\ \text{m/s})\mathbf{i}}$

30.62 At equilibrium, $\frac{F_B}{\ell} = \frac{\mu_0 I_A I_B}{2\pi a} = \frac{mg}{\ell}$ or $I_B = \frac{2\pi a(m/\ell)g}{\mu_0 I_A}$

$$I_B = \frac{2\pi(0.0250\text{ m})(0.0100\text{ kg/m})(9.80\text{ m/s}^2)}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(150\text{ A})} = \boxed{81.7\text{ A}}$$

30.63 (a) The magnetic field due to an infinite sheet of charge (or the magnetic field at points near a large sheet of charge) is given by $B = \mu_0 J_s/2$. The current density $J_s = I/\ell$ and in this case the equivalent current of the moving charged belt is



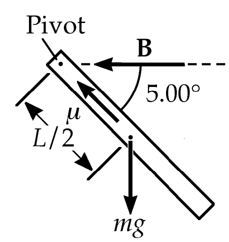
$$I = \frac{dq}{dt} = \frac{d}{dt}(\sigma \ell x) = \sigma \ell v; \qquad v = \frac{dx}{dt}$$

Therefore, $J_s = \sigma v$ and $\boxed{B = \frac{\mu_0 \sigma v}{2}}$

(b) If the sheet is positively charged and moving in the direction shown, the magnetic field is $\boxed{\text{out of the page, parallel to the roller axes.}}$

30.64 $$C = \frac{TM}{B} = \frac{(4.00\text{ K})(10.0\%)(8.00\times10^{27}\text{ atoms / m}^3)(5.00)(9.27\times10^{-24}\text{ J / T}^2)}{5.00\text{ T}} = \boxed{2.97\times10^4\ \frac{\text{K}\cdot\text{J}}{\text{T}^2\cdot\text{m}^3}}$$

30.65 At equilibrium, $\Sigma\tau = +|\mu\times\mathbf{B}| - mg\left(\frac{L}{2}\cos 5.00°\right) = 0,$



or $\mu B\sin 5.00° = \frac{mgL}{2}\cos 5.00°$

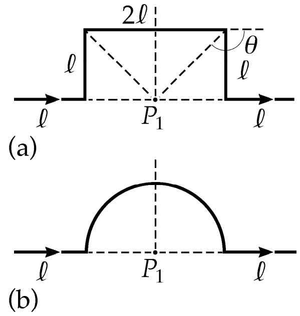
Therefore, $B = \frac{mgL}{2\mu\tan 5.00°} = \frac{(0.0394\text{ kg})(9.80\text{ m/s}^2)(0.100\text{ m})}{2(7.65\text{ J/T})\tan 5.00°}$

$B = \boxed{28.8\text{ mT}}$

30.66 The central wire creates field $\mathbf{B} = \mu_0 I_1/2\pi R$ counterclockwise. The curved portions of the loop feels no force since $\ell\times\mathbf{B} = 0$ there. The straight portions both feel $I\ell\times\mathbf{B}$ forces to the right, amounting to

$$\mathbf{F}_B = I_2\,2L\,\frac{\mu_0 I_1}{2\pi R} = \boxed{\frac{\mu_0 I_1 I_2 L}{\pi R}\text{ to the right}}$$

30.67 When the conductor is in the rectangular shape shown in figure (a), the segments carrying current straight toward or away from point P_1 do not contribute to the magnetic field at P_1. Each of the other four setions of length ℓ makes an equal contribution to the total field into the page at P_1. To find the contribution of the horizontal section of current in the upper right, we use



$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) \quad \text{with} \quad a = \ell,\ \theta_1 = 90°,\ \text{and}\ \ \theta_2 = 135°$$

So $$B_1 = \frac{4\mu_0 I}{4\pi \ell}\left(0 - \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\sqrt{2}\ \pi \ell}$$

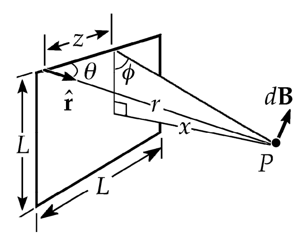
When the conductor is in the shape of a circular arc, the magnitude or the field at the center is given by Equation 30.6, $B = \frac{\mu_0 I}{4\pi R}\theta$. From the geometry in this case, we find $R = \frac{4\ell}{\pi}$ and $\theta = \pi$.

Therefore, $B_2 = \frac{\mu_0 I \pi}{4\pi(4\ell/\pi)} = \frac{\mu_0 I \pi}{16\ell}$; so that $\boxed{\frac{B_1}{B_2} = \frac{8\sqrt{2}}{\pi^2}}$

30.68 $$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(9.00\times10^3\right)\left(1.50\times10^{-8}\right)}{4\pi\times10^{-7}} = \boxed{675\ \text{A}}$$

Flow of $\boxed{\text{positive current is downward } \textit{or} \text{ negative charge flows upward}}$.

30.69 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $L/2$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by Equation 30.3.



$$\mathbf{B}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{1}\times\sim}{r^2}$$

From the figure we see that

$$r = \sqrt{x^2 + (L^2/4) + z^2} \qquad \text{and} \qquad \left|d\mathbf{1}\times\sim\right| = dz\sin\theta = dz\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}$$

By symmetry all components of the field $\mathbf{B}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$B_{0x} = B_0 \cos\phi$ where $\cos\phi = \dfrac{L/2}{\sqrt{L^2/4 + x^2}}$.

Therefore, $\mathbf{B}_{0x} = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_0^{L/2} \frac{\sin\theta \cos\phi \, dz}{r^2}$ and $B = 8B_{0x}$.

Using the expressions given above for sin θ cos ϕ, and r, we find

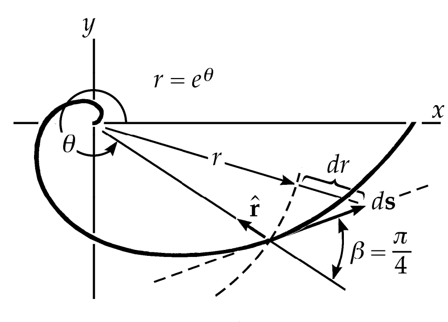
$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2 + \dfrac{L^2}{4}\right)\sqrt{x^2 + \dfrac{L^2}{2}}}$$

30.70 (a) From Equation 30.10, the magnetic field produced by one loop at the center of the second loop is given by $B = \dfrac{\mu_0 I R^2}{2x^3} = \dfrac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3} = \dfrac{\mu_0 \mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu = I\left(\pi R^2\right)$. Therefore,

$$|F_x| = \mu\frac{dB}{dx} = \mu\left(\frac{\mu_0 \mu}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0\left(\pi R^2 I\right)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}$$

(b) $$|F_x| = \frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4} = \frac{3\pi}{2}\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(10.0\ \text{A})^2\left(5.00\times10^{-3}\ \text{m}\right)^4}{\left(5.00\times10^{-2}\ \text{m}\right)^4} = \boxed{5.92\times10^{-8}\ \text{N}}$$

30.71 There is no contribution from the straight portion of the wire since $d\mathbf{s}\times \sim = 0$. For the field of the spiral,



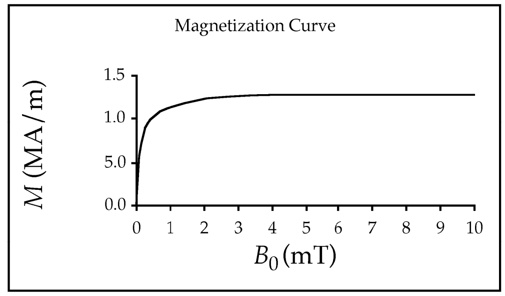
$$d\mathbf{B} = \frac{\mu_0 I}{(4\pi)}\frac{(d\mathbf{s}\times\sim)}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{|d\mathbf{s}|\sin\theta|\sim|}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}\,dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}\,dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\pi}$$

Substitute $r = e^\theta$: $B = -\dfrac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\dfrac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^0\right] = \boxed{\dfrac{\mu_0 I}{4\pi}\left(1 - e^{-2\pi}\right)}$ (out of the page)

30.72 (a) $\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M}$

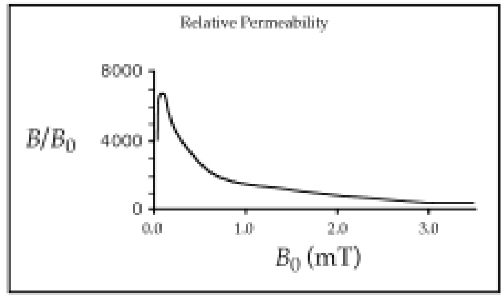


$$\mathbf{M} = \frac{\mathbf{B} - \mathbf{B}_0}{\mu_0} \quad \text{and} \quad M = \frac{|\mathbf{B} - \mathbf{B}_0|}{\mu_0}$$

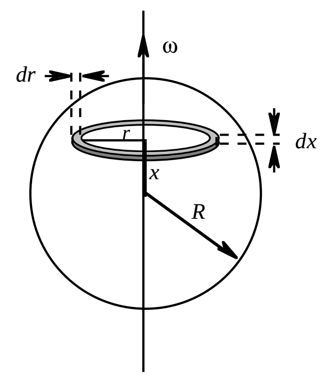
Assuming that **B** and $\mathbf{B}_0$ are parallel, this becomes $M = (B - B_0)/\mu_0$

The magnetization curve gives a plot of M versus B_0.

(b) The second graph is a plot of the relative permeability (B/B_0) as a function of the applied field B_0.



30.73 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The contribution to the field from each ring is



$$dB = \frac{\mu_0 r^2\, dI}{2\left(x^2 + r^2\right)^{3/2}} \qquad \text{where} \qquad dI = \frac{dQ}{t} = \frac{\omega\, dQ}{2\pi}$$

$$dQ = \rho\, dV = \rho(2\pi r\, dr)(dx)$$

$$dB = \frac{\mu_0 \rho \omega r^3\, dr\, dx}{2\left(x^2 + r^2\right)^{3/2}} \quad \text{where} \quad \rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$

$$B = \int_{x=-R}^{+R} \int_{r=0}^{\sqrt{R^2 - x^2}} \frac{\mu_0 \rho \omega}{2} \frac{r^3\, dr\, dx}{\left(x^2 + r^2\right)^{3/2}}$$

Let $v = r^2 + x^2$, $\quad dv = 2r\, dr$, and $\quad r^2 = v - x^2$

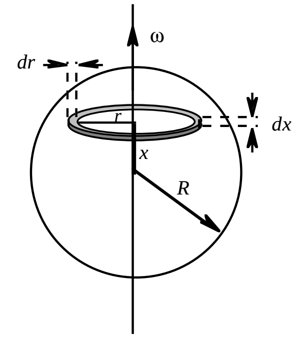
$$B = \int_{x=-R}^{+R} \int_{v=x^2}^{R^2} \frac{\mu_0 \rho \omega}{2} \frac{\left(v - x^2\right) dv}{2 v^{3/2}} dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[\int_{v=x^2}^{R^2} v^{-1/2}\, dv - x^2 \int_{v=x^2}^{R^2} v^{-3/2}\, dv \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2v^{1/2}\Big|_{x^2}^{R^2} + \left(2x^2\right) v^{-1/2}\Big|_{x^2}^{R^2} \right] dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2\left(R - |x|\right) + 2x^2\left(\frac{1}{R} - \frac{1}{|x|}\right) \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{-R}^{R} \left[2\frac{x^2}{R} - 4|x| + 2R \right] dx = \frac{2\mu_0 \rho \omega}{4} \int_{0}^{R} \left[2\frac{x^2}{R} - 4x + 2R \right] dx$$

$$B = \frac{2\mu_0 \rho \omega}{4} \left(\frac{2R^3}{3R} - \frac{4R^2}{2} + 2R^2 \right) = \boxed{\frac{\mu_0 \rho \omega R^2}{3}}$$

30.74 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The current associated with each rotating ring of charge is



$$dI = \frac{dQ}{t} = \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right]$$

The magnetic moment contributed by this ring is

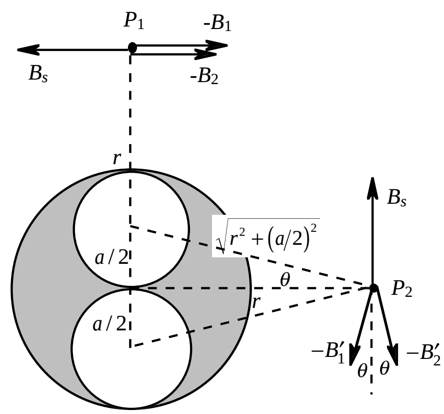
$$d\mu = A(dI) = \pi r^2 \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right] = \pi\omega\rho r^3\,dr\,dx$$

$$\mu = \pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}} r^3\,dr\right]dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu = \frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4-2R^2x^2+x^4\right)dx = \frac{\pi\omega\rho}{4}\left[R^4(2R)-2R^2\left(\frac{2R^3}{3}\right)+\frac{2R^5}{5}\right]$$

$$\mu = \frac{\pi\omega\rho}{4}R^5\left(2-\frac{4}{3}+\frac{2}{5}\right) = \frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right) = \boxed{\frac{4\pi\omega\rho R^5}{15}} \quad \text{up}$$

30.75 Note that the current I exists in the conductor with a current density $J = I/A$, where



$$A = \pi\left[a^2 - a^2/4 - a^2/4\right] = \pi a^2/2$$

Therefore, $J = 2I/\pi a^2$.

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $a/2$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1, $B_s = \dfrac{\mu_0 J(\pi a^2)}{2\pi r}$, $B_1 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r-a/2)}$, and $B_2 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r+a/2)}$.

$$B = B_s - B_1 - B_2 = \frac{\mu_0 J\pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4(r-a/2)} - \frac{1}{4(r+a/2)}\right]$$

$$B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2-a^2-2r^2}{4r\left(r^2-a^2/4\right)}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2-a^2}{4r^2-a^2}\right]\text{ directed to the left}}$$

(b) At point P_2, $B_s = \frac{\mu_0 J(\pi a^2)}{2\pi r}$ and $B_1' = B_2' = \frac{\mu_0 J\pi(a/2)^2}{2\pi\sqrt{r^2 + (a/2)^2}}$.

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B = B_s - B_1'\cos\theta - B_2'\cos\theta = \frac{\mu_0 J(\pi a^2)}{2\pi r} - 2\left(\frac{\mu_0 J\pi a^2/4}{2\pi\sqrt{r^2 + a^2/4}}\right)\frac{r}{\sqrt{r^2 + a^2/4}}$$

$$B = \frac{\mu_0 J(\pi a^2)}{2\pi r}\left[1 - \frac{r^2}{2(r^2 + a^2/4)}\right] = \frac{\mu_0(2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2 + a^2}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right] \text{ directed toward the top of the page}}$$

Chapter 31 Solutions

31.1 $\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(NBA)}{\Delta t} = \boxed{500\ \text{mV}}$

31.2 $\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = 1.60\ \text{mV}$ and $I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60\ \text{mV}}{2.00\ \Omega} = \boxed{0.800\ \text{mA}}$

31.3 $\mathcal{E} = -N \frac{\Delta BA\cos\theta}{\Delta t} = -NB \pi r^2 \left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$

$= -25.0\left(50.0\times 10^{-6}\ \text{T}\right)\pi\left(0.500\ \text{m}\right)^2\left(\frac{\cos 180° - \cos 0}{0.200\ \text{s}}\right)$

$\text{E} = \boxed{+9.82\ \text{mV}}$

31.4 (a) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = \boxed{\frac{AB_{\max}}{\tau} e^{-t/\tau}}$

(b) $\mathcal{E} = \frac{(0.160\ \text{m}^2)(0.350\ \text{T})}{2.00\ \text{s}} e^{-4.00/2.00} = \boxed{3.79\ \text{mV}}$

(c) At $t = 0$, $\mathcal{E} = \boxed{28.0\ \text{mV}}$

31.5 $|\mathcal{E}| = N\frac{d\Phi_B}{dt} = \frac{\Delta(NBA)}{\Delta t} = 3.20\ \text{kV}$ so $I = \frac{\mathcal{E}}{R} = \boxed{160\ \text{A}}$

Goal Solution

A strong electromagnet produces a uniform field of 1.60 T over a cross-sectional area of 0.200 m^2. A coil having 200 turns and a total resistance of 20.0 Ω is placed around the electromagnet. The current in the electromagnet is then smoothly decreased until it reaches zero in 20.0 ms. What is the current induced in the coil?

G: A strong magnetic field turned off in a short time (20.0 ms) will produce a large emf, maybe on the order of 1 kV. With only 20.0 Ω of resistance in the coil, the induced current produced by this emf will probably be larger than 10 A but less than 1000 A.

O: According to Faraday's law, if the magnetic field is reduced uniformly, then a constant emf will be produced. The definition of resistance can be applied to find the induced current from the emf.

A: Noting unit conversions from $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} = -N\left(\frac{0 - B_i A\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})\left(0.200\text{ m}^2\right)(\cos 0°)}{20.0\times10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s / C}\cdot\text{m}}{\text{T}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3200\text{ V}$$

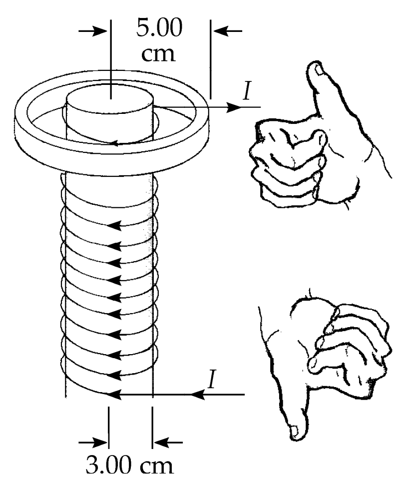
$$I = \frac{\mathcal{E}}{R} = \frac{3200\text{ V}}{20.0\ \Omega} = 160\text{ A}$$

L: This is a large current, as we expected. The positive sign is indicative that the induced electric field is in the positive direction around the loop (as defined by the area vector for the loop).

31.6 $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -\dfrac{N(BA - 0)}{\Delta t}$

$$\Delta t = \frac{NBA}{|\mathcal{E}|} = \frac{NB(\pi r^2)}{\mathcal{E}} = \frac{500(0.200)\pi(5.00\times10^{-2})^2}{10.0\times10^3} = \boxed{7.85\times10^{-5}\text{ s}}$$

31.7 $|\mathcal{E}| = \dfrac{d(BA)}{dt} = 0.500\ \mu_0 nA\dfrac{dI}{dt} = 0.480\times10^{-3}\text{ V}$



(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \dfrac{4.80\times10^{-4}}{3.00\times10^{-4}} = \boxed{1.60\text{ A}}$

(b) $B_{\text{ring}} = \dfrac{\mu_0 I}{2r_{\text{ring}}} = \boxed{20.1\ \mu\text{T}}$

(c) Coil's field points downward, and is increasing, so

$\boxed{B_{\text{ring}}\text{ points upward}}$

31.8 $|\mathcal{E}| = \dfrac{d(BA)}{dt} = 0.500\ \mu_0 nA\dfrac{dI}{dt} = 0.500\ \mu_0 n\pi r_2^2\dfrac{\Delta I}{\Delta t}$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 n \pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}}$

(b) $B = \dfrac{\mu_0 I}{2r_1} = \boxed{\dfrac{\mu_0^2 n \pi r_2^2}{4r_1 R}\dfrac{\Delta I}{\Delta t}}$

(c) The coil's field points downward, and is increasing, so $\boxed{B_{\text{ring}} \text{ points upward}}$.

31.9 (a) $d\Phi_B = \mathbf{B}\cdot d\mathbf{A} = \dfrac{\mu_0 I}{2\pi x}L dx$: $\Phi_B = \displaystyle\int_{x=h}^{h+w} \frac{\mu_0 I L}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 I L}{2\pi}\ln\left(\frac{h+w}{h}\right)}$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

$$\mathcal{E} = -\frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)\left(10.0\ \frac{\text{A}}{\text{s}}\right) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle (first figure, above). As it increases, the rectangle wants to produce its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second figure, above).

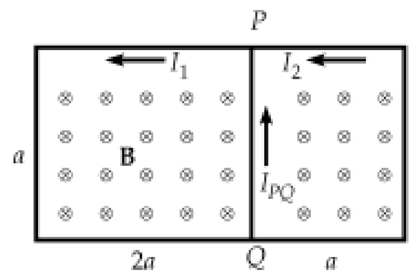
31. 10 $\Phi_B = (\mu_0 n I) A_{\text{solenoid}}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)(600\ \text{A/s})\cos(120t)$$

$$\mathcal{E} = -15.0\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^3/\text{m}\right)\pi(0.0200\ \text{m})^2(600\ \text{A/s})\cos(120t)$$

$$\boxed{\mathcal{E} = -14.2\cos(120t)\ \text{mV}}$$

31.11 For a counterclockwise trip around the left-hand loop, with $B = At$



$$\frac{d}{dt}\left[At(2a^2)\cos 0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[Ata^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in QP

Thus, $2Aa^2 - 5R(I_{PQ} + I_2) - I_{PQ}R = 0$

and $\quad Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \tfrac{5}{3}(Aa^2 + I_{PQ}R) = 0$$

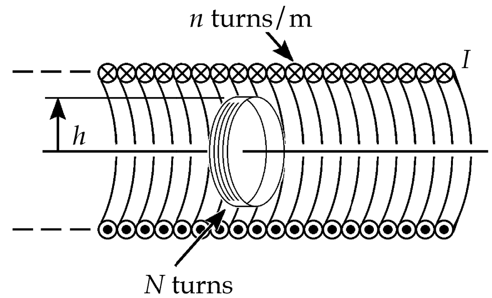
$$I_{PQ} = \frac{Aa^2}{23R}$$ upward, and since $R = (0.100\ \Omega/\text{m})(0.650\ \text{m}) = 0.0650\ \Omega$

$$I_{PQ} = \frac{(1.00\times10^{-3}\ \text{T/s})(0.650\ \text{m})^2}{23(0.0650\ \Omega)} = \boxed{283\ \mu\text{A upward}}$$

31.12 $$\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N(0.0100 + 0.0800t)A$$

At $t = 5.00$ s, $\quad \mathcal{E} = 30.0(0.410\ \text{T})\left[\pi(0.0400\ \text{m})^2\right] = \boxed{61.8\ \text{mV}}$

31.13



$$B = \mu_0 n I = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)$$

$$\Phi_B = \int B\,dA = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)\int dA$$

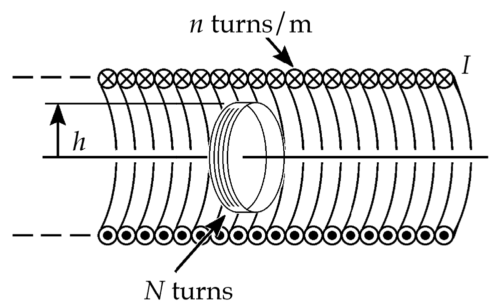
$$\Phi_B = \mu_0 n (30.0\text{ A})\left(1 - e^{-1.60t}\right)\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n (30.0\text{ A})\pi R^2 (1.60) e^{-1.60t}$$

$$\mathcal{E} = -(250)(4\pi\times10^{-7}\text{ N/A}^2)(400\text{ m}^{-1})(30.0\text{ A})\left[\pi(0.0600\text{ m})^2\right]1.60\text{ s}^{-1}e^{-1.60t}$$

$$\mathcal{E} = \boxed{(68.2\text{ mV})e^{-1.60t}\ \text{ counterclockwise}}$$

31.14



$$B = \mu_0 n I = \mu_0 n I_{\max}(1 - e^{-\alpha t})$$

$$\Phi_B = \int B\,dA = \mu_0 n I_{\max}(1 - e^{-\alpha t})\int dA$$

$$\Phi_B = \mu_0 n I_{\max}(1 - e^{-\alpha t})\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n I_{\max}\pi R^2 \alpha e^{-\alpha t}$$

$$\mathcal{E} = \boxed{N\mu_0 n I_{\max}\pi R^2 \alpha e^{-\alpha t}\ \text{ counterclockwise}}$$

31.15

$$\mathcal{E} = \frac{d}{dt}(NB\ell^2\cos\theta) = \frac{N\ell^2\,\Delta B\cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\mathcal{E}\,\Delta t}{N\,\Delta B\cos\theta}} = \sqrt{\frac{(80.0\times10^{-3}\text{ V})(0.400\text{ s})}{(50)(600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T})\cos(30.0°)}} = 1.36\text{ m}$$

Length = $4\ell N$ = 4(1.36 m)(50) = $\boxed{272\text{ m}}$

Goal Solution

A coil formed by wrapping 50.0 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.400 s, an emf of 80.0 mV is induced in the coil. What is the total length of the wire?

G: If we assume that this square coil is some reasonable size between 1 cm and 1 m across, then the total length of wire would be between 2 m and 200 m.

O: The changing magnetic field will produce an emf in the coil according to Faraday's law of induction. The constant area of the coil can be found from the change in flux required to produce the emf.

A: By Faraday's law, $$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta) = -NA\cos\theta\frac{dB}{dt}$$

For magnitudes, $$|\mathcal{E}| = NA\cos\theta\left(\frac{\Delta B}{\Delta t}\right)$$

and the area is $$A = \frac{|\mathcal{E}|}{N\cos\theta\left(\frac{\Delta B}{\Delta t}\right)} = \frac{80.0\times10^{-3}\text{ V}}{50(\cos 30.0°)\left(\frac{600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T}}{0.400\text{ s}}\right)} = 1.85\text{ m}^2$$

Each side of the coil has length $d = \sqrt{A}$, so the total length of the wire is

$$L = N(4d) = 4N\sqrt{A} = (4)(50)\sqrt{1.85\text{ m}^2} = 272\text{ m}$$

L: The total length of wire is slightly longer than we predicted. With $d = 1.36$ m, a normal person could easily step through this large coil! As a bit of foreshadowing to a future chapter on AC circuits, an even bigger coil with more turns could be hidden in the ground below high-power transmission lines so that a significant amount of power could be "stolen" from the electric utility. There is a story of one man who did this and was arrested when investigators finally found the reason for a large power loss in the transmission lines!

31.16 The average induced emf is given by $$\mathcal{E} = -N\left(\frac{\Delta\Phi_B}{\Delta t}\right)$$

Here $N = 1$, and $$\Delta\Phi_B = B(A_{\text{square}} - A_{\text{circle}})$$

with $$A_{\text{circle}} = \pi r^2 = \pi(0.500\text{ m})^2 = 0.785\text{ m}^2$$

Also, the circumference of the circle is $2\pi r = 2\pi(0.500\text{ m}) = 3.14$ m

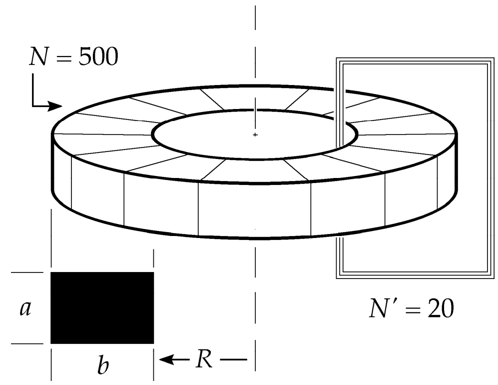
Thus, each side of the square has a length $$L = \frac{3.14\text{ m}}{4} = 0.785\text{ m},$$

and $$A_{\text{square}} = L^2 = 0.617\text{ m}^2$$

So $$\Delta\Phi_B = (0.400\text{ T})(0.617\text{ m}^2 - 0.785\text{ m}^2) = -0.0672\text{ T}\cdot\text{m}^2$$

The average induced emf is therefore: $$\mathcal{E} = -\frac{-0.0672\text{ T}\cdot\text{m}^2}{0.100\text{ s}} = \boxed{0.672\text{ V}}$$

31.17 In a toroid, all the flux is confined to the inside of the toroid.



$$B = \frac{\mu_0 N I}{2\pi r} = \frac{500\,\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B\,dA = \frac{500\,\mu_0 I_{\max}}{2\pi}\sin\omega t \int \frac{dz\,dr}{r}$$

$$\Phi_B = \frac{500\,\mu_0 I_{\max}}{2\pi} a \sin\omega t \ln\left(\frac{b+R}{R}\right)$$

$$\mathcal{E} = N' \frac{d\Phi_B}{dt} = 20\left(\frac{500\,\mu_0 I_{\max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos\omega t$$

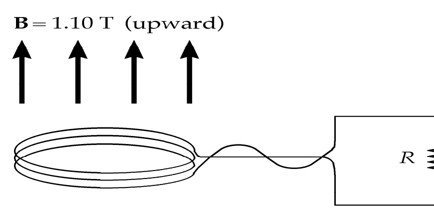
$$\mathcal{E} = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\ \frac{\text{N}}{\text{A}^2}\right)(50.0\ \text{A})\left(377\ \frac{\text{rad}}{\text{s}}\right)(0.0200\ \text{m})\ln\left(\frac{(3.00+4.00)\ \text{cm}}{4.00\ \text{cm}}\right)\cos\omega t = \boxed{(0.422\ \text{V})\cos\omega t}$$

31.18 The field inside the solenoid is: $B = \mu_0 n I = \mu_0\left(\frac{N}{\ell}\right) I$

Thus, through the single-turn loop $\Phi_B = BA_{\text{solenoid}} = \mu_0\left(\frac{N}{\ell}\right)\left(\pi r^2\right) I$

and the induced emf in the loop is $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\mu_0\left(\frac{N}{\ell}\right)\left(\pi r^2\right)\left(\frac{\Delta I}{\Delta t}\right) = \boxed{-\frac{\mu_0 N \pi r^2}{\ell}\left(\frac{I_2 - I_1}{\Delta t}\right)}$

31.19



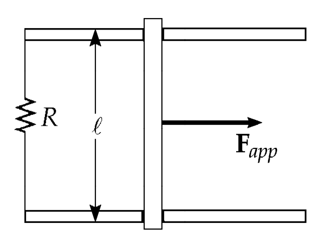
$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad IR = -N\frac{d\Phi_B}{dt}$$

$$I\,dt = -\frac{N}{R}\,d\Phi_B \qquad \int I\,dt = -\frac{N}{R}\int d\Phi_B$$

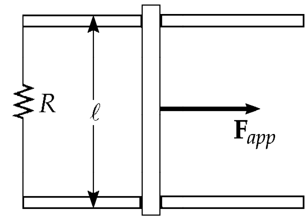
$$Q = -\frac{N}{R}\Delta\Phi_B = -\frac{N}{R}A\left(B_f - B_i\right)$$

$$Q = -\left(\frac{200}{5.00\ \Omega}\right)(100\times10^{-4}\ \text{m}^2)(-1.10-1.10)\ \text{T} = \boxed{0.880\ \text{C}}$$

31.20 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$



$\boxed{v = 1.00\ \text{m/s}}$



31.21 (a) $|\mathbf{F}_B| = I|\boldsymbol{\ell} \times \mathbf{B}| = I\ell B$. When $I = \mathcal{E}/R$ and $\mathcal{E} = B\ell v$, we get

$$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00\text{ N}$$

The applied force is $\boxed{\text{3.00 N to the right}}$

(b) $\mathcal{P} = I^2 R = \frac{B^2\ell^2 v^2}{R} = 6.00\text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00\text{ W}}$

***31.22** $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$ so $B = \frac{IR}{\ell v}$

(a) $F_B = \frac{I^2 \ell R}{\ell v}$ and $I = \sqrt{\frac{F_B v}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2 R = \boxed{2.00\text{ W}}$

(c) For constant force, $\mathcal{P} = \mathbf{F} \cdot \mathbf{v} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$

31.23 The downward component of **B**, perpendicular to **v**, is $(50.0 \times 10^{-6}\text{ T})\sin 58.0° = 4.24 \times 10^{-5}\text{ T}$

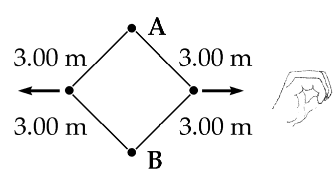
$$\mathcal{E} = B\ell v = (4.24 \times 10^{-5}\text{ T})(60.0\text{ m})(300\text{ m/s}) = \boxed{0.763\text{ V}}$$

The $\boxed{\text{left wing tip is positive}}$ relative to the right.

31.24 $\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right)$

$$\mathcal{E} = -1(0.100\text{ T})\cos 0°\ \frac{(3.00\text{ m} \times 3.00\text{ m}\sin 60.0°) - (3.00\text{ m})^2}{0.100\text{ s}} = 1.21\text{ V}$$

$$I = \frac{1.21\text{ V}}{10.0\ \Omega} = \boxed{0.121\text{ A}}$$



The flux is into the page and decreasing. The loop makes its own magnetic field into the page by carrying $\boxed{\text{clockwise}}$ current.

31.25 $\omega = (2.00\text{ rev/s})(2\pi\text{ rad/rev}) = (4.00)\pi\text{ rad/s}$

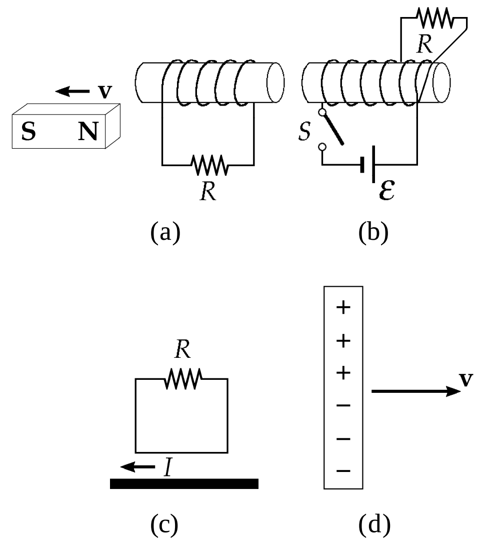
$$\mathcal{E} = \tfrac{1}{2}B\omega\ell^2 = \boxed{2.83\text{ mV}}$$

31.26 (a) $\mathbf{B}_{ext} = B_{ext}\,\mathbf{i}$ and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,\mathbf{i}$ (to the right). Therefore, the current is **to the right** in the resistor.

(b) $\mathbf{B}_{ext} = B_{ext}\,(-\mathbf{i})$ increases; therefore, the induced field $\mathbf{B}_0 = B_0\,(+\mathbf{i})$ is to the right, and the current is **to the right** in the resistor.

(c) $\mathbf{B}_{ext} = B_{ext}\,(-\mathbf{k})$ into the paper and B_{ext} decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\,(-\mathbf{k})$ into the paper. Therefore, the current is **to the right** in the resistor.

(d) By the Lorentz force law, $F_B = q(\mathbf{v} \times \mathbf{B})$. Therefore, a positive charge will move to the top of the bar if $\mathbf{B}$ is **into the paper**.



31.27 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$

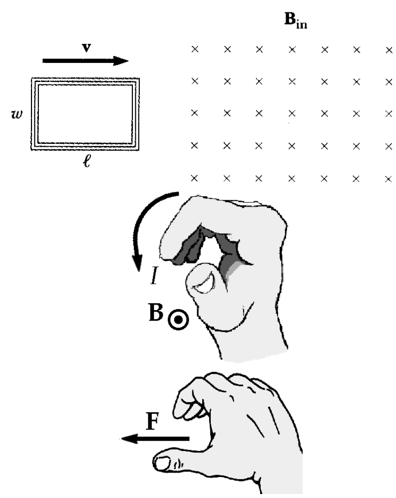
The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv,$$

so the current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

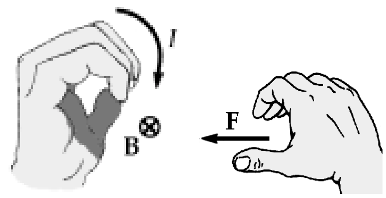
$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R}\text{ to the left}}$$



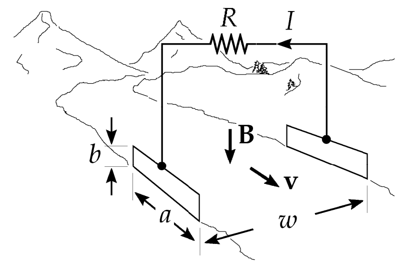
(b) Once the coil is entirely inside the field, $\Phi_B = NBA =$ constant, so $\mathcal{E} = 0$, $I = 0$, and $F = \boxed{0}$.

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current flows clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$



31.28 (a) Motional emf $\mathcal{E} = Bwv$ appears in the conducting water. Its resistance, if the plates are submerged, is



$$\frac{\rho L}{A} = \frac{\rho w}{ab}$$

Kirchhoff's loop theorem says $Bwv - IR - \dfrac{I\rho w}{ab} = 0$

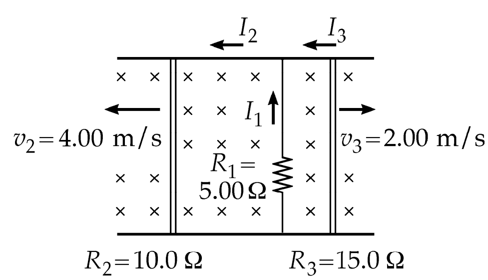
$$I = \frac{Bwv}{R + \dfrac{\rho w}{ab}} = \frac{abvB}{\rho + \dfrac{abR}{w}}$$

(b) $$I_{sc} = \frac{(100 \text{ m})(5.00 \text{ m})(3.00 \text{ m/s})(50.0 \times 10^{-6} \text{ T})}{100\ \Omega \cdot \text{m}} = \boxed{0.750 \text{ mA}}$$

31.29 Look in the direction of *ba*. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counterclockwise current. Therefore, current must flow from *b* to *a* through the resistor. Hence, $V_a - V_b$ will be $\boxed{\text{negative}}$.

31.30 $$\mathcal{E} = \frac{1}{2}B\omega l^2 = \boxed{0.259 \text{ mV}}$$

31.31 Name the currents as shown in the diagram:



Left loop: $+Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $+Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$$

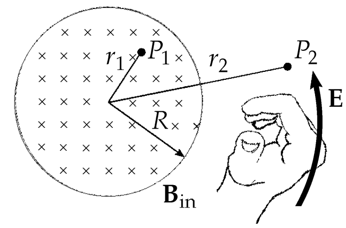
So, $$Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.0100\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m / s})(15.0\,\Omega) - (2.00\text{ m / s})(10.0\,\Omega)}{(5.00\,\Omega)(10.0\,\Omega) + (5.00\,\Omega)(15.0\,\Omega) + (10.0\,\Omega)(15.0\,\Omega)}\right] = \boxed{145\,\mu\text{A}}$$

upward

31.32 (a) $\frac{dB}{dt} = 6.00t^2 - 8.00t \quad |\mathcal{E}| = \frac{d\Phi_B}{dt}$



At $t = 2.00$ s, $E = \frac{\pi R^2(dB/dt)}{2\pi r_2} = \frac{8.00\pi(0.0250)^2}{2\pi(0.0500)}$

$F = qE = \boxed{8.00 \times 10^{-21}\text{ N}}$ clockwise for electron

(b) When $6.00t^2 - 8.00t = 0$, $t = \boxed{1.33\text{ s}}$

31.33 $\frac{dB}{dt} = 0.0600t \qquad |\mathcal{E}| = \frac{d\Phi_B}{dt}$

At $t = 3.00$ s, $E = \pi r_1^2\left(\frac{dB}{2\pi r_1\, dt}\right) = \boxed{1.80 \times 10^{-3}\text{ N/C perpendicular to } r_1 \text{ and counterclockwise}}$

***31.34** $\mathcal{E} = \frac{d\Phi_B}{dt} = \pi r^2\left(\frac{dB}{dt}\right) = \oint \mathbf{E}\cdot d\mathbf{1}$

$E(2\pi R) = \pi r^2 \frac{dB}{dt}$, or $E = \left(\frac{\pi r^2}{2\pi R}\right)\frac{dB}{dt}$

$B = \mu_0 nI \qquad \frac{dB}{dt} = \mu_0 n \frac{dI}{dt}$

$I = 3.00e^{0.200t} \qquad \frac{dI}{dt} = 0.600e^{0.200t}$

At $t = 10.0$ s, $E = \frac{\pi r^2}{2\pi R}(\mu_0 n)(0.600e^{0.200t})$

becomes $E = \frac{(0.0200\text{ m})^2}{2(0.0500\text{ m})}(4\pi \times 10^{-7}\text{ N / A}^2)(1000\text{ turns / m})(0.600)e^{2.00} = \boxed{2.23 \times 10^{-5}\text{ N / C}}$

31.35 (a) $\oint \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\Phi_B}{dt}\right|$

$2\pi r E = (\pi r^2)\frac{dB}{dt}$ so $E = \boxed{(9.87 \text{ mV/m}) \cos (100\, \pi t)}$

(b) The E field is always opposite to increasing B. $\therefore$ $\boxed{\text{clockwise}}$

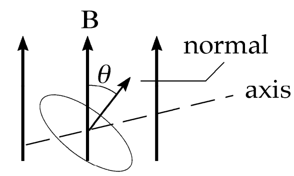
31.36 For the alternator, $\omega = 3000 \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 314\text{ rad/s}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -250\frac{d}{dt}\left[(2.50\times10^{-4}\text{ T}\cdot\text{m}^2)\cos(314\,t\text{ / s})\right] = +250(2.50\times10^{-4}\text{ T}\cdot\text{m}^2)(314\text{/s})\sin(314t)$$

(a) $\boxed{\mathcal{E} = (19.6\text{ V})\sin(314t)}$

(b) $\boxed{\mathcal{E}_{\max} = 19.6\text{ V}}$

31.37 (a) $\mathcal{E}_{\max} = NAB\omega = (1000)(0.100)(0.200)(120\pi) = \boxed{7.54\text{ kV}}$



(b) $\mathcal{E}(t) = -NBA\omega\cdot\sin\omega t = -NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta| = 1$, or $\theta = \pm\dfrac{\pi}{2}$,

so the $\boxed{\text{plane of coil is parallel to }\mathbf{B}}$

31.38 Let θ represent the angle through which the coil turns, starting from $\theta = 0$ at an instant when the horizontal component of the Earth's field is perpendicular to the area. Then,

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -NBA\frac{d}{dt}\cos\omega t = +NBA\omega\sin\omega t$$

Here $\sin\omega t$ oscillates between +1 and –1, so the spinning coil generates an alternating voltage with amplitude

$$\mathcal{E}_{\max} = NBA\omega = NBA2\pi f = 100(2.00\times10^{-5}\text{ T})(0.200\text{ m})^2(1500)\frac{2\pi\text{ rad}}{60.0\text{ s}} = \boxed{12.6\text{ mV}}$$

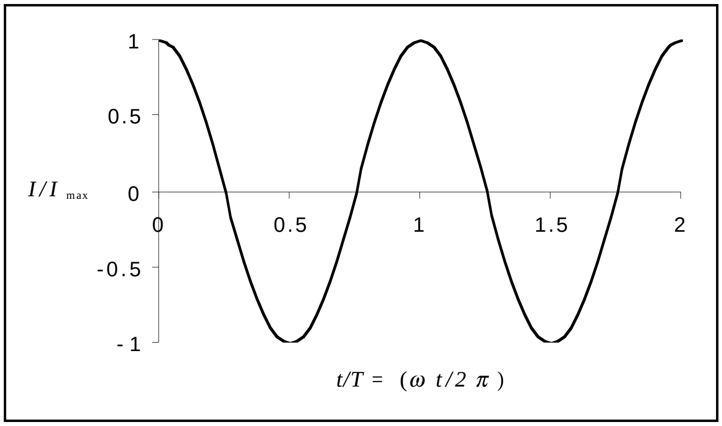
31.39 $B = \mu_0 nI = \left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(200\text{ m}^{-1}\right)(15.0\text{ A}) = 3.77\times10^{-3}\text{ T}$

For the small coil, $\Phi_B = N\mathbf{B}\cdot\mathbf{A} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$

Thus, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$

$$\mathcal{E} = (30.0)\left(3.77\times10^{-3}\text{ T}\right)\pi(0.0800\text{ m})^2\left(4.00\pi\text{ s}^{-1}\right)\sin(4.00\pi t) = \boxed{(28.6\text{ mV})\sin(4.00\pi t)}$$

31.40 As the magnet rotates, the flux through the coil varies sinusoidally in time with $\Phi_B = 0$ at $t = 0$. Choosing the flux as positive when the field passes from left to right through the area of the coil, the flux at any time may be written as $\Phi_B = -\Phi_{max}\sin\omega t$ so the induced emf is given by



$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega\Phi_{max}\cos\omega t.$$

The current in the coil is then $I = \frac{\mathcal{E}}{R} = \frac{\omega\Phi_{max}}{R}\cos\omega t = \boxed{I_{max}\cos\omega t}$

31.41 (a) $F = NI\ell B$

$\tau_{max} = 2Fr = NI\ell wB = \boxed{0.640\ \text{N}\cdot\text{m}}$

(b) $\mathcal{P} = \tau\omega = (0.640\ \text{N}\cdot\text{m})(120\pi\ \text{rad/s})$

$\mathcal{P}_{max} = \boxed{241\ \text{W}}$ (about $\frac{1}{3}$ hp)

31.42 (a) $\mathcal{E}_{max} = BA\omega = B\left(\frac{1}{2}\pi R^2\right)\omega$

$\mathcal{E}_{max} = (1.30\ \text{T})\frac{\pi}{2}(0.250\ \text{m})^2\left(4.00\pi\ \frac{\text{rad}}{\text{s}}\right)$

$\mathcal{E}_{max} = \boxed{1.60\ \text{V}}$

(b) $\bar{\mathcal{E}} = \int_0^{2\pi}\frac{\mathcal{E}}{2\pi}d\theta = \frac{BA\omega}{2\pi}\int_0^{2\pi}\sin\theta\, d\theta = \boxed{0}$

(c) The maximum and average $\mathcal{E}$ would remain unchanged.

(d) See Figure 1 at the right.

(e) See Figure 2 at the right.

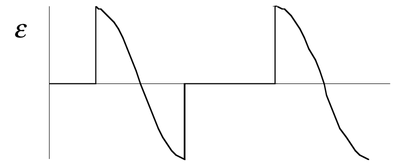


Figure 1

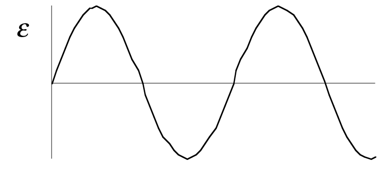


Figure 2

31.43 (a) $\Phi_B = BA\cos\theta = BA\cos\omega t = (0.800\ \text{T})(0.0100\ \text{m}^2)\cos 2\pi(60.0)t = \boxed{(8.00\ \text{mT}\cdot\text{m}^2)\cos(377t)}$

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = \boxed{(3.02 \text{ V}) \sin(377t)}$

(c) $I = \mathcal{E}R = \boxed{(3.02 \text{ A}) \sin(377t)}$

(d) $P = I^2R = \boxed{(9.10 \text{ W}) \sin^2(377t)}$

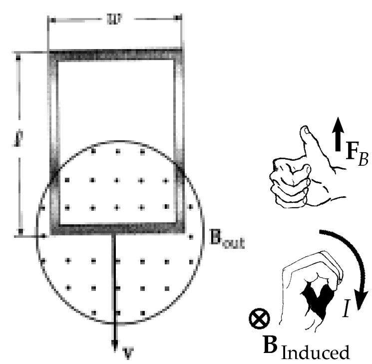
(e) $P = Fv = \tau\omega$ so $\tau = \frac{P}{\omega} = \boxed{(24.1 \text{ mN} \cdot \text{m}) \sin^2(377t)}$

31.44 At terminal speed, the upward magnetic force exerted on the lower edge of the loop must equal the weight of the loop. That is,

$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_t}{R}\right)wB = \frac{B^2w^2v_t}{R}$$

Thus,

$$B = \sqrt{\frac{MgR}{w^2v_t}} = \sqrt{\frac{(0.150 \text{ kg})(9.80 \text{ m/s}^2)(0.750\ \Omega)}{(1.00 \text{ m})^2(2.00 \text{ m/s})}} = \boxed{0.742 \text{ T}}$$



31.45 See the figure above with Problem 31.44.

(a) At terminal speed, $Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_t}{R}\right)wB = \frac{B^2w^2v_t}{R}$

or $\boxed{v_t = \frac{MgR}{B^2w^2}}$

(b) The emf is directly proportional to v_t, but the current is inversely proportional to R. A large R means a small current at a given speed, so the loop must travel faster to get $F_m = mg$.

(c) At given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small B, the speed must increase to compensate for both the small B and also the current, so $v_t \propto B^2$.

***31.46** The current in the magnet creates an upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of **B** increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being counterclockwise as the picture correctly shows.

31.47 $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

$$\mathbf{a} = \frac{e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \quad \text{where} \quad \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\mathbf{j} + 200(0.300)\mathbf{k}$$

$$\mathbf{a} = \frac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}}[50.0\mathbf{j} - 80.0\mathbf{j} + 60.0\mathbf{k}] = 9.58 \times 10^{7}[-30.0\mathbf{j} + 60.0\mathbf{k}]$$

$$\mathbf{a} = 2.87 \times 10^{9}[-\mathbf{j} + 2\mathbf{k}] \text{ m/s}^2 = \boxed{(-2.87 \times 10^{9}\,\mathbf{j} + 5.75 \times 10^{9}\,\mathbf{k}) \text{ m/s}^2}$$

31.48 $\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$ so $\mathbf{a} = \frac{-e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$ where $\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10.0 & 0 & 0 \\ 0 & 0 & 0.400 \end{vmatrix} = -4.00\mathbf{j}$

$$\mathbf{a} = \frac{\left(-1.60 \times 10^{-19}\right)}{9.11 \times 10^{-31}}[2.50\,\mathbf{i} + 5.00\,\mathbf{j} - 4.00\,\mathbf{j}] = \left(-1.76 \times 10^{11}\right)[2.50\,\mathbf{i} + 1.00\,\mathbf{j}]$$

$$\mathbf{a} = \boxed{\left(-4.39 \times 10^{11}\,\mathbf{i} - 1.76 \times 10^{11}\,\mathbf{j}\right) \text{ m/s}^2}$$

***31.49** $\mathcal{E} = -N\frac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0^\circ \frac{dB}{dt}$

$$\mathcal{E} = -(30.0)\pi\left(2.70 \times 10^{-3} \text{ m}\right)^2(1)\frac{d}{dt}\left[50.0 \text{ mT} + (3.20 \text{ mT})\sin\left(2\pi\ 523t/\text{s}\right)\right]$$

$$\mathcal{E} = -(30.0)\pi\left(2.70 \times 10^{-3} \text{ m}\right)^2\left(3.20 \times 10^{-3} \text{ T}\right)(2\pi)(523/\text{s})\cos\left(2\pi\ 523t/\text{s}\right)$$

$$\mathcal{E} = \boxed{-\left(7.22 \times 10^{-3} \text{ V}\right)\cos\left(2\pi\ 523t/\text{s}\right)}$$

***31.50** (a) Doubling the number of turns.

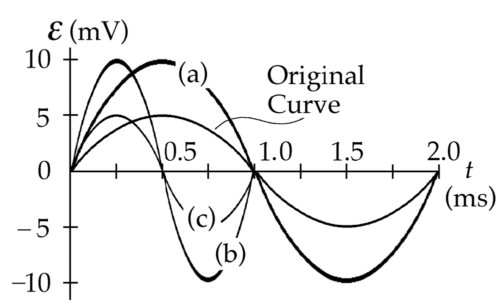
Amplitude doubles: period unchanged

(b) Doubling the angular velocity.

doubles the amplitude: cuts the period in half

(c) Doubling the angular velocity while reducing the number of turns to one half the original value.

Amplitude unchanged: cuts the period in half

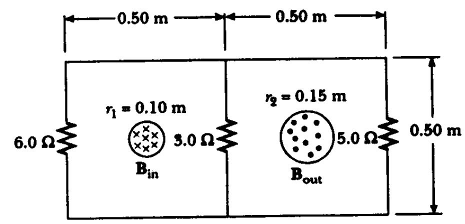


***31.51** $\mathcal{E} = -N\dfrac{\Delta}{\Delta t}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0^\circ \dfrac{\Delta B}{\Delta t} = -1\left(0.00500\ \text{m}^2\right)(1)\left(\dfrac{1.50\ \text{T} - 5.00\ \text{T}}{20.0 \times 10^{-3}\ \text{s}}\right) = 0.875\ \text{V}$

(a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.875\ \text{V}}{0.0200\ \Omega} = \boxed{43.8\ \text{A}}$

(b) $\mathcal{P} = \mathcal{E}I = (0.875\ \text{V})(43.8\ \text{A}) = \boxed{38.3\ \text{W}}$

31.52 In the loop on the left, the induced emf is



$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.100\ \text{m})^2(100\ \text{T/s}) = \pi\ \text{V}$$

and it attempts to produce a counterclockwise current in this loop.

In the loop on the right, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi(0.150\ \text{m})^2(100\ \text{T/s}) = 2.25\pi\ \text{V}$$

and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00-Ω resistor, I_2 flows down through the 5.00-Ω resistor, and that I_3 flows up through the 3.00-Ω resistor.

From Kirchhoff's point rule: $I_3 = I_1 + I_2$ (1)

Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$ (2)

Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ (3)

Solving these three equations simultaneously,

$I_1 = \boxed{0.0623\ \text{A}}$, $I_2 = \boxed{0.860\ \text{A}}$, and $I_3 = \boxed{0.923\ \text{A}}$

***31.53** The emf induced between the ends of the moving bar is

$$\mathcal{E} = B\ell v = (2.50\ \text{T})(0.350\ \text{m})(8.00\ \text{m/s}) = 7.00\ \text{V}$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives: $+7.00\text{ V} - I_1(2.00\ \Omega) = 0$ $I_1 = \boxed{3.50\text{ A}}$

and $+7.00\text{ V} - I_3(5.00\ \Omega) = 0$ $I_3 = \boxed{1.40\text{ A}}$

(b) The total power dissipated in the resistors of the circuit is

$$\mathcal{P} = \mathcal{E}I_1 + \mathcal{E}I_3 = \mathcal{E}(I_1 + I_3) = (7.00\text{ V})(3.50\text{ A} + 1.40\text{ A}) = \boxed{34.3\text{ W}}$$

(c) Method 1: The current in the sliding conductor is downward with value $I_2 = 3.50\text{ A} + 1.40\text{ A} = 4.90\text{ A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\text{ A})(0.350\text{ m})(2.50\text{ T}) = 4.29\text{ N}$ directed toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\text{ N}}$ to the left to keep the bar moving.

Method 2: The agent moving the bar must supply the power according to $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = Fv\cos 0^\circ$. The force required is then:

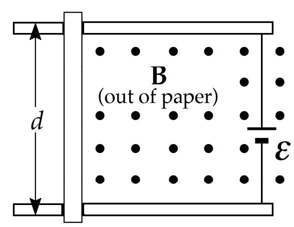
$$F = \frac{\mathcal{P}}{v} = \frac{34.3\text{ W}}{8.00\text{ m/s}} = \boxed{4.29\text{ N}}$$

***31.54** Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB(\pi r^2)\left(\frac{\cos 180^\circ - \cos 0^\circ}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)(10^{-3}\text{ T})\pi(0.0150\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right) \quad \boxed{\sim 10^{-4}\text{ V}}$$

31.55 $I = \dfrac{\mathcal{E} + \mathcal{E}_{\text{Induced}}}{R}$ and $\mathcal{E}_{\text{Induced}} = -\dfrac{d}{dt}(BA)$



$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}(\mathcal{E} + \mathcal{E}_{\text{Induced}}) = \frac{Bd}{mR}(\mathcal{E} - Bvd)$$

To solve the differential equation, let $u = (\mathcal{E} - Bvd)$, $\dfrac{du}{dt} = -Bd\dfrac{dv}{dt}$.

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u \quad \text{so} \quad \int_{u_0}^{u}\frac{du}{u} = -\int_{t=0}^{t}\frac{(Bd)^2}{mR}dt$$

Integrating from $t = 0$ to $t = t$, $\ln\dfrac{u}{u_0} = -\dfrac{(Bd)^2}{mR}t$ or $\dfrac{u}{u_0} = e^{-B^2d^2t/mR}$

Since $v = 0$ when $t = 0$, $u_0 = \mathcal{E}$ and $u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR} \qquad \text{and} \qquad \boxed{v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$$

31.56 (a) For maximum induced emf, with positive charge at the top of the antenna,

$\mathbf{F}_+ = q_+ (\mathbf{v} \times \mathbf{B})$, so the auto must move east

(b) $\mathcal{E} = Blv = (5.00 \times 10^{-5}\text{ T})(1.20\text{ m})\left(\dfrac{65.0 \times 10^3\text{ m}}{3600\text{ s}}\right)\cos 65.0° = \boxed{4.58 \times 10^{-4}\text{ V}}$

31.57 $I = \dfrac{\mathcal{E}}{R} = \dfrac{B}{R}\dfrac{|\Delta A|}{\Delta t}$

so $q = I\,\Delta t = \dfrac{(15.0\ \mu\text{T})(0.200\text{ m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

Goal Solution

The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0\ \mu\text{T}$, as shown in Figure P31.57. The total resistance of the loop and the wires connecting it to the galvanometer is $0.500\ \Omega$. If the loop is suddenly collapsed by horizontal forces as shown, what total charge passes through the galvanometer?

G: For the situation described, the maximum current is probably less than 1 mA. So if the loop is closed in 0.1 s, then the total charge would be

$$Q = I\Delta t = (1\text{ mA})(0.1\text{ s}) = 100\ \mu\text{C}$$

O: We do not know how quickly the loop is collapsed, but we can find the total charge by integrating the change in magnetic flux due to the change in area of the loop ($a^2 \to 0$).

A: $$Q = \int I dt = \int \frac{\mathcal{E}\,dt}{R} = \frac{1}{R}\int -\left(\frac{d\Phi_B}{dt}\right)dt = -\frac{1}{R}\int d\Phi_B = -\frac{1}{R}\int d(BA) = -\frac{B}{R}\int_{A_1=a^2}^{A_2=0} dA$$

$$Q = -\frac{B}{R}A\Big]_{A_1=a^2}^{A_2=0} = \frac{Ba^2}{R} = \frac{(15.0\times 10^{-6}\text{ T})(0.200\text{ m})^2}{0.500\ \Omega} = 1.20\times 10^{-6}\text{ C}$$

L: The total charge is less than the maximum charge we predicted, so the answer seems reasonable. It is interesting that this charge can be calculated without knowing either the current or the time to collapse the loop. **Note**: We ignored the internal resistance of the galvanometer. D'Arsonval galvanometers typically have an internal resistance of 50 to 100 Ω, significantly more than the resistance of the wires given in the problem. A proper solution that includes R_G would reduce the total charge by about 2 orders of magnitude ($Q \sim 0.01\ \mu\text{C}$).

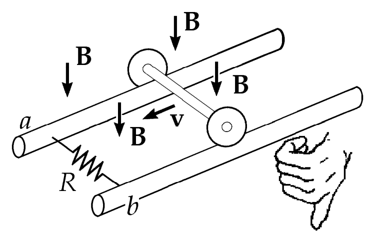
***31.58** (a) $I = \dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$ so $\int dq = \dfrac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge through the circuit will be $|Q| = \dfrac{N}{R}(\Phi_2 - \Phi_1)$

(b) $Q = \dfrac{N}{R}\left[BA\cos 0 - BA\cos\left(\dfrac{\pi}{2}\right)\right] = \dfrac{BAN}{R}$

so $B = \dfrac{RQ}{NA} = \dfrac{(200\ \Omega)(5.00\times 10^{-4}\ \text{C})}{(100)(40.0\times 10^{-4}\ \text{m}^2)} = \boxed{0.250\ \text{T}}$

31.59 (a) $\mathcal{E} = B\ell v = 0.360$ V $\quad I = \dfrac{\mathcal{E}}{R} = \boxed{0.900\ \text{A}}$

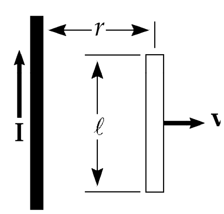


(b) $F_B = I\ell B = \boxed{0.108\ \text{N}}$

(c) Since the magnetic flux $\mathbf{B}\cdot\mathbf{A}$ is in effect decreasing, the induced current flow through R is from b to a. $\boxed{\text{Point } b}$ is at higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of ab. Here counterclockwise current will flow to produce upward magnetic field. The in R is still from b to a.

31.60 $\mathcal{E} = B\ell v$ at a distance r from wire



$$|\mathcal{E}| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

31.61 (a) At time t, the flux through the loop is $\Phi_B = BA\cos\theta = (a+bt)(\pi r^2)\cos 0° = \pi(a+bt)r^2$

At $t = 0$, $\Phi_B = \boxed{\pi a r^2}$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\pi r^2\dfrac{d(a+bt)}{dt} = \boxed{-\pi b r^2}$

(c) $I = \dfrac{\mathcal{E}}{R} = \boxed{-\dfrac{\pi b r^2}{R}}$

(d) $\mathcal{P} = \mathcal{E}I = \left(-\dfrac{\pi b r^2}{R}\right)\left(-\pi b r^2\right) = \boxed{\dfrac{\pi^2 b^2 r^4}{R}}$

31.62 $\mathcal{E} = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K$

(a) $Q = C\mathcal{E} = \boxed{C\pi a^2 K}$

(b) **B** into the paper is decreasing; therefore, current will attempt to counteract this. Positive charge will go to $\boxed{\text{upper plate}}$.

(c) The changing magnetic field through the enclosed area $\boxed{\text{induces an electric field}}$, surrounding the **B**-field, and this pushes on charges in the wire.

31.63 The flux through the coil is $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta = BA\cos\omega t$. The induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t.$$

(a) $\mathcal{E}_{\max} = NBA\omega = 60.0(1.00\text{ T})\left(0.100\times 0.200\text{ m}^2\right)(30.0\text{ rad/s}) = \boxed{36.0\text{ V}}$

(b) $\frac{d\Phi_B}{dt} = \frac{\mathcal{E}}{N}$, thus $\left|\frac{d\Phi_B}{dt}\right|_{\max} = \frac{\mathcal{E}_{\max}}{N} = \frac{36.0\text{ V}}{60.0} = 0.600\text{ V} = \boxed{0.600\text{ Wb/s}}$

(c) At $t = 0.0500$ s, $\omega t = 1.50$ rad and $\mathcal{E} = \mathcal{E}_{\max}\sin(1.50\text{ rad}) = (36.0\text{ V})\sin(1.50\text{ rad}) = \boxed{35.9\text{ V}}$

(d) The torque on the coil at any time is $\tau = |\mu\times\mathbf{B}| = |NI\mathbf{A}\times\mathbf{B}| = (NAB)I|\sin\omega t| = \left(\frac{\mathcal{E}_{\max}}{\omega}\right)\left(\frac{\mathcal{E}}{R}\right)|\sin\omega t|$

When $\mathcal{E} = \mathcal{E}_{\max}$, $\sin\omega t = 1.00$ and $\tau = \frac{\mathcal{E}_{\max}^2}{\omega R} = \frac{(36.0\text{ V})^2}{(30.0\text{ rad/s})(10.0\ \Omega)} = \boxed{4.32\text{ N}\cdot\text{m}}$

31.64 (a) We use $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.

Taking $a = 5.00\times 10^{-3}$ m to be the radius of the washer, and $h = 0.500$ m,

$$\Delta\Phi_B = B_2 A - B_1 A = A(B_2 - B_1) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right) = \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 a h I}{2(h+a)}$$

The time for the washer to drop a distance h (from rest) is: $\Delta t = \sqrt{\frac{2h}{g}}$

Therefore, $\mathcal{E} = \frac{\mu_0 a h I}{2(h+a)\Delta t} = \frac{\mu_0 a h I}{2(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 a I}{2(h+a)}\sqrt{\frac{gh}{2}}$

and $\mathcal{E} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(5.00\times 10^{-3}\text{ m})(10.0\text{ A})}{2(0.500\text{ m} + 0.00500\text{ m})}\sqrt{\frac{(9.80\text{ m/s}^2)(0.500\text{ m})}{2}} = \boxed{97.4\text{ nV}}$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

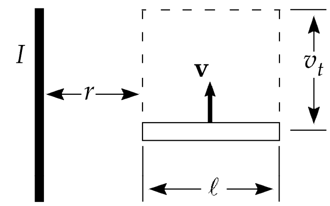
31.65

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200\left(50.0\times10^{-6}\ \text{T}\right)\left(\cos 62.0^\circ\right)\left(\frac{39.0\times10^{-4}\ \text{m}^2}{1.80\ \text{s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

31.66

Find an expression for the flux through a rectangular area "swept out" by the bar in time t. The magnetic field at a distance x from wire is



$B = \dfrac{\mu_0 I}{2\pi x}$ and $\Phi_B = \int B\,dA$. Therefore,

$\Phi_B = \dfrac{\mu_0 I v t}{2\pi}\displaystyle\int_r^{r+\ell}\frac{dx}{x}$ where vt is the distance the bar has moved in time t.

Then, $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \boxed{\dfrac{\mu_0 I v}{2\pi}\ln\left(1+\dfrac{\ell}{r}\right)}$

31.67

The magnetic field at a distance x from a long wire is $B = \dfrac{\mu_0 I}{2\pi x}$. Find an expression for the flux through the loop.

$d\Phi_B = \dfrac{\mu_0 I}{2\pi x}(\ell\,dx)$ so $\Phi_B = \dfrac{\mu_0 I \ell}{2\pi}\displaystyle\int_r^{r+w}\frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(1+\frac{w}{r}\right)$

Therefore, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 I \ell v}{2\pi r}\dfrac{w}{(r+w)}$ and $I = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 I \ell v}{2\pi R r}\dfrac{w}{(r+w)}}$

31.68

As the wire falls through the magnetic field, a motional emf $\mathcal{E} = B\ell v$ is induced in it. Thus, a counterclockwise induced current of $I = \mathcal{E}/R = B\ell v/R$ flows in the circuit. The falling wire is carrying a current toward the left through the magnetic field. Therefore, it experiences an upward magnetic force given by $F_B = I\ell B = B^2\ell^2 v/R$. The wire will have attained terminal speed when the magnitude of this magnetic force equals the weight of the wire.

Thus, $\dfrac{B^2\ell^2 v_t}{R} = mg$, or the terminal speed is $v_t = \boxed{\dfrac{mgR}{B^2\ell^2}}$

31.69

$\Phi_B = (6.00t^3 - 18.0t^2)\ \text{T}\cdot\text{m}^2$ and $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$

Maximum $\mathcal{E}$ occurs when $\dfrac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$, which gives $t = 1.00$ s.

Therefore, the maximum current (at $t = 1.00$ s) is $I = \dfrac{\mathcal{E}}{R} = \dfrac{(-18.0 + 36.0)\text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$

31.70 For the suspended mass, M: $\Sigma F = Mg - T = Ma$

For the sliding bar, m: $\Sigma F = T - I\ell B = ma$, where $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$

$$Mg - \frac{B^2\ell^2 v}{R} = (m+M)a \qquad \text{or} \qquad a = \frac{dv}{dt} = \frac{Mg}{m+M} - \frac{B^2\ell^2 v}{R(M+m)}$$

$$\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt \qquad \text{where} \qquad \alpha = \frac{Mg}{M+m} \quad \text{and} \quad \beta = \frac{B^2\ell^2}{R(M+m)}$$

Therefore, the velocity varies with time as $v = \dfrac{\alpha}{\beta}(1 - e^{-\beta t}) = \boxed{\dfrac{MgR}{B^2\ell^2}\left[1 - e^{-B^2\ell^2 t/R(M+m)}\right]}$

***31.71** (a) $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -NA\dfrac{dB}{dt} = -NA\dfrac{d}{dt}(\mu_0 nI)$

where A = area of coil, N = number of turns in coil, and n = number of turns per unit length in solenoid. Therefore,

$$|\mathcal{E}| = N\mu_0 An\frac{d}{dt}\left[4\sin(120\pi t)\right] = N\mu_0 An(480\pi)\cos(120\pi t)$$

$$|\mathcal{E}| = 40(4\pi\times 10^{-7})\left[\pi(0.0500\text{ m})^2\right](2.00\times 10^3)(480\pi)\cos(120\pi t) = \boxed{(1.19\text{ V})\cos(120\pi t)}$$

(b) $I = \dfrac{\Delta V}{R}$ and $\mathcal{P} = \Delta VI = \dfrac{(1.19\text{ V})^2\cos^2(120\pi t)}{(8.00\ \Omega)}$

From $\cos^2\theta = \dfrac{1}{2} + \dfrac{1}{2}\cos 2\theta$, the average value of $\cos^2\theta$ is $\dfrac{1}{2}$, so $\bar{\mathcal{P}} = \dfrac{1}{2}\dfrac{(1.19\text{ V})^2}{(8.00\ \Omega)} = \boxed{88.5\text{ mW}}$

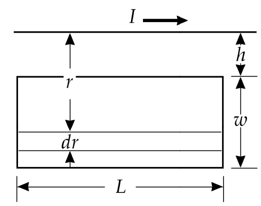
31.72 The induced emf is $\mathcal{E} = B\ell v$ where $B = \dfrac{\mu_0 I}{2\pi y}$, $v = v_i + gt = \left(9.80\text{ m/s}^2\right)t$, and

$y = y_i - \frac{1}{2}gt^2 = 0.800\text{ m} - \left(4.90\text{ m/s}^2\right)t^2$.

$$\mathcal{E} = \frac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)(200\text{ A})}{2\pi\left[0.800\text{ m} - \left(4.90\text{ m/s}^2\right)t^2\right]}(0.300\text{ m})\left(9.80\text{ m/s}^2\right)t = \boxed{\frac{\left(1.18\times 10^{-4}\right)t}{\left[0.800 - 4.90t^2\right]}\text{ V}}$$

At $t = 0.300$ s, $\mathcal{E} = \dfrac{\left(1.18\times 10^{-4}\right)(0.300)}{\left[0.800 - 4.90(0.300)^2\right]}\text{ V} = \boxed{98.3\ \mu\text{V}}$

31.73 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \mu_0 I / 2\pi r$. Thus, the flux linkage is



$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi)$$

Finally, the induced emf is $\mathcal{E} = -\frac{\mu_0 NI_{max}L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t + \phi)$

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\right)(100)(50.0)(0.200\text{ m})(200\pi\text{ s}^{-1})}{2\pi}\ln\left(1+\frac{5.00\text{ cm}}{5.00\text{ cm}}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = \boxed{-(87.1\text{ mV})\cos(200\pi t + \phi)}$$

The term $\sin(\omega t + \phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.100 rad or less. Thus, the current does not change appreciably during a time interval

$$t < \frac{0.100}{(200\pi\text{ s}^{-1})} = 1.60\times10^{-4}\text{ s}.$$

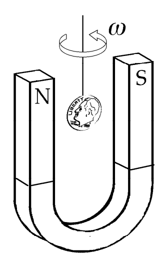
We define a critical length, $ct = (3.00\times10^8\text{ m/s})(1.60\times10^{-4}\text{ s}) = 4.80\times10^4\text{ m}$ equal to the distance to which field changes could be propagated during an interval of 1.60×10^{-4} s. This length is so much larger than any dimension of the coilor its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

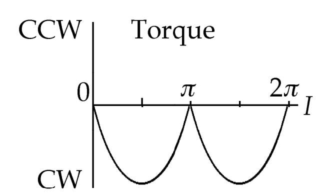
If the frequency ω were much larger, say, $200\pi\times10^5\text{ s}^{-1}$, the corresponding critical length would be only 48.0 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a "rule of thumb" we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \omega/2\pi$, that are less than about 10^6 Hz.

31.74 $\Phi_B = BA\cos\theta$ $\quad \frac{d\Phi_B}{dt} = -\omega BA\sin\theta$;

$I \propto -\sin\theta$

$\tau \propto IB\sin\theta$ $\boxed{\propto -\sin^2\theta}$





31.75 The area of the tent that is effective in intercepting magnetic field lines is the area perpendicular to the direction of the magnetic field. This is the same as the base of the tent. In the initial configuration, this is

$$A_1 = L(2L\cos\theta) = 2(1.50\text{ m})^2\cos 60.0° = 2.25\text{ m}^2$$

After the tent is flattened, $A_2 = L(2L) = 2L^2 = 2(1.50\text{ m})^2 = 4.50\text{ m}^2$

The average induced emf is: $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{B(\Delta A)}{\Delta t} = -\frac{(0.300\text{ T})(4.50-2.25)\text{ m}^2}{0.100\text{ s}} = \boxed{-6.75\text{ V}}$

Chapter 32 Solutions

***32.1** $|\overline{\mathcal{E}}| = L\frac{\Delta I}{\Delta t} = (3.00\times 10^{-3}\text{ H})\left(\frac{1.50\text{ A} - 0.200\text{ A}}{0.200\text{ s}}\right) = 1.95\times 10^{-2}\text{ V} = \boxed{19.5\text{ mV}}$

32.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m / A})(70.0)^2(\pi)(6.50\times 10^{-3}\text{ m})^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}$$

32.3 $|\overline{\mathcal{E}}| = +L\left(\frac{\Delta I}{\Delta t}\right) = (2.00\text{ H})\left(\frac{0.500\text{ A}}{0.0100\text{ s}}\right) = \boxed{100\text{ V}}$

32.4 $L = \mu_0 n^2 A\ell$ so $n = \sqrt{\frac{L}{\mu_0 A\ell}} = \boxed{7.80\times 10^3\text{ turns / m}}$

32.5 $L = \frac{N\Phi_B}{I} \rightarrow \Phi_B = \frac{LI}{N} = \boxed{240\text{ nT}\cdot\text{m}^2}$ (through each turn)

32.6 $|\mathcal{E}| = L\frac{dI}{dt}$ where $L = \frac{\mu_0 N^2 A}{\ell}$

Thus, $|\mathcal{E}| = \left(\frac{\mu_0 N^2 A}{\ell}\right)\frac{dI}{dt} = \frac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)(300)^2\left(\pi\times 10^{-4}\text{ m}^2\right)}{0.150\text{ m}}(10.0\text{ A/s}) = \boxed{2.37\text{ mV}}$

32.7 $\mathcal{E}_{\text{back}} = -\mathcal{E} = L\frac{dI}{dt} = L\frac{d}{dt}(I_{\max}\sin\omega t) = L\omega I_{\max}\cos\omega t = (10.0\times 10^{-3})(120\pi)(5.00)\cos\omega t$

$\mathcal{E}_{\text{back}} = (6.00\pi)\cos(120\pi t) = \boxed{(18.8\text{ V})\cos(377t)}$

***32.8** From $|\mathcal{E}| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\mathcal{E}}{\left(\frac{\Delta I}{\Delta t}\right)} = \frac{24.0 \times 10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40 \times 10^{-3}\text{ H}$

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40 \times 10^{-3}\text{ H})(4.00\text{ A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$

32.9 $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0(420)^2(3.00 \times 10^{-4})}{0.160} = 4.16 \times 10^{-4}\text{ H}$

$\mathcal{E} = -L\frac{dI}{dt} \rightarrow \frac{dI}{dt} = \frac{-\mathcal{E}}{L} = \frac{-175 \times 10^{-6}\text{ V}}{4.16 \times 10^{-4}\text{ H}} = \boxed{-0.421\text{ A/s}}$

32.10 The induced emf is $\mathcal{E} = -L\frac{dI}{dt}$, where the self-inductance of a solenoid is given by $L = \frac{\mu_0 N^2 A}{l}$.

Thus, $\frac{dI}{dt} = -\frac{\mathcal{E}}{L} = \boxed{-\frac{\mathcal{E}l}{\mu_0 N^2 A}}$

32.11 $|\mathcal{E}| = L\frac{dI}{dt} = (90.0 \times 10^{-3})\frac{d}{dt}(t^2 - 6t)\text{ V}$

(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360\text{ mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180\text{ mV}}$

(c) $\mathcal{E} = (90.0 \times 10^{-3})(2t - 6) = 0$ when $\boxed{t = 3.00\text{ s}}$

32.12 (a) $B = \mu_0 nI = \mu_0\left(\frac{450}{0.120}\right)(0.0400\text{ mA}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = \boxed{3.33 \times 10^{-8}\text{ T}\cdot\text{m}^2}$

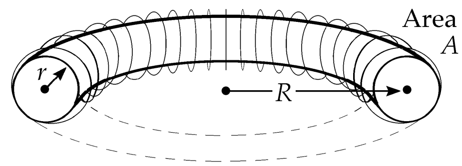
(c) $L = \frac{N\Phi_B}{I} = \boxed{0.375\text{ mH}}$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current}}$

32.13 (a) $L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 (120)^2 \pi (5.00 \times 10^{-3})^2}{0.0900} = \boxed{15.8\ \mu\text{H}}$

(b) $\Phi_B' = \frac{\mu_m}{\mu_0} \Phi_B \rightarrow L = \frac{\mu_m N^2 A}{\ell} = 800(1.58 \times 10^{-5}\ \text{H}) = \boxed{12.6\ \text{mH}}$

32.14 $L = \frac{N\Phi_B}{I} = \frac{NBA}{I} \approx \frac{NA}{I} \cdot \frac{\mu_0 NI}{2\pi R} = \boxed{\frac{\mu_0 N^2 A}{2\pi R}}$



32.15 $\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L \frac{dI}{dt}$

$dI = -\frac{\mathcal{E}_0}{L} e^{-kt} dt$

If we require $I \rightarrow 0$ as $t \rightarrow \infty$, the solution is $I = \frac{\mathcal{E}_0}{kL} e^{-kt} = \frac{dq}{dt}$

$Q = \int I\, dt = \int_0^\infty \frac{\mathcal{E}_0}{kL} e^{-kt} dt = -\frac{\mathcal{E}_0}{k^2 L}$ $\boxed{|Q| = \frac{\mathcal{E}_0}{k^2 L}}$

32.16 $I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$

$0.900 \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left[1 - e^{-R(3.00\ \text{s})/2.50\ \text{H}}\right]$

$\exp\left(-\frac{R(3.00\ \text{s})}{2.50\ \text{H}}\right) = 0.100$

$R = \frac{2.50\ \text{H}}{3.00\ \text{s}} \ln 10.0 = \boxed{1.92\ \Omega}$

32.17 $\tau = \frac{L}{R} = 0.200$ s: $\frac{I}{I_{\max}} = 1 - e^{-t/\tau}$

(a) $0.500 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 2.00 = \boxed{0.139 \text{ s}}$

(b) $0.900 = 1 - e^{-t/0.200} \rightarrow t = \tau \ln 10.0 = \boxed{0.461 \text{ s}}$

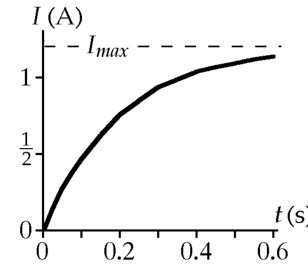


Figure for Goal Solution

Goal Solution

A 12.0-V battery is about to be connected to a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. How long will it take the current to reach (a) 50.0% and (b) 90.0% of its final value?

G: The time constant for this circuit is $\tau = L/R = 0.2$ s, which means that in 0.2 s, the current will reach $1/e$ = 63% of its final value, as shown in the graph to the right. We can see from this graph that the time to reach 50% of $I_{\max}$ should be slightly less than the time constant, perhaps about 0.15 s, and the time to reach $0.9I_{\max}$ should be about $2.5\tau = 0.5$ s.

O: The exact times can be found from the equation that describes the rising current in the above graph and gives the current as a function of time for a known emf, resistance, and time constant. We set time $t = 0$ to be the moment the circuit is first connected.

A: At time t, $I(t) = \frac{\mathcal{E}(1 - e^{-t/\tau})}{R}$

where, after a long time, $I_{\max} = \frac{\mathcal{E}(1 - e^{-\infty})}{R} = \frac{\mathcal{E}}{R}$

At $I(t) = 0.500 I_{\max}$, $(0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}(1 - e^{-t/0.200 \text{ s}})}{R}$ so $0.500 = 1 - e^{-t/0.200 \text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/2.00 \text{ s}}\right) = \ln(0.500)$

and solving for t, $-\frac{t}{0.200 \text{ s}} = -0.693$ or $t = 0.139$ s

(b) Similarly, to reach 90% of $I_{\max}$, $0.900 = 1 - e^{-t/\tau}$ and $t = -\tau \ln(1 - 0.900)$

Thus, $t = -(0.200 \text{ s})\ln(0.100) = 0.461$ s

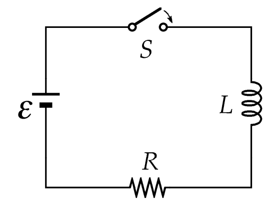
L: The calculated times agree reasonably well with our predictions. We must be careful to avoid confusing the equation for the rising current with the similar equation for the falling current. Checking our answers against predictions is a safe way to prevent such mistakes.

32.18 Taking $\tau = L/R$, $I = I_0 e^{-t/\tau}$: $\dfrac{dI}{dt} = I_0 e^{-t/\tau}\left(-\dfrac{1}{\tau}\right)$

$IR + L\dfrac{dI}{dt} = 0$ will be true if $I_0 R e^{-t/\tau} + L\left(I_0 e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right) = 0$

Because $\tau = L/R$, we have agreement with 0 = 0

***32.19** (a) $\tau = L/R = 2.00 \times 10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$



(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\ \text{V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\ \text{A}}$

(c) $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\ \text{V}}{4.00\ \Omega} = \boxed{1.50\ \text{A}}$

(d) $0.800 = 1 - e^{-t/2.00\ \text{ms}} \rightarrow t = -(2.00\ \text{ms})\ln(0.200) = \boxed{3.22\ \text{ms}}$

***32.20** $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \dfrac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02\ \text{A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2\ \text{V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8\ \text{V}}$

32.21 (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\ \text{A}) = 16.0\ \text{V}$ and
$\Delta V_L = \mathcal{E} - \Delta V_R = 36.0\ \text{V} - 16.0\ \text{V} = 20.0\ \text{V}$

Therefore, $\dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0\ \text{V}}{20.0\ \text{V}} = \boxed{0.800}$

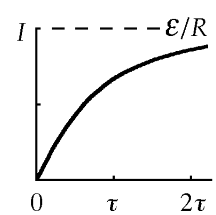


Figure for Goal Solution

(b) $\Delta V_R = IR = (4.50\ \text{A})(8.00\ \Omega) = 36.0\ \text{V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

Goal Solution

For the *RL* circuit shown in Figure P32.19, let L = 3.00 H, R = 8.00 Ω, and $\mathcal{E}$ = 36.0 V. (a) Calculate the ratio of the potential difference across the resistor to that across the inductor when I = 2.00 A. (b) Calculate the voltage across the inductor when I = 4.50 A.

G: The voltage across the resistor is proportional to the current, $\Delta V_R = IR$, while the voltage across the inductor is proportional to the **rate of change** in the current, $\mathcal{E}_L = -L\,dI/dt$. When the switch is first closed, the voltage across the inductor will be large as it opposes the sudden change in current. As the current approaches its steady state value, the voltage across the resistor increases and the inductor's emf decreases. The maximum current will be $\mathcal{E}/R$ = 4.50 A, so when I = 2.00 A, the resistor and inductor will share similar voltages at this mid-range current, but when I = 4.50 A, the entire circuit voltage will be across the resistor, and the voltage across the inductor will be zero.

O: We can use the definition of resistance to calculate the voltage across the resistor for each current. We will find the voltage across the inductor by using Kirchhoff's loop rule.

A: (a) When $I = 2.00$ A, the voltage across the resistor is $\Delta V_R = IR = (2.00\text{ A})(8.00\ \Omega) = 16.0\text{ V}$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 16.0\text{ V} - \mathcal{E}_L = 0$ so $\mathcal{E}_L = 20.0\text{ V}$ and $\dfrac{\Delta V_R}{\mathcal{E}_L} = \dfrac{16.0\text{ V}}{20.0\text{ V}} = 0.800$

(b) Similarly, for $I = 4.50$ A, $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 36.0\text{ V} - \mathcal{E}_L = 0$ so $\mathcal{E}_L = 0$

L: We see that when $I = 2.00$ A, $\Delta V_R < \mathcal{E}_L$, but they are similar in magnitude as expected. Also as predicted, the voltage across the inductor goes to zero when the current reaches its maximum value. A worthwhile exercise would be to consider the ratio of these voltages for several different times after the switch is reopened.

***32.22** After a long time, 12.0 V = (0.200 A)R Thus, R = 60.0 Ω. Now, $\tau = \dfrac{L}{R}$ gives

$L = \tau R = (5.00 \times 10^{-4}\text{ s})(60.0\text{ V/A}) = \boxed{30.0\text{ mH}}$

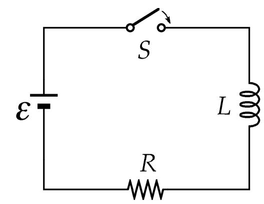
32.23 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\dfrac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\dfrac{1}{\tau}\right)$

$\tau = \dfrac{L}{R} = \dfrac{15.0\text{ H}}{30.0\ \Omega} = 0.500\text{ s}$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^{-t/\tau}$ and $I_{\max} = \dfrac{\mathcal{E}}{R}$

(a) $t = 0$: $\dfrac{dI}{dt} = \dfrac{R}{L} I_{\max} e^{0} = \dfrac{\mathcal{E}}{L} = \dfrac{100\text{ V}}{15.0\text{ H}} = \boxed{6.67\text{ A/s}}$

(b) $t = 1.50$ s: $\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{L} e^{-t/\tau} = (6.67\text{ A/s})e^{-1.50/(0.500)} = (6.67\text{ A/s})e^{-3.00} = \boxed{0.332\text{ A/s}}$

32.24 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$



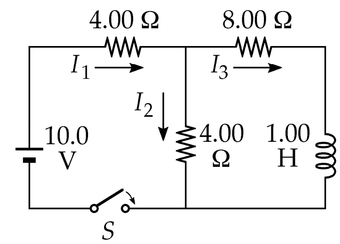
$0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

$0.0200 = e^{-3.00\times10^{-3}/\tau}$

$\tau = -\dfrac{3.00\times10^{-3}}{\ln(0.0200)} = 7.67\times10^{-4}\ \text{s}$

$\tau = L/R$, so $L = \tau R = (7.67\times10^{-4})(10.0) =$ $\boxed{7.67\ \text{mH}}$

32.25 Name the currents as shown. By Kirchhoff's laws:



$I_1 = I_2 + I_3$ (1)

$+10.0\ \text{V} - 4.00 I_1 - 4.00 I_2 = 0$ (2)

$+10.0\ \text{V} - 4.00 I_1 - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0$ (3)

From (1) and (2), $+10.0 - 4.00 I_1 - 4.00 I_1 + 4.00 I_3 = 0$ and $I_1 = 0.500 I_3 + 1.25\ \text{A}$

Then (3) becomes $10.0\ \text{V} - 4.00(0.500 I_3 + 1.25\ \text{A}) - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0$

$$(1.00\ \text{H})(dI_3/dt) + (10.0\ \Omega)I_3 = 5.00\ \text{V}$$

We solve the differential equation using Equations 32.6 and 32.7:

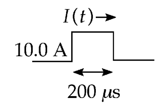
$I_3(t) = \dfrac{5.00\ \text{V}}{10.0\ \Omega}\left[1 - e^{-(10.0\ \Omega)t/1.00\ \text{H}}\right] = \boxed{(0.500\ \text{A})\left[1 - e^{-10t/\text{s}}\right]}$

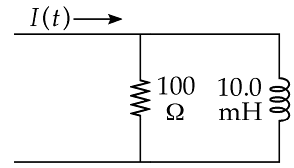
$I_1 = 1.25 + 0.500 I_3 = \boxed{1.50\ \text{A} - (0.250\ \text{A})e^{-10t/\text{s}}}$

32.26 (a) Using $\tau = RC = \dfrac{L}{R}$, we get $R = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{3.00\ \text{H}}{3.00\times10^{-6}\ \text{F}}} = 1.00\times10^{3}\ \Omega =$ $\boxed{1.00\ \text{k}\Omega}$

(b) $\tau = RC = (1.00\times10^{3}\ \Omega)(3.00\times10^{-6}\ \text{F}) = 3.00\times10^{-3}\ \text{s} =$ $\boxed{3.00\ \text{ms}}$

32.27 For $t \le 0$, the current in the inductor is zero. At $t = 0$, it starts to grow from zero toward 10.0 A with time constant $\tau = L/R = (10.0\text{ mH})/(100\ \Omega) = 1.00 \times 10^{-4}$ s.





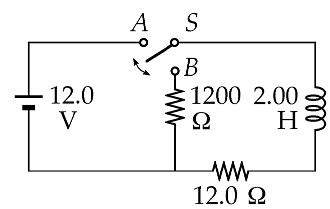
For $0 \le t \le 200\ \mu\text{s}$, $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \boxed{(10.00\text{ A})\left(1 - e^{-10000t/\text{s}}\right)}$

At $t = 200\ \mu\text{s}$, $I = (10.00\text{ A})\left(1 - e^{-2.00}\right) = 8.65$ A

Thereafter, it decays exponentially as $I = I_0 e^{-t'/\tau}$, so for $t \ge 200\ \mu\text{s}$,

$$I = (8.65\text{ A})e^{-10000(t - 200\ \mu\text{s})/\text{s}} = (8.65\text{ A})e^{-10000t/\text{s} + 2.00} = \left(8.65e^{2.00}\text{ A}\right)e^{-10000t/\text{s}} = \boxed{(63.9\text{ A})e^{-10000t/\text{s}}}$$

32.28 (a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{12.0\text{ V}}{12.0\ \Omega} = \boxed{1.00\text{ A}}$



(b) Initial current is 1.00 A, : $\Delta V_{12} = (1.00\text{ A})(12.00\ \Omega) = \boxed{12.0\text{ V}}$

$\Delta V_{1200} = (1.00\text{ A})(1200\ \Omega) = \boxed{1.20\text{ kV}}$

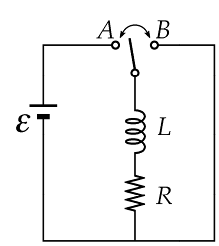
$\Delta V_L = \boxed{1.21\text{ kV}}$

(c) $I = I_{\max} e^{-Rt/L}$: $\dfrac{dI}{dt} = -I_{\max}\dfrac{R}{L} e^{-Rt/L}$ and $-L\dfrac{dI}{dt} = \Delta V_L = I_{\max} R e^{-Rt/L}$

Solving $12.0\text{ V} = (1212\text{ V})e^{-1212t/2.00}$ so $9.90 \times 10^{-3} = e^{-606t}$

Thus, $\boxed{t = 7.62\text{ ms}}$

32.29 $\tau = \dfrac{L}{R} = \dfrac{0.140}{4.90} = 28.6\text{ ms}$; $I_{\max} = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{4.90\ \Omega} = 1.22$ A



(a) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$ so $0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

$e^{-t/\tau} = 0.820$ $t = -\tau \ln(0.820) = \boxed{5.66\text{ ms}}$

(b) $I = I_{\max}\left(1 - e^{-\frac{10.0}{0.0286}}\right) = (1.22\text{ A})\left(1 - e^{-350}\right) = \boxed{1.22\text{ A}}$

(c) $I = I_{\max} e^{-t/\tau}$ and $0.160 = 1.22e^{-t/\tau}$ so $t = -\tau \ln(0.131) = \boxed{58.1\text{ ms}}$

32.30 (a) For a series connection, both inductors carry equal currents at every instant, so dI/dt is the same for both. The voltage across the pair is

$$L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} \qquad \text{so} \qquad \boxed{L_{\text{eq}} = L_1 + L_2}$$

(b) $$L_{\text{eq}}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = \Delta V_L \qquad \text{where } I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

Thus, $$\frac{\Delta V_L}{L_{\text{eq}}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2} \qquad \text{and} \qquad \boxed{\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

(c) $$L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI}{dt} + IR_1 + L_2\frac{dI}{dt} + IR_2$$

Now I and dI/dt are separate quantities under our control, so functional equality requires both

$$\boxed{L_{\text{eq}} = L_1 + L_2 \quad \text{and} \quad R_{\text{eq}} = R_1 + R_2}$$

(d) $$\Delta V = L_{\text{eq}}\frac{dI}{dt} + R_{\text{eq}}I = L_1\frac{dI_1}{dt} + R_1 I_1 = L_2\frac{dI_2}{dt} + R_2 I_2 \qquad \text{where} \qquad I = I_1 + I_2 \quad \text{and} \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

We may choose to keep the currents constant in time. Then, $$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We may choose to make the current swing through 0. Then, $$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

This equivalent coil with resistance will be equivalent to the pair of real inductors for all other currents as well.

32.31 $$L = \frac{N\Phi_B}{I} = \frac{200(3.70 \times 10^{-4})}{1.75} = 42.3\text{ mH} \qquad \text{so} \qquad U = \frac{1}{2}LI^2 = \frac{1}{2}(0.423\text{ H})(1.75\text{ A})^2 = \boxed{0.0648\text{ J}}$$

32.32 (a) The magnetic energy density is given by

$$u = \frac{B^2}{2\mu_0} = \frac{(4.50\text{ T})^2}{2(1.26 \times 10^{-6}\text{ T}\cdot\text{m/A})} = \boxed{8.06 \times 10^6\text{ J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = (8.06 \times 10^6\text{ J/m}^3)\left[(0.260\text{ m})\pi(0.0310\text{ m})^2\right] = \boxed{6.32\text{ kJ}}$$

32.33 $L = \mu_0 \dfrac{N^2 A}{\ell} = \mu_0 \dfrac{(68.0)^2\, \pi(0.600 \times 10^{-2})^2}{0.0800} = 8.21\, \mu\text{H}$

$U = \frac{1}{2} L I^2 = \frac{1}{2}(8.21 \times 10^{-6}\text{ H})(0.770\text{ A})^2 = \boxed{2.44\ \mu\text{J}}$

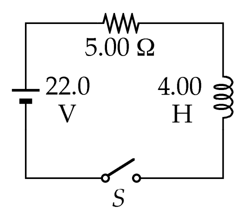
32.34 (a) $U = \frac{1}{2} L I^2 = \frac{1}{2} L\left(\dfrac{\mathcal{E}}{2R}\right)^2 = \dfrac{L\mathcal{E}^2}{8R^2} = \dfrac{(0.800)(500)^2}{8(30.0)^2} = \boxed{27.8\text{ J}}$

(b) $I = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right]$ so $\dfrac{\mathcal{E}}{2R} = \left(\dfrac{\mathcal{E}}{R}\right)\left[1 - e^{-(R/L)t}\right] \rightarrow e^{-(R/L)t} = \dfrac{1}{2}$

$\dfrac{R}{L} t = \ln 2$ so $t = \dfrac{L}{R}\ln 2 = \dfrac{0.800}{30.0}\ln 2 = \boxed{18.5\text{ ms}}$

32.35 $u = \epsilon_0 \dfrac{E^2}{2} = \boxed{44.2\text{ nJ/m}^3}$ $u = \dfrac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

***32.36** (a) $U = \frac{1}{2} L I^2 = \frac{1}{2}(4.00\text{ H})(0.500\text{ A})^2 = \boxed{0.500\text{ J}}$



(b) $\dfrac{dU}{dt} = LI = (4.00\text{ H})(1.00\text{ A}) = 4.00\text{ J/s} = \boxed{4.00\text{ W}}$

(c) $\mathcal{P} = (\Delta V)I = (22.0\text{ V})(0.500\text{ A}) = \boxed{11.0\text{ W}}$

32.37 From Equation 32.7, $I = \dfrac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$

(a) The maximum current, after a long time t, is $I = \dfrac{\mathcal{E}}{R} = 2.00\text{ A}.$

At that time, the inductor is fully energized and $\mathcal{P} = I(\Delta V) = (2.00\text{ A})(10.0\text{ V}) = \boxed{20.0\text{ W}}$

(b) $\mathcal{P}_{\text{lost}} = I^2 R = (2.00\text{ A})^2(5.00\ \Omega) = \boxed{20.0\text{ W}}$

(c) $\mathcal{P}_{\text{inductor}} = I(\Delta V_{\text{drop}}) = \boxed{0}$

(d) $U = \dfrac{LI^2}{2} = \dfrac{(10.0\text{ H})(2.00\text{ A})^2}{2} = \boxed{20.0\text{ J}}$

32.38 We have $u = \epsilon_0 \frac{E^2}{2}$ and $u = \frac{B^2}{2\mu_0}$

Therefore $\epsilon_0 \frac{E^2}{2} = \frac{B^2}{2\mu_0}$ so $B^2 = \epsilon_0 \mu_0 E^2$

$$B = E\sqrt{\epsilon_0 \mu_0} = \frac{6.80 \times 10^5 \text{ V / m}}{3.00 \times 10^8 \text{ m / s}} = \boxed{2.27 \times 10^{-3} \text{ T}}$$

32.39 The total magnetic energy is the volume integral of the energy density, $u = \frac{B^2}{2\mu_0}$

Because B changes with position, u is not constant. For $B = B_0 (R / r)^2$, $u = \left(\frac{B_0{}^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2\, dr$, so the energy stored in it is

$$dU = u\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0{}^2 R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U = \boxed{\frac{2\pi B_0^2 R^3}{\mu_0}} = \frac{2\pi(5.00 \times 10^{-5} \text{ T})^2 (6.00 \times 10^6 \text{ m})^3}{(1.26 \times 10^{-6} \text{ T} \cdot \text{m / A})} = \boxed{2.70 \times 10^{18} \text{ J}}$$

32.40 $I_1(t) = I_{\max} e^{-\alpha t} \sin \omega t$ with $I_{\max} = 5.00$ A, $\alpha = 0.0250 \text{ s}^{-1}$, and $\omega = 377$ rad/s.

$$\frac{dI_1}{dt} = I_{\max} e^{-\alpha t}\left(-\alpha \sin \omega t + \omega \cos \omega t\right)$$

At $t = 0.800$ s, $$\frac{dI_1}{dt} = \left(5.00 \text{ A/s}\right) e^{-0.0200}\left[-(0.0250)\sin\left(0.800(377)\right) + 377\cos\left(0.800(377)\right)\right]$$

$$\frac{dI_1}{dt} = 1.85 \times 10^3 \text{ A/s}$$

Thus, $\mathcal{E}_2 = -M\frac{dI_1}{dt}$: $$M = \frac{-\mathcal{E}_2}{dI_1/dt} = \frac{+3.20 \text{ V}}{1.85 \times 10^3 \text{ A/s}} = \boxed{1.73 \text{ mH}}$$

32.41 $\mathcal{E}_2 = -M\dfrac{dI_1}{dt} = -(1.00\times10^{-4}\text{ H})(1.00\times10^{4}\text{ A / s})\cos(1000t)$

$(\mathcal{E}_2)_{\max} = \boxed{1.00\text{ V}}$

32.42 $M = \left|\dfrac{\mathcal{E}_2}{dI_1/dt}\right| = \dfrac{96.0\text{ mV}}{1.20\text{ A / s}} = \boxed{80.0\text{ mH}}$

32.43 (a) $M = \dfrac{N_B\Phi_{BA}}{I_A} = \dfrac{700(90.0\times10^{-6})}{3.50} = \boxed{18.0\text{ mH}}$

(b) $L_A = \dfrac{\Phi_A}{I_A} = \dfrac{400(300\times10^{-6})}{3.50} = \boxed{34.3\text{ mH}}$

(c) $\mathcal{E}_B = -M\dfrac{dI_A}{dt} = -(18.0\text{ mH})(0.500\text{ A / s}) = \boxed{-9.00\text{ mV}}$

32.44 $M = \dfrac{N_2\Phi_{12}}{I_1} = \dfrac{N_2(B_1A_1)}{I_1} = \dfrac{N_2\left[(\mu_0 n_1 I_1)A_1\right]}{I_1} = N_2\mu_0 n_1 A_1$

$M = (1.00)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(\dfrac{70.0}{0.0500\text{ m}}\right)\left[\pi\left(5.00\times10^{-3}\text{ m}\right)^2\right] = \boxed{138\text{ nH}}$

32.45 B at center of (larger) loop: $B_1 = \dfrac{\mu_0 I_1}{2R}$

(a) $M = \dfrac{\Phi_2}{I_1} = \dfrac{B_1A_2}{I_1} = \dfrac{(\mu_0 I_1 / 2R)(\pi r^2)}{I_1} = \boxed{\dfrac{\mu_0\pi r^2}{2R}}$

(b) $M = \dfrac{\mu_0\pi(0.0200)^2}{2(0.200)} = \boxed{3.95\text{ nH}}$

***32.46** Assume the long wire carries current I. Then the magnitude of the magnetic field it generates at distance x from the wire is $B = \mu_0 I / 2\pi x$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

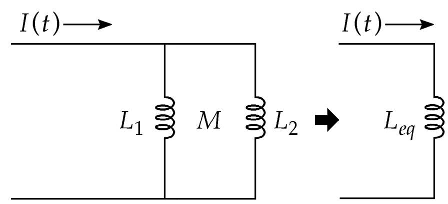
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \int B\,dA = \int B(\ell\,dx) = \frac{\mu_0 I \ell}{2\pi} \int_{0.400\text{ mm}}^{1.70\text{ mm}} \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 \mu_0 I \ell}{2\pi I} \ln\left(\frac{1.70}{0.400}\right) = \frac{N_2 \mu_0 \ell}{2\pi}(1.45) = \frac{1(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(2.70 \times 10^{-3}\text{ m})}{2\pi}(1.45)$$

$$M = 7.81 \times 10^{-10}\text{ H} = \boxed{781\text{ pH}}$$

32.47 With $I = I_1 + I_2$, the voltage across the pair is:



(a) (b)

$$\Delta V = -L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2 \frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{\text{eq}} \frac{dI}{dt}$$

So, $$-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2 \frac{dI_2}{dt} + \frac{M(\Delta V)}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$$

$$(-L_1 L_2 + M^2)\frac{dI_2}{dt} = \Delta V(L_1 - M) \qquad [1]$$

By substitution, $$-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$(-L_1 L_2 + M^2)\frac{dI_1}{dt} = \Delta V(L_2 - M) \qquad [2]$$

Adding [1] to [2], $$(-L_1 L_2 + M^2)\frac{dI}{dt} = \Delta V(L_1 + L_2 - 2M)$$

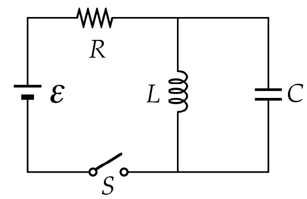
So, $$L_{\text{eq}} = -\frac{\Delta V}{dI/dt} = \boxed{\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

32.48 At different times, $(U_C)_{\max} = (U_L)_{\max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$

$$I_{\max} = \sqrt{\frac{C}{L}}(\Delta V)_{\max} = \sqrt{\frac{1.00 \times 10^{-6}\text{ F}}{10.0 \times 10^{-3}\text{ H}}}(40.0\text{ V}) = \boxed{0.400\text{ A}}$$

32.49 $\left[\frac{1}{2}C(\Delta V)^2\right]_{\max} = \left(\frac{1}{2}LI^2\right)_{\max}$ so $(\Delta V_C)_{\max} = \sqrt{\frac{L}{C}}\,I_{\max} = \sqrt{\frac{20.0\times 10^{-3}\text{ H}}{0.500\times 10^{-6}\text{ F}}}\,(0.100\text{ A}) = \boxed{20.0\text{ V}}$

32.50 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{\max} = \mathcal{E}/R$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop.



We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_{\max}^2$.

Then, $L = \frac{C(\Delta V)^2}{I_{\max}^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \frac{(0.500\times 10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}$

32.51 $C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi\cdot 6.30\times 10^6)^2(1.05\times 10^6)} = \boxed{608\text{ pF}}$

Goal Solution

A fixed inductance $L = 1.05\ \mu$H is used in series with a variable capacitor in the tuning section of a radio. What capacitance tunes the circuit to the signal from a station broadcasting at 6.30 MHz?

G: It is difficult to predict a value for the capacitance without doing the calculations, but we might expect a typical value in the μF or pF range.

O: We want the resonance frequency of the circuit to match the broadcasting frequency, and for a simple RLC circuit, the resonance frequency only depends on the magnitudes of the inductance and capacitance.

A: The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$

Thus, $$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left[(2\pi)(6.30\times 10^6\text{ Hz})\right]^2(1.05\times 10^{-6}\text{ H})} = 608\text{ pF}$$

L: This is indeed a typical capacitance, so our calculation appears reasonable. However, you probably would not hear any familiar music on this broadcast frequency. The frequency range for FM radio broadcasting is 88.0 – 108.0 MHz, and AM radio is 535 – 1605 kHz. The 6.30 MHz frequency falls in the Maritime Mobile SSB Radiotelephone range, so you might hear a ship captain instead of Top 40 tunes! This and other information about the radio frequency spectrum can be found on the National Telecommunications and Information Administration (NTIA) website, which at the time of this printing was at http://www.ntia.doc.gov/osmhome/allochrt.html

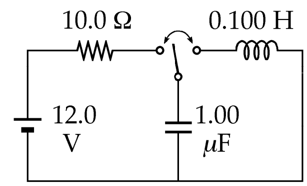
32.52 $f = \dfrac{1}{2\pi\sqrt{LC}}$: $L = \dfrac{1}{(2\pi f)^2 C} = \dfrac{1}{(2\pi \cdot 120)^2 (8.00\times10^{-6})} = \boxed{0.220 \text{ H}}$

32.53 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.0820 \text{ H})(17.0\times10^{-6} \text{ F})}} = \boxed{135 \text{ Hz}}$

(b) $Q = Q_{\max}\cos\omega t = (180\ \mu C)\cos(847\times 0.00100) = \boxed{119\ \mu\text{C}}$

(c) $I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114 \text{ mA}}$

32.54 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.100 \text{ H})(1.00\times10^{-6} \text{ F})}} = \boxed{503 \text{ Hz}}$



(b) $Q = C\mathcal{E} = (1.00\times10^{-6} \text{ F})(12.0 \text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_{\max}^2$

$I_{\max} = \mathcal{E}\sqrt{\dfrac{C}{L}} = 12 \text{ V}\sqrt{\dfrac{1.00\times10^{-6} \text{ F}}{0.100 \text{ H}}} = \boxed{37.9 \text{ mA}}$

(d) At all times $U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00\times10^{-6} \text{ F})(12.0 \text{ V})^2 = \boxed{72.0\ \mu\text{J}}$

32.55 $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(3.30 \text{ H})(840\times10^{-12} \text{ F})}} = 1.899\times10^4 \text{ rad s}$

$Q = Q_{\max}\cos\omega t, \quad I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$

(a) $U_C = \dfrac{Q^2}{2C} = \dfrac{\left(\left[105\times10^{-6}\right]\cos\left[\left(1.899\times10^4 \text{ rad/s}\right)\left(2.00\times10^{-3} \text{ s}\right)\right]\right)^2}{2\left(840\times10^{-12}\right)} = \boxed{6.03 \text{ J}}$

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q_{\max}^2\sin^2(\omega t) = \dfrac{Q_{\max}^2\sin^2(\omega t)}{2C}$

$U_L = \dfrac{\left(105\times10^{-6} \text{ C}\right)^2\sin^2\left[\left(1.899\times10^4 \text{ rad/s}\right)\left(2.00\times10^{-3} \text{ s}\right)\right]}{2\left(840\times10^{-12} \text{ F}\right)} = \boxed{0.529 \text{ J}}$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56 \text{ J}}$

32.56 (a) $$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{\left(2.20\times10^{-3}\right)\left(1.80\times10^{-6}\right)} - \left(\frac{7.60}{2\left(2.20\times10^{-3}\right)}\right)^2} = 1.58\times10^4 \text{ rad / s}$$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \boxed{2.51 \text{ kHz}}$

(b) $R_c = \sqrt{\frac{4L}{C}} = \boxed{69.9\ \Omega}$

32.57 (a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)(0.100\times10^{-6})}} = \boxed{4.47 \text{ krad / s}}$$

(b) $$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \boxed{4.36 \text{ krad / s}}$$

(c) $$\frac{\Delta\omega}{\omega_0} = \boxed{2.53\% \text{ lower}}$$

32.58 Choose to call positive current clockwise in Figure 32.19. It drains charge from the capacitor according to $I = -dQ/dt$. A clockwise trip around the circuit then gives

$$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

$$+\frac{Q}{C} + \frac{dQ}{dt}R + L\frac{d}{dt}\frac{dQ}{dt} = 0,$$ identical with Equation 32.29.

32.59 (a) $Q = Q_{\max} e^{-\frac{Rt}{2L}} \cos \omega_d t$ so $I_{\max} \propto e^{-\frac{Rt}{2L}}$

$0.500 = e^{-\frac{Rt}{2L}}$ and $\frac{Rt}{2L} = -\ln(0.500)$

$$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) $U_0 \propto Q_{\max}^2$ and $U = 0.500U_0$ so $Q = \sqrt{0.500}\, Q_{\max} = 0.707Q_{\max}$

$$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)}$$ (half as long)

32.60 With $Q = Q_{\max}$ at $t = 0$, the charge on the capacitor at any time is $Q = Q_{\max}\cos\omega t$ where $\omega = 1/\sqrt{LC}$. The energy stored in the capacitor at time t is then

$$U = \frac{Q^2}{2C} = \frac{Q_{\max}^2}{2C}\cos^2\omega t = U_0\cos^2\omega t.$$

When $U = \frac{1}{4}U_0$, $\cos\omega t = \frac{1}{2}$ and $\omega t = \frac{1}{3}\pi$ rad

Therefore, $\frac{t}{\sqrt{LC}} = \frac{\pi}{3}$ or $\frac{t^2}{LC} = \frac{\pi^2}{9}$

The inductance is then: $L = \boxed{\frac{9t^2}{\pi^2 C}}$

32.61 (a) $\mathcal{E}_L = -L\frac{dI}{dt} = -(1.00\text{ mH})\frac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$

(b) $Q = \int_0^t I\,dt = \int_0^t (20.0t)\,dt = 10.0t^2$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\frac{Q^2}{2C} \ge \frac{1}{2}LI^2$, or $\frac{\left(-10.0t^2\right)^2}{2\left(1.00\times10^{-6}\right)} \ge \frac{1}{2}\left(1.00\times10^{-3}\right)(20.0t)^2$,

then $100t^4 \ge \left(400\times10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$

32.62 (a) $\mathcal{E}_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = \boxed{-LK}$

(b) $I = \frac{dQ}{dt}$, so $Q = \int_0^t I\,dt = \int_0^t Kt\,dt = \frac{1}{2}Kt^2$

and $\Delta V_C = \frac{-Q}{C} = \boxed{-\frac{Kt^2}{2C}}$

(c) When $\frac{1}{2}C\left(\Delta V_C\right)^2 = \frac{1}{2}LI^2$, $\frac{1}{2}C\left(\frac{K^2t^4}{4C^2}\right) = \frac{1}{2}L\left(K^2t^2\right)$

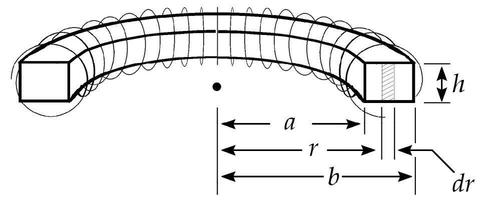
Thus $t = \boxed{2\sqrt{LC}}$

32.63 $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}LI^2$ so $I = \sqrt{\frac{3Q^2}{4CL}}$

The flux through each turn of the coil is $\Phi_B = \frac{LI}{N} = \boxed{\frac{Q}{2N}\sqrt{\frac{3L}{C}}}$

where N is the number of turns.

32.64 Equation 30.16: $B = \frac{\mu_0 NI}{2\pi r}$



(a) $\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r} h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$

$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$

(b) $L = \frac{\mu_0 (500)^2 (0.0100)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$

(c) $L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0 (500)^2}{2\pi}\left(\frac{2.00\times 10^{-4}\ \text{m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$

***32.65** (a) At the center, $B = \frac{N\mu_0 IR^2}{2(R^2+0^2)^{3/2}} = \frac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B \approx BA\cos\theta = \frac{N\mu_0 I}{2R}\pi R^2 \cos 0° = \frac{\pi}{2} N\mu_0 IR$

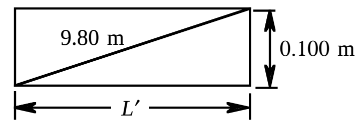
When the current it carries changes, $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} \approx -N\frac{\pi}{2}N\mu_0 R\frac{dI}{dt} = -L\frac{dI}{dt}$

so $\boxed{L \approx \frac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r \approx 3(0.3\ \text{m})$, so $r \approx 0.14$ m; $L \approx \frac{\pi}{2}\ 1^2\left(4\pi\times 10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\ 0.14\ \text{m} = 2.8\times 10^{-7}\ \text{H}\ \boxed{\sim 100\ \text{nH}}$

(c) $\frac{L}{R} \approx \frac{2.8\times 10^{-7}\ \text{V}\cdot\text{s/A}}{270\ \text{V/A}} = 1.0\times 10^{-9}\ \text{s}\ \boxed{\sim 1\ \text{ns}}$

32.66 (a) If unrolled, the wire forms the diagonal of a 0.100 m (10.0 cm) rectangle as shown. The length of this rectangle is



$$L' = \sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}$$

The mean circumference of each turn is $C = 2\pi r'$, where $r' = \frac{24.0 + 0.644}{2}$ mm is the mean radius of each turn. The number of turns is then:

$$N = \frac{L'}{C} = \frac{\sqrt{(9.80\text{ m})^2 - (0.100\text{ m})^2}}{2\pi\left(\frac{24.0+0.644}{2}\right)\times 10^{-3}\text{ m}} = \boxed{127}$$

(b) $$R = \frac{\rho \ell}{A} = \frac{(1.70\times 10^{-8}\ \Omega\cdot\text{m})(10.0\text{ m})}{\pi(0.322\times 10^{-3}\text{ m})^2} = \boxed{0.522\ \Omega}$$

(c) $$L = \frac{\mu N^2 A}{\ell'} = \frac{800\mu_0}{\ell'}\left(\frac{L'}{C}\right)^2 \pi (r')^2$$

$$L = \frac{800(4\pi\times 10^{-7})}{0.100\text{ m}}\left(\frac{\sqrt{(9.80\text{ m})^2-(0.100\text{ m})^2}}{\pi(24.0+0.644)\times 10^{-3}\text{ m}}\right)^2 \pi\left[\left(\frac{24.0+0.644}{2}\right)\times 10^{-3}\text{ m}\right]^2$$

$$L = 7.68\times 10^{-2}\text{ H} = \boxed{76.8\text{ mH}}$$

32.67 From Ampere's law, the magnetic field at distance $r \le R$ is found as:

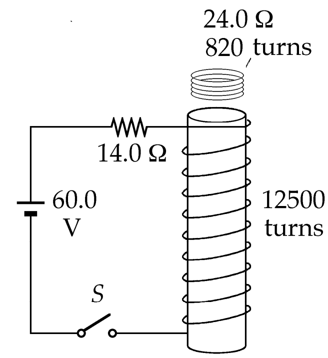
$$B(2\pi r) = \mu_0 J(\pi r^2) = \mu_0\left(\frac{I}{\pi R^2}\right)(\pi r^2), \quad \text{or} \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

This is independent of the radius of the wire.

32.68 The primary circuit (containing the battery and solenoid) is an RL circuit with $R = 14.0\ \Omega$, and



$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7}\right)\left(12\ 500\right)^2\left(1.00\times10^{-4}\right)}{0.0700} = 0.280\ \text{H}$$

(a) The time for the current to reach 63.2% of the maximum value is the time constant of the circuit:

$$\tau = \frac{L}{R} = \frac{0.280\ \text{H}}{14.0\ \Omega} = 0.0200\ \text{s} = \boxed{20.0\ \text{ms}}$$

(b) The solenoid's average back emf is $\left|\bar{\mathcal{E}}_L\right| = L\left(\frac{\Delta I}{\Delta t}\right) = L\left(\frac{I_f - 0}{\Delta t}\right)$

where $I_f = 0.632\, I_{\max} = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\ \text{V}}{14.0\ \Omega}\right) = 2.71\ \text{A}$

Thus, $\left|\bar{\mathcal{E}}_L\right| = \left(0.280\ \text{H}\right)\left(\frac{2.71\ \text{A}}{0.0200\ \text{s}}\right) = \boxed{37.9\ \text{V}}$

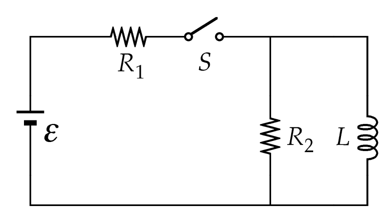
(c) The average rate of change of flux through each turn of the overwrapped concentric coil is the same as that through a turn on the solenoid:

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{\mu_0 n(\Delta I)A}{\Delta t} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(12\,500/0.0700\ \text{m}\right)\left(2.71\ \text{A}\right)\left(1.00\times10^{-4}\ \text{m}^2\right)}{0.0200\ \text{s}} = \boxed{3.04\ \text{mV}}$$

(d) The magnitude of the average induced emf in the coil is $\left|\mathcal{E}_L\right| = N\left(\Delta\Phi_B/\Delta t\right)$ and magnitude of the average induced current is

$$I = \frac{\left|\mathcal{E}_L\right|}{R} = \frac{N}{R}\left(\frac{\Delta\Phi_B}{\Delta t}\right) = \frac{820}{24.0\ \Omega}\left(3.04\times10^{-3}\ \text{V}\right) = 0.104\ \text{A} = \boxed{104\ \text{mA}}$$

32.69 Left-hand loop: $\mathcal{E} - (I + I_2)R_1 - I_2 R_2 = 0$



Outside loop: $\mathcal{E} - (I + I_2)R_1 - L\frac{dI}{dt} = 0$

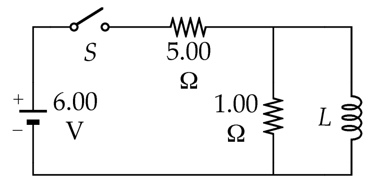
Eliminating I_2 gives $\mathcal{E}' - IR' - L\frac{dI}{dt} = 0$

This is of the same form as Equation 32.6, so its solution is of the same form as Equation 32.7: $I(t) = \frac{\mathcal{E}'}{R'}\left(1 - e^{-R't/L}\right)$

But $R' = R_1R_2/\left(R_1 + R_2\right)$ and $\mathcal{E}' = R_2\mathcal{E}/\left(R_1 + R_2\right)$, so $\frac{\mathcal{E}'}{R'} = \frac{\mathcal{E}R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)} = \frac{\mathcal{E}}{R_1}$

Thus $I(t) = \frac{\mathcal{E}}{R_1}\left(1 - e^{-R't/L}\right)$

32.70 When switch is closed, steady current $I_0 = 1.20$ A. When the switch is opened after being closed a long time, the current in the right loop is



$$I = I_0 e^{-R_2 t/L}$$

so $\quad e^{Rt/L} = \frac{I_0}{I} \quad$ and $\quad \frac{Rt}{L} = \ln\left(\frac{I_0}{I}\right)$

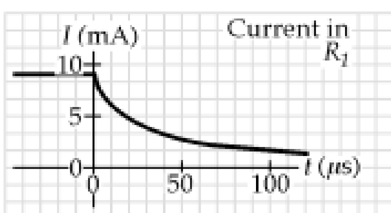
Therefore, $\quad L = \frac{R_2 t}{\ln(I_0/I)} = \frac{(1.00\ \Omega)(0.150\ \text{s})}{\ln(1.20\ \text{A}/0.250\ \text{A})} = 0.0956\ \text{H} = \boxed{95.6\ \text{mH}}$

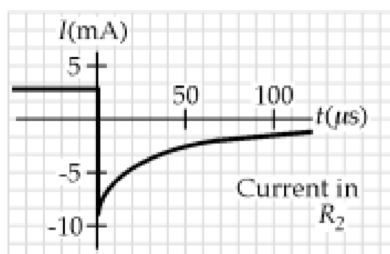
32.71 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\mathcal{E}_0 - \left[(2.00 + 6.00) \times 10^3\ \Omega\right]\left(9.00 \times 10^{-3}\ \text{A}\right) = 0$$

$$+\mathcal{E}_0 = \boxed{72.0\ \text{V with end } b \text{ at the higher potential}}$$

(b)





(c) After the switch is opened, the current around the outer loop decays as

$I = I_{max} e^{-Rt/L} \quad$ with $\quad I_{max} = 9.00$ mA, $\quad R = 8.00$ kΩ, $\quad$ and $\quad L = 0.400$ H

Thus, when the current has reached a value $I = 2.00$ mA, the elapsed time is:

$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_{max}}{I}\right) = \left(\frac{0.400\ \text{H}}{8.00 \times 10^3\ \Omega}\right)\ln\left(\frac{9.00}{2.00}\right) = 7.52 \times 10^{-5}\ \text{s} = \boxed{75.2\ \mu\text{s}}$$

32.72 (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0, \quad I_C = \mathcal{E}_0/R, \quad I_R = \mathcal{E}_0/R}$$

$$\boxed{\Delta V_L = \mathcal{E}_0, \quad \Delta V_C = 0, \quad \Delta V_R = \mathcal{E}_0}$$

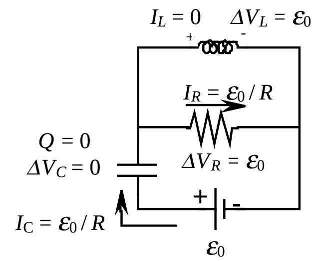


Figure (a)

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0, \quad I_C = 0, \quad I_R = 0}$$

$$\boxed{\Delta V_L = 0, \quad \Delta V_C = \mathcal{E}_0, \quad \Delta V_R = 0}$$

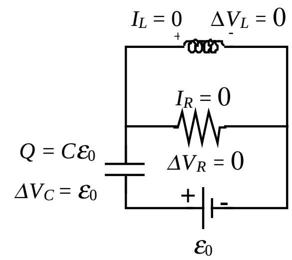


Figure (b)

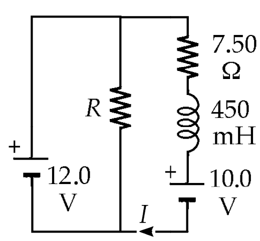
32.73 When the switch is closed, as shown in Figure (a), the current in the inductor is I:

$12.0 - 7.50I - 10.0 = 0 \rightarrow I = 0.267$ A

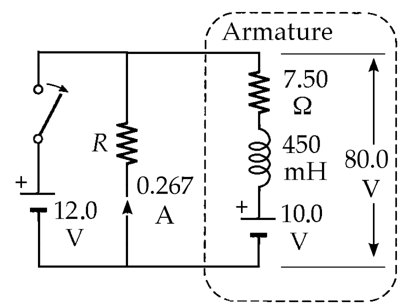
When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $(0.267 \text{ A})R \le 80.0$ V

$$\boxed{R \le 300\ \Omega}$$



(a)



(b)

Goal Solution

To prevent damage from arcing in an electric motor, a discharge resistor is sometimes placed in parallel with the armature. If the motor is suddenly unplugged while running, this resistor limits the voltage that appears across the armature coils. Consider a 12.0-V dc motor with an armature that has a resistance of 7.50 Ω and an inductance of 450 mH. Assume that the back emf in the armature coils is 10.0 V when the motor is running at normal speed. (The equivalent circuit for the armature is shown in Figure P32.73.) Calculate the maximum resistance R that limits the voltage across the armature to 80.0 V when the motor is unplugged.

G: We should expect R to be significantly greater than the resistance of the armature coil, for otherwise a large portion of the source current would be diverted through R and much of the total power would be wasted on heating this discharge resistor.

O: When the motor is unplugged, the 10-V back emf will still exist for a short while because the motor's inertia will tend to keep it spinning. Now the circuit is reduced to a simple series loop with an emf, inductor, and two resistors. The current that was flowing through the armature coil must now flow through the discharge resistor, which will create a voltage across R that we wish to limit to 80 V. As time passes, the current will be reduced by the opposing back emf, and as the motor slows down, the back emf will be reduced to zero, and the current will stop.

A: The steady-state coil current when the switch is closed is found from applying Kirchhoff's loop rule to the outer loop:

$$+12.0\text{ V} - I(7.50\ \Omega) - 10.0\text{ V} = 0$$

so $$I = \frac{2.00\text{ V}}{7.50\ \Omega} = 0.267\text{ A}$$

We then require that $$\Delta V_R = 80.0\text{ V} = (0.267\text{ A})R$$

so $$R = \frac{\Delta V_R}{I} = \frac{80.0\text{ V}}{0.267\text{ A}} = 300\ \Omega$$

L: As we expected, this discharge resistance is considerably greater than the coil's resistance. Note that while the motor is running, the discharge resistor turns $P = (12\text{ V})^2/300\ \Omega = 0.48\text{ W}$ of power into heat (or wastes 0.48 W). The source delivers power at the rate of about $P = IV = [0.267\text{ A} + (12\text{ V}/300\ \Omega)](12\text{ V}) = 3.68\text{ W}$, so the discharge resistor wastes about 13% of the total power. For a sense of perspective, this 4-W motor could lift a 40-N weight at a rate of 0.1 m/s.

32.74 (a) $$L_1 = \frac{\mu_0 N_1^2 A}{\ell_1} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)^2(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-4}\text{ H} = \boxed{251\ \mu\text{H}}$$

(b) $$M = \frac{N_2\Phi_2}{I_1} = \frac{N_2\Phi_1}{I_1} = \frac{N_2 BA}{I_1} = \frac{N_2[\mu_0(N_1/\ell_1)I_1]A}{I_1} = \frac{\mu_0 N_1 N_2 A}{\ell_1}$$

$$M = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1000)(100)(1.00\times10^{-4}\text{ m}^2)}{0.500\text{ m}} = 2.51\times10^{-5}\text{ H} = \boxed{25.1\ \mu\text{H}}$$

(c) $$\mathcal{E}_1 = -M\frac{dI_2}{dt}, \quad \text{or} \quad I_1R_1 = -M\frac{dI_2}{dt} \quad \text{and} \quad I_1 = \frac{dQ_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$Q_1 = -\frac{M}{R_1}\int_0^{t_f} dI_2 = -\frac{M}{R_1}(I_{2f} - I_{2i}) = -\frac{M}{R_1}(0 - I_{2i}) = \frac{M I_{2i}}{R_1}$$

$$Q_1 = \frac{(2.51\times10^{-5}\text{ H})(1.00\text{ A})}{1000\ \Omega} = 2.51\times10^{-8}\text{ C} = \boxed{25.1\text{ nC}}$$

32.75 (a) It has a magnetic field, and it stores energy, so $L = \dfrac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi = LI$ so $L = \dfrac{\Phi}{I} = \dfrac{1}{I}\int_{y=a}^{w-a} B\,da$

$$L = \frac{1}{I}\int_a^{w-a} x\,dy\left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(w-y)}\right) = \frac{2}{I}\int \frac{\mu_0 I x}{2\pi y}dy = \frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}$$

Thus $L = \dfrac{\mu_0 x}{\pi}\ln\left(\dfrac{w-a}{a}\right)$

32.76 For an RL circuit, $I(t) = I_{\max} e^{-\frac{R}{L}t}$: $\dfrac{I(t)}{I_{\max}} = 1 - 10^{-9} = e^{-\frac{R}{L}t} \cong 1 - \dfrac{R}{L}t$

$\dfrac{R}{L}t = 10^{-9}$ so $R_{\max} = \dfrac{(3.14\times10^{-8})(10^{-9})}{(2.50\text{ yr})(3.16\times10^{7}\text{ s/yr})} = \boxed{3.97\times10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least $10^{-6}\ \Omega$).

32.77 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

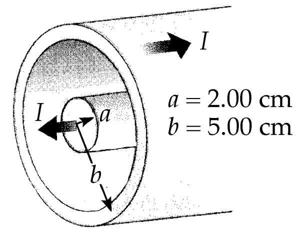
Then one wire creates a field of $B = \dfrac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\dfrac{\mu_0 I}{2\pi r}\sin 90° = \dfrac{\mu_0 \ell I^2}{2\pi r}$

Solving for the force, $F = (4\pi\times10^{-7}\text{ N/A}^2)\dfrac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{(2\pi)(0.250\text{ m})} = \boxed{2000\text{ N}}$

32.78 $\mathcal{P} = I(\Delta V)$



$$I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times 10^9 \text{ W}}{200\times 10^3 \text{ V}} = 5.00\times 10^3 \text{ A}$$

From Ampere's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \dfrac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.0200$ m, $I_{\text{enclosed}} = 5.00\times 10^3$ A and

$$B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.0200 \text{ m})} = 0.0500 \text{ T} = \boxed{50.0 \text{ mT}}$$

(b) At $r = b = 0.0500$ m, $I_{\text{enclosed}} = I = 5.00\times 10^3$ A and

$$B = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})}{2\pi(0.0500 \text{ m})} = 0.0200 \text{ T} = \boxed{20.0 \text{ mT}}$$

(c) $$U = \int u\,dV = \int_{r=a}^{r=b} \frac{[B(r)]^2 (2\pi r \ell dr)}{2\mu_0} = \frac{\mu_0 I^2 \ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U = \frac{(4\pi\times 10^{-7} \text{ T}\cdot\text{m/A})(5.00\times 10^3 \text{ A})^2(1000\times 10^3 \text{ m})}{4\pi}\ln\left(\frac{5.00 \text{ cm}}{2.00 \text{ cm}}\right) = 2.29\times 10^6 \text{ J} = \boxed{2.29 \text{ MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w. It carries a current of

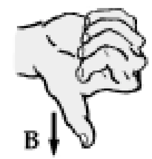
$$(5.00\times 10^3 \text{ A})\left(\frac{w}{2\pi(0.0500 \text{ m})}\right)$$

and experiences an outward force $F = I\ell B\sin\theta = \dfrac{(5.00\times 10^3 \text{ A})w}{2\pi(0.0500 \text{ m})}\ell(20.0\times 10^{-3} \text{ T})\sin 90.0°$

The pressure on it is $P = \dfrac{F}{A} = \dfrac{F}{w\ell} = \dfrac{(5.00\times 10^3 \text{ A})(20.0\times 10^{-3} \text{ T})}{2\pi(0.0500 \text{ m})} = \boxed{318 \text{ Pa}}$

***32.79** (a) $B = \frac{\mu_0 NI}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1400)(2.00\ \text{A})}{1.20\ \text{m}} = \boxed{2.93 \times 10^{-3}\ \text{T (upward)}}$

(b) $u = \frac{B^2}{2\mu_0} = \frac{\left(2.93 \times 10^{-3}\ \text{T}\right)^2}{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)} = 3.42\ \frac{\text{J}}{\text{m}^3}\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right) = 3.42\ \frac{\text{N}}{\text{m}^2} = \boxed{3.42\ \text{Pa}}$



(c) To produce a downward magnetic field, the surface of the super conductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi\left(1.10 \times 10^{-2}\ \text{m}\right)^2\right] = \boxed{1.30 \times 10^{-3}\ \text{N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; see problem 12 in Chapter 21.

Chapter 33 Solutions

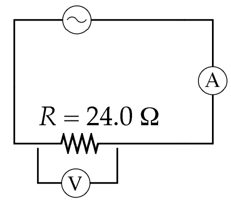
33.1 $\Delta v(t) = \Delta V_{\max} \sin(\omega t) = \sqrt{2}\ \Delta V_{\text{rms}} \sin(\omega t) = 200\sqrt{2}\ \sin[2\pi(100t)] = \boxed{(283\text{ V})\sin(628t)}$

33.2 $\Delta V_{\text{rms}} = \dfrac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$

(a) $\mathcal{P} = \dfrac{(\Delta V_{\text{rms}})^2}{R} \to R = \dfrac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$

(b) $R = \dfrac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

33.3 Each meter reads the rms value.



$\Delta V_{\text{rms}} = \dfrac{100\text{ V}}{\sqrt{2}} = \boxed{70.7\text{ V}}$

$I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{70.7\text{ V}}{24.0\ \Omega} = \boxed{2.95\text{ A}}$

33.4 (a) $\Delta v_R = \Delta V_{\max} \sin \omega t$

$\Delta v_R = 0.250(\Delta V_{\max})$, so $\sin \omega t = 0.250$, or $\omega t = \sin^{-1}(0.250)$

The smallest angle for which this is true is $\omega t = 0.253$ rad. Thus, if $t = 0.0100$ s,

$\omega = \dfrac{0.253\text{ rad}}{0.0100\text{ s}} = \boxed{25.3\text{ rad/s}}$

(b) The second time when $\Delta v_R = 0.250(\Delta V_{\max})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253\text{ rad} = 2.89\text{ rad}$ (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin\theta$ from trigonometry). Thus,

$t = \dfrac{2.89\text{ rad}}{25.3\text{ rad/s}} = \boxed{0.114\text{ s}}$

33.5 $i_R = I_{\max} \sin \omega t$ becomes $0.600 = \sin(\omega\ 0.00700)$

Thus, $(0.00700)\omega = \sin^{-1}(0.600) = 0.644$

and $\omega = 91.9\text{ rad/s} = 2\pi f$ so $\boxed{f = 14.6\text{ Hz}}$

33.6 $\mathcal{P} = I_{rms}(\Delta V_{rms})$ and $\Delta V_{rms} = 120\text{ V}$ for each bulb (parallel circuit), so:

$$I_1 = I_2 = \frac{\mathcal{P}_1}{\Delta V_{rms}} = \frac{150\text{ W}}{120\text{ V}} = \boxed{1.25\text{ A}}, \quad \text{and} \quad R_1 = \frac{\Delta V_{rms}}{I_1} = \frac{120\text{ V}}{1.25\text{ A}} = \boxed{96.0\ \Omega} = R_2$$

$$I_3 = \frac{\mathcal{P}_3}{\Delta V_{rms}} = \frac{100\text{ W}}{120\text{ V}} = \boxed{0.833\text{ A}}, \quad \text{and} \quad R_3 = \frac{\Delta V_{rms}}{I_3} = \frac{120\text{ V}}{0.833\text{ A}} = \boxed{144\ \Omega}$$

33.7 $\Delta V_{max} = 15.0\text{ V}$ and $R_{total} = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$

$$I_{max} = \frac{\Delta V_{max}}{R_{total}} = \frac{15.0\text{ V}}{18.6\ \Omega} = 0.806\text{ A} = \sqrt{2}\ I_{rms}$$

$$\mathcal{P}_{speaker} = I_{rms}^2 R_{speaker} = \left(\frac{0.806\text{ A}}{\sqrt{2}}\right)^2 (10.4\ \Omega) = \boxed{3.38\text{ W}}$$

33.8 For $I_{max} = 80.0\text{ mA}$, $I_{rms} = \dfrac{80.0\text{ mA}}{\sqrt{2}} = 56.6\text{ mA}$

$$(X_L)_{min} = \frac{V_{rms}}{I_{rms}} = \frac{50.0\text{ V}}{0.0566\text{ A}} = 884\ \Omega$$

$$X_L = 2\pi f L \to L = \frac{X_L}{2\pi f} \geq \frac{884\ \Omega}{2\pi(20.0)} \geq \boxed{7.03\text{ H}}$$

33.9 (a) $X_L = \dfrac{\Delta V_{max}}{I_{max}} = \dfrac{100}{7.50} = 13.3\ \Omega$

$$L = \frac{X_L}{\omega} = \frac{13.3}{2\pi(50.0)} = 0.0424\text{ H} = \boxed{42.4\text{ mH}}$$

(b) $X_L = \dfrac{\Delta V_{max}}{I_{max}} = \dfrac{100}{2.50} = 40.0\ \Omega$

$$\omega = \frac{X_L}{L} = \frac{40.0}{42.4 \times 10^{-3}} = \boxed{942\text{ rad/s}}$$

33.10 At 50.0 Hz, $X_L = 2\pi(50.0\text{ Hz})L = 2\pi(50.0\text{ Hz})\left(\dfrac{X_L\big|_{60.0\text{ Hz}}}{2\pi(60.0\text{ Hz})}\right) = \dfrac{50.0}{60.0}(54.0\ \Omega) = 45.0\ \Omega$

$$I_{max} = \frac{\Delta V_{max}}{X_L} = \frac{\sqrt{2}(\Delta V_{rms})}{X_L} = \frac{\sqrt{2}(100\text{ V})}{45.0\ \Omega} = \boxed{3.14\text{ A}}$$

33.11 $$i_L(t) = \frac{\Delta V_{\max}}{\omega L}\sin\left(\omega t - \pi/2\right) = \frac{(80.0\text{ V})\sin\left[(65.0\pi)(0.0155) - \pi/2\right]}{(65.0\pi\text{ rad/s})\left(70.0\times10^{-3}\text{ H}\right)}$$

$$i_L(t) = (5.60\text{ A})\sin(1.59\text{ rad}) = \boxed{5.60\text{ A}}$$

33.12 $$\omega = 2\pi f = 2\pi(60.0/\text{s}) = 377\text{ rad/s}$$

$$X_L = \omega L = (377/\text{s})(0.0200\text{ V}\cdot\text{s/A}) = 7.54\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{120\text{ V}}{7.54\ \Omega} = 15.9\text{ A}$$

$$I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}\,(15.9\text{ A}) = 22.5\text{ A}$$

$$i(t) = I_{\max}\sin\omega t = (22.5\text{ A})\sin\left(\frac{2\pi(60.0)}{\text{s}}\cdot\frac{1\text{ s}}{180}\right) = (22.5\text{ A})\sin 120° = 19.5\text{ A}$$

$$U = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\left(0.0200\,\frac{\text{V}\cdot\text{s}}{\text{A}}\right)(19.5\text{ A})^2 = \boxed{3.80\text{ J}}$$

33.13 $$L = \frac{N\Phi_B}{I}$$ where Φ_B is the flux through each turn. $$N\Phi_{B,\max} = LI_{B,\max} = \frac{X_L}{\omega}\frac{\left(\Delta V_{L,\max}\right)}{X_L}$$

$$N\Phi_{B,\max} = \frac{\sqrt{2}\left(\Delta V_{L,\text{rms}}\right)}{2\pi f} = \frac{120\text{ V}\cdot\text{s}}{\sqrt{2}\,\pi(60.0)}\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right) = \boxed{0.450\text{ T}\cdot\text{m}^2}$$

33.14 (a) $$X_C = \frac{1}{2\pi fC}:\qquad \frac{1}{2\pi f(22.0\times10^{-6})} < 175\ \Omega$$

$$\frac{1}{2\pi(22.0\times10^{-6})(175)} < f \qquad \boxed{f > 41.3\text{ Hz}}$$

(b) $$X_C \propto \frac{1}{C},\quad\text{so}\quad X(44) = \tfrac{1}{2}X(22):\quad \boxed{X_C < 87.5\ \Omega}$$

33.15 $$I_{\max} = \sqrt{2}\,I_{\text{rms}} = \frac{\sqrt{2}\left(\Delta V_{\text{rms}}\right)}{X_C} = \sqrt{2}\left(\Delta V_{\text{rms}}\right)2\pi fC$$

(a) $$I_{\max} = \sqrt{2}\,(120\text{ V})2\pi(60.0/\text{s})(2.20\times10^{-6}\text{ C/V}) = \boxed{141\text{ mA}}$$

(b) $$I_{\max} = \sqrt{2}\,(240\text{ V})2\pi(50.0/\text{s})(2.20\times10^{-6}\text{ F}) = \boxed{235\text{ mA}}$$

33.16 $Q_{\max} = C(\Delta V_{\max}) = C\left[\sqrt{2}(\Delta V_{\text{rms}})\right] = \boxed{\sqrt{2}\,C(\Delta V_{\text{rms}})}$

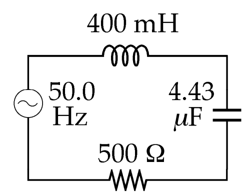
33.17 $I_{\max} = (\Delta V_{\max})\omega C = (48.0\text{ V})(2\pi)(90.0\text{ s}^{-1})(3.70\times10^{-6}\text{ F}) = \boxed{100\text{ mA}}$

33.18 $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(60.0\text{ / s})(1.00\times10^{-3}\text{ C / V})} = 2.65\ \Omega$

$v_C(t) = \Delta V_{\max}\sin\omega t$, to be zero at $t = 0$

$i_C = \dfrac{\Delta V_{\max}}{X_C}\sin(\omega t + \phi) = \dfrac{\sqrt{2}\,(120\text{ V})}{2.65\ \Omega}\sin\left[2\pi\dfrac{60\text{ s}^{-1}}{180\text{ s}^{-1}} + 90.0°\right] = (64.0\text{ A})\sin(120° + 90.0°) = \boxed{-32.0\text{ A}}$

33.19 (a) $X_L = \omega L = 2\pi(50.0)(400\times10^{-3}) = 126\ \Omega$



$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(50.0)(4.43\times10^{-6})} = 719\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{500^2 + (126 - 719)^2} = 776\ \Omega$

$\Delta V_{\max} = I_{\max} Z = (250\times10^{-3})(776) = \boxed{194\text{ V}}$

(b) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{126 - 719}{500}\right) = \boxed{-49.9°}$ Thus, the $\boxed{\text{Current leads the voltage.}}$

33.20 $\omega L = \dfrac{1}{\omega C} \rightarrow \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(57.0\times10^{-6})(57.0\times10^{-6})}} = 1.75\times10^{4}\text{ rad / s}$

$f = \dfrac{\omega}{2\pi} = \boxed{2.79\text{ kHz}}$

33.21 (a) $X_L = \omega L = 2\pi(50.0\text{ s}^{-1})(250\times10^{-3}\text{ H}) = \boxed{78.5\ \Omega}$

(b) $X_C = \dfrac{1}{\omega C} = \left[2\pi(50.0\text{ s}^{-1})(2.00\times10^{-6}\text{ F})\right]^{-1} = \boxed{1.59\text{ k}\Omega}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \boxed{1.52\text{ k}\Omega}$

(d) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{210\text{ V}}{1.52\times10^{3}\ \Omega} = \boxed{138\text{ mA}}$

(e) $\phi = \tan^{-1}\left[\dfrac{X_L - X_C}{R}\right] = \tan^{-1}(-10.1) = \boxed{-84.3°}$

33.22 (a) $Z = \sqrt{R^2 + \left(X_L - X_C\right)} = \sqrt{68.0^2 + (16.0 - 101)^2} = \boxed{109\ \Omega}$

$X_L = \omega L = (100)(0.160) = 16.0\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(100)\left(99.0 \times 10^{-6}\right)} = 101\ \Omega$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{16.0 - 101}{68.0} = -1.25$:

$\phi = -0.896\ \text{rad} = -51.3°$

$\boxed{I_{\max} = 0.367\ \text{A}}$ $\boxed{\omega = 100\ \text{rad/s}}$ $\boxed{\phi = -0.896\ \text{rad} = -51.3°}$

33.23 $X_L = 2\pi f L = 2\pi(60.0)(0.460) = 173\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0)\left(21.0 \times 10^{-6}\right)} = 126\ \Omega$

(a) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega} = 0.314$

$\phi = 0.304\ \text{rad} = \boxed{17.4°}$

(b) Since $X_L > X_C$, ϕ is positive; so $\boxed{\text{voltage leads the current}}$.

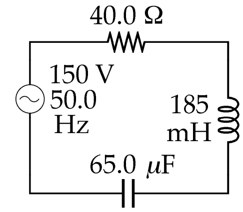
33.24 $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0\ \text{Hz})(20.0 \times 10^{-12}\ \text{F})} = 1.33 \times 10^8\ \Omega$

$Z = \sqrt{(50.0 \times 10^3\ \Omega)^2 + (1.33 \times 10^8\ \Omega)^2} \approx 1.33 \times 10^8\ \Omega$

$I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{5000\ \text{V}}{1.33 \times 10^8\ \Omega} = 3.77 \times 10^{-5}\ \text{A}$

$\left(\Delta V_{\text{rms}}\right)_{\text{body}} = I_{\text{rms}} R_{\text{body}} = (3.77 \times 10^{-5}\ \text{A})(50.0 \times 10^3\ \Omega) = \boxed{1.88\ \text{V}}$

33.25 $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)(65.0\times10^{-6})} = 49.0\ \Omega$



$X_L = \omega L = 2\pi(50.0)(185\times10^{-3}) = 58.1\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40.0)^2 + (58.1 - 49.0)^2} = 41.0\ \Omega$

$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150}{41.0} = 3.66\ \text{A}$

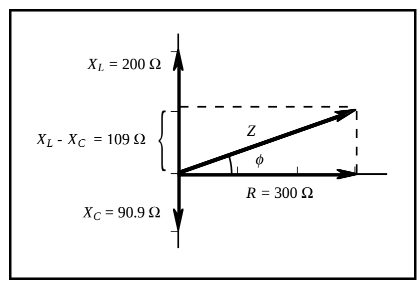
(a) $\Delta V_R = I_{\max} R = (3.66)(40) =$ $\boxed{146\ \text{V}}$

(b) $\Delta V_L = I_{\max} X_L = (3.66)(58.1) = 212.5 =$ $\boxed{212\ \text{V}}$

(c) $\Delta V_C = I_{\max} X_C = (3.66)(49.0) = 179.1\ \text{V} =$ $\boxed{179\ \text{V}}$

(d) $\Delta V_L - \Delta V_C = 212.5 - 179.1 =$ $\boxed{33.4\ \text{V}}$

33.26 $R = 300\ \Omega$

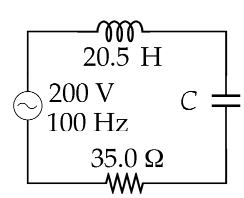


$X_L = \omega L = 2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)(0.200\ \text{H}) = 200\ \Omega$

$X_C = \frac{1}{\omega C} = \left[2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)\left(11.0\times10^{-6}\ \text{F}\right)\right]^{-1} = 90.9\ \Omega$

$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = 319\ \Omega$ and $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 20.0°$

33.27 (a) $X_L = 2\pi(100\ \text{Hz})(20.5\ \text{H}) = 1.29\times10^4\ \Omega$



$Z = \frac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \frac{200\ \text{V}}{4.00\ \text{A}} = 50.0\ \Omega$

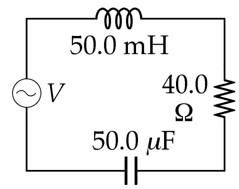
$\left(X_L - X_C\right)^2 = Z^2 - R^2 = (50.0\ \Omega)^2 - (35.0\ \Omega)^2$

$X_L - X_C = 1.29\times10^4\ \Omega - \frac{1}{2\pi(100\ \text{Hz})C} = \pm 35.7\ \Omega$ $\boxed{\text{C = 123 nF or 124 nF}}$

(b) $\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L = (4.00\ \text{A})\left(1.29\times10^4\ \Omega\right) =$ $\boxed{51.5\ \text{kV}}$

Notice that this is a very large voltage!

33.28



$X_L = \omega L = [(1000\ /\ \text{s})(0.0500\ \text{H})] = 50.0\ \Omega$

$X_C = 1\ /\ \omega C = [(1000\ /\ \text{s})(50.0 \times 10^{-6}\ \text{F})]^{-1} = 20.0\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \sqrt{(40.0)^2 + (50.0 - 20.0)^2} = 50.0\ \Omega$

(a) $I_{\text{rms}} = (\Delta V_{\text{rms}})\ /\ Z = 100\ \text{V}\ /\ 50.0\ \Omega$

$I_{\text{rms}} = \boxed{2.00\ \text{A}}$

$\phi = \text{Arctan}\left(\dfrac{X_L - X_C}{R}\right)$

$\phi = \text{Arctan}\ \dfrac{30.0\ \Omega}{40.0\ \Omega} = 36.9°$

(b) $\mathcal{P} = (\Delta V_{\text{rms}}) I_{\text{rms}} \cos\phi = 100\ \text{V}(2.00\ \text{A}) \cos 36.9° = \boxed{160\ \text{W}}$

(c) $\mathcal{P}_R = I_{\text{rms}}^2 R = (2.00\ \text{A})^2\ 40.0\ \Omega = \boxed{160\ \text{W}}$

33.29 $\omega = 1000\ \text{rad/s},\quad R = 400\ \Omega,\quad C = 5.00 \times 10^{-6}\ \text{F},\quad L = 0.500\ \text{H}$

$\Delta V_{\text{max}} = 100\ \text{V},\quad \omega L = 500\ \Omega,\ \left(\dfrac{1}{\omega C}\right) = 200\ \Omega$

$Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \sqrt{400^2 + 300^2} = 500\ \Omega$

$I_{\text{max}} = \dfrac{\Delta V_{\text{max}}}{Z} = \dfrac{100}{500} = 0.200\ \text{A}$

The average power dissipated in the circuit is $\mathcal{P} = I_{\text{rms}}^2 R = \left(\dfrac{I_{\text{max}}^2}{2}\right) R$

$\mathcal{P} = \dfrac{(0.200\ \text{A})^2}{2}(400\ \Omega) = \boxed{8.00\ \text{W}}$

Goal Solution

An ac voltage of the form $\Delta v = (100\text{ V})\sin(1000\,t)$ is applied to a series *RLC* circuit. If R = 400 Ω, C = 5.00 μF, and L = 0.500 H, what is the average power delivered to the circuit?

G: Comparing $\Delta v = (100\text{ V})\sin(1000\,t)$ with $\Delta v = \Delta V_{\max}\sin\omega t$, we see that

$$\Delta V_{\max} = 100\text{ V} \quad \text{and} \quad \omega = 1000\text{ s}^{-1}$$

Only the resistor takes electric energy out of the circuit, but the capacitor and inductor will impede the current flow and therefore reduce the voltage across the resistor. Because of this impedance, the average power dissipated by the resistor must be less than the maximum power from the source:

$$P_{\max} = \frac{(\Delta V_{\max})^2}{2R} = \frac{(100\text{ V})^2}{2(400\ \Omega)} = 12.5\text{ W}$$

O: The actual power dissipated by the resistor can be found from $P = I_{rms}^2 R$, where $I_{rms} = \Delta V_{rms} / Z$.

A: $\Delta V_{rms} = \dfrac{100}{\sqrt{2}} = 70.7\text{ V}$

In order to calculate the impedance, we first need the capacitive and inductive reactances:

$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ s}^{-1})(5.00\times10^{-6}\text{ F})} = 200\ \Omega \quad \text{and} \quad X_L = \omega L = \left(1000\text{ s}^{-1}\right)(0.500\text{ H}) = 500\ \Omega$$

Then, $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(400\ \Omega)^2 + (500\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{70.7\text{ V}}{500\ \Omega} = 0.141\text{ A} \quad \text{and} \quad P = I_{\text{rms}}^2 R = (0.141\text{ A})^2(400\ \Omega) = 8.00\text{ W}$$

L: The power dissipated by the resistor is less than 12.5 W, so our answer appears to be reasonable. As with other *RLC* circuits, the power will be maximized at the resonance frequency where $X_L = X_C$ so that $Z = R$. Then the average power dissipated will simply be the 12.5 W we calculated first.

33.30 $$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{or} \quad (X_L - X_C) = \sqrt{Z^2 - R^2}$$

$$(X_L - X_C) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1^\circ$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\text{ V}}{75.0\ \Omega} = 2.80\text{ A}$$

$$P = (\Delta V_{\text{rms}})I_{\text{rms}}\cos\phi = (210\text{ V})(2.80\text{ A})\cos(53.1^\circ) = \boxed{353\text{ W}}$$

33.31 (a) $\mathcal{P} = I_{\text{rms}}(\Delta V_{\text{rms}})\cos\phi = (9.00)(180)\cos(-37.0°) = 1.29\times10^3\ \text{W}$

$\mathcal{P} = I_{\text{rms}}^2 R$ so $1.29\times10^3 = (9.00)^2 R$ and $R = \boxed{16.0\ \Omega}$

(b) $\tan\phi = \dfrac{X_L - X_C}{R}$ becomes $\tan(-37.0°) = \dfrac{X_L - X_C}{16}$: so $X_L - X_C = \boxed{-12.0\ \Omega}$

***33.32** $X_L = \omega L = 2\pi(60.0\ /\ \text{s})(0.0250\ \text{H}) = 9.42\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0)^2 + (9.42)^2}\ \Omega = 22.1\ \Omega$

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{120\ \text{V}}{22.1\ \Omega} = \boxed{5.43\ \text{A}}$

(b) $\phi = \tan^{-1}(9.42\ /\ 20.0) = 25.2°$ so power factor $= \cos\phi = \boxed{0.905}$

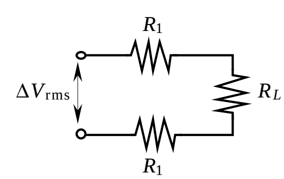
(c) We require $\phi = 0$. Thus, $X_L = X_C$: $9.42\ \Omega = \dfrac{1}{2\pi(60.0\ \text{s}^{-1})C}$

and $C = \boxed{281\ \mu\text{F}}$

(d) $\mathcal{P}_b = \mathcal{P}_d$ or $(\Delta V_{\text{rms}})_b(I_{\text{rms}})_b\cos\phi_b = \dfrac{(\Delta V_{\text{rms}})_d^2}{R}$

$(\Delta V_{\text{rms}})_d = \sqrt{R(\Delta V_{\text{rms}})_b(I_{\text{rms}})_b\cos\phi_b} = \sqrt{(20.0\ \Omega)(120\ \text{V})(5.43\ \text{A})(0.905)} = \boxed{109\ \text{V}}$

33.33 Consider a two-wire transmission line:



$I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}} = \dfrac{100\times10^6\ \text{W}}{50.0\times10^3\ \text{V}} = 2.00\times10^3\ \text{A}$

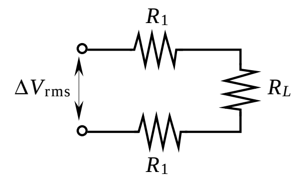
loss $= (0.0100)\mathcal{P} = I_{\text{rms}}^2 R_{\text{line}} = I_{\text{rms}}^2(2R_1)$

Thus, $R_1 = \dfrac{(0.0100)\mathcal{P}}{2I_{\text{max}}^2} = \dfrac{(0.0100)(100\times10^6\ \text{W})}{2(2.00\times10^3\ \text{A})^2} = 0.125\ \Omega$

But $R_1 = \dfrac{\rho \ell}{A}$ or $A = \dfrac{\pi d^2}{4} = \dfrac{\rho \ell}{R_1}$

Therefore $d = \sqrt{\dfrac{4\rho \ell}{\pi R_1}} = \sqrt{\dfrac{4(1.70\times10^{-8}\ \Omega\cdot\text{m})(100\times10^3\ \text{m})}{\pi(0.125\ \Omega)}} = 0.132\ \text{m} = \boxed{132\ \text{mm}}$

33.34 Consider a two-wire transmission line:



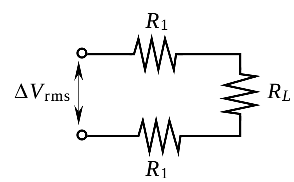
$$I_{\text{rms}} = \frac{\mathcal{P}}{\Delta V_{\text{rms}}} \quad \text{and} \quad \text{power loss} = I_{\text{rms}}^2 R_{\text{line}} = \frac{\mathcal{P}}{100}$$

$$\text{Thus, } \left(\frac{\mathcal{P}}{\Delta V_{\text{rms}}}\right)^2 (2R_1) = \frac{\mathcal{P}}{100} \quad \text{or} \quad R_1 = \frac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$$

$$R_1 = \frac{\rho d}{A} = \frac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}} \quad \text{or} \quad A = \frac{\pi(2r)^2}{4} = \frac{200\rho \mathcal{P} d}{(\Delta V_{\text{rms}})^2}$$

and the diameter is $$2r = \boxed{\sqrt{\frac{800\rho \mathcal{P} d}{\pi(\Delta V_{\text{rms}})^2}}}$$

33.35 One-half the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The power supply sees resistance



$$\left[\frac{1}{2R} + \frac{1}{2R}\right]^{-1} = R \quad \text{and the power is} \quad \frac{(\Delta V_{\text{rms}})^2}{R}$$

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has zero resistance. The equivalent resistance is then

$$R_{\text{eq}} = R + \left[\frac{1}{3R} + \frac{1}{R}\right]^{-1} = \frac{7R}{4} \quad \text{and} \quad \mathcal{P} = \frac{(\Delta V_{\text{rms}})^2}{R_{\text{eq}}} = \frac{4(\Delta V_{\text{rms}})^2}{7R}$$

The overall time average power is: $$\frac{\left[(\Delta V_{\text{rms}})^2/R\right] + \left[4(\Delta V_{\text{rms}})^2/7R\right]}{2} = \boxed{\frac{11(\Delta V_{\text{rms}})^2}{14R}}$$

33.36 At resonance, $$\frac{1}{2\pi f C} = 2\pi f L \quad \text{and} \quad \frac{1}{(2\pi f)^2 L} = C$$

The range of values for C is $\boxed{46.5\text{ pF to }419\text{ pF}}$

33.37 $$\omega_0 = 2\pi(99.7\times 10^6) = 6.26\times 10^8 \text{ rad / s} = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(6.26\times 10^8)^2(1.40\ \times 10^{-6})} = \boxed{1.82\text{ pF}}$$

33.38 $L = 20.0\text{ mH},\ C = 1.00\times10^{-7},\ R = 20.0\ \Omega,\ \Delta V_{\max} = 100\text{ V}$

(a) The resonant frequency for a series –*RLC* circuit is $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \boxed{3.56\text{ kHz}}$

(b) At resonance, $I_{\max} = \frac{\Delta V_{\max}}{R} = \boxed{5.00\text{ A}}$

(c) From Equation 33.36, $Q = \frac{\omega_0 L}{R} = \boxed{22.4}$

(d) $\Delta V_{L,\max} = X_L I_{\max} = \omega_0 L I_{\max} = \boxed{2.24\text{ kV}}$

33.39 The resonance frequency is $\omega_0 = 1/\sqrt{LC}$. Thus, if $\omega = 2\omega_0$,

$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$ and $X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25(L/C)}$ so $I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$

and the energy dissipated in one period is $Q = \mathcal{P}\Delta t$:

$$Q = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2 C + 2.25L}\left(\pi\sqrt{LC}\right) = \frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2 C + 9.00L}$$

With the values specified for this circuit, this gives:

$$Q = \frac{4\pi(50.0\text{ V})^2(10.0\ \Omega)\left(100\times10^{-6}\text{ F}\right)^{3/2}\left(10.0\times10^{-3}\text{ H}\right)^{1/2}}{4(10.0\ \Omega)^2\left(100\times10^{-6}\text{ F}\right) + 9.00\left(10.0\times10^{-3}\text{ H}\right)} = \boxed{242\text{ mJ}}$$

33.40 The resonance frequency is $\omega_0 = 1/\sqrt{LC}$. Thus, if $\omega = 2\omega_0$,

$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$ and $X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$

Then $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25(L/C)}$ so $I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$

and the energy dissipated in one period is

$$Q = \mathcal{P}\Delta t = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2 C + 2.25L}\left(\pi\sqrt{LC}\right) = \boxed{\frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2 C + 9.00L}}$$

***33.41** For the circuit of problem 22, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(160\times10^{-3}\text{ H})(99.0\times10^{-6}\text{ F})}} = 251\text{ rad s}$

$$Q = \frac{\omega_0 L}{R} = \frac{(251\text{ rad/s})(160\times10^{-3}\text{ H})}{68.0\ \Omega} = \boxed{0.591}$$

For the circuit of problem 23, $Q = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{150\ \Omega}\sqrt{\frac{460\times10^{-3}\text{ H}}{21.0\times10^{-6}\text{ F}}} = \boxed{0.987}$

The $\boxed{\text{circuit of problem 23}}$ has a sharper resonance.

33.42 (a) $\Delta V_{2,\text{rms}} = \frac{1}{13}(120\text{ V}) = \boxed{9.23\text{ V}}$

(b) $\Delta V_{1,\text{rms}} I_{1,\text{rms}} = \Delta V_{2,\text{rms}} I_{2,\text{rms}}$

$(120\text{ V})(0.350\text{ A}) = (9.23\text{ V})I_{2,\text{rms}}$

$I_{2,\text{rms}} = \frac{42.0\text{ W}}{9.23\text{ V}} = \boxed{4.55\text{ A}}$ for a transformer with no energy loss

(c) $P = \boxed{42.0\text{ W}}$ from (b)

33.43 $(\Delta V_{\text{out}})_{\max} = \frac{N_2}{N_1}(\Delta V_{\text{in}})_{\max} = \left(\frac{2000}{350}\right)(170\text{ V}) = 971\text{ V}$

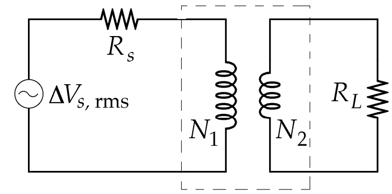
$(\Delta V_{\text{out}})_{\text{rms}} = \frac{(971\text{ V})}{\sqrt{2}} = \boxed{687\text{ V}}$

33.44 (a) $(\Delta V_{2,\text{ rms}}) = \frac{N_2}{N_1}(\Delta V_{1,\text{ rms}})$ $N_2 = \frac{(2200)(80)}{110} = \boxed{1600\text{ windings}}$

(b) $I_{1,\text{rms}}(\Delta V_{1,\text{rms}}) = I_{2,\text{rms}}(\Delta V_{2,\text{rms}})$ $I_{1,\text{rms}} = \frac{(1.50)(2200)}{110} = \boxed{30.0\text{ A}}$

(c) $0.950\, I_{1,\text{rms}}(\Delta V_{1,\text{rms}}) = I_{2,\text{rms}}(\Delta V_{2,\text{rms}})$ $I_{1,\text{rms}} = \frac{(1.20)(2200)}{110(0.950)} = \boxed{25.3\text{ A}}$

33.45 The rms voltage across the transformer primary is



$$\frac{N_1}{N_2}\left(\Delta V_{2,\mathrm{rms}}\right)$$

so the source voltage is $\Delta V_{s,\mathrm{rms}} = I_{1,\mathrm{rms}} R_s + \frac{N_1}{N_2}\left(\Delta V_{2,\mathrm{rms}}\right)$

The secondary current is $\frac{\left(\Delta V_{2,\mathrm{rms}}\right)}{R_L}$, so the primary current is $\frac{N_2}{N_1}\frac{\left(\Delta V_{2,\mathrm{rms}}\right)}{R_L} = I_{1,\mathrm{rms}}$

Then $\Delta V_{s,\mathrm{rms}} = \frac{N_2\left(\Delta V_{2,\mathrm{rms}}\right)R_s}{N_1 R_L} + \frac{N_1\left(\Delta V_{2,\mathrm{rms}}\right)}{N_2}$

and $R_s = \frac{N_1 R_L}{N_2\left(\Delta V_{2,\mathrm{rms}}\right)}\left(\Delta V_{s,\mathrm{rms}} - \frac{N_1\left(\Delta V_{2,\mathrm{rms}}\right)}{N_2}\right) = \frac{5(50.0\ \Omega)}{2(25.0\ \mathrm{V})}\left(80.0\ \mathrm{V} - \frac{5(25.0\ \mathrm{V})}{2}\right) = \boxed{87.5\ \Omega}$

33.46 (a) $\Delta V_{2,\mathrm{rms}} = \frac{N_2}{N_1}\left(\Delta V_{1,\mathrm{rms}}\right)$ $\quad \frac{N_2}{N_1} = \frac{\Delta V_{2,\mathrm{rms}}}{\Delta V_{1,\mathrm{rms}}} = \frac{10.0\times 10^3\ \mathrm{V}}{120\ \mathrm{V}} = \boxed{83.3}$

(b) $I_{2,\mathrm{rms}}\left(\Delta V_{2,\mathrm{rms}}\right) = 0.900\, I_{1,\mathrm{rms}}\left(\Delta V_{1,\mathrm{rms}}\right)$

$I_{2,\mathrm{rms}}\left(10.0\times 10^3\ \mathrm{V}\right) = 0.900\left(\frac{120\ \mathrm{V}}{24.0\ \Omega}\right)(120\ \mathrm{V})$ $\quad I_{2,\mathrm{rms}} = \boxed{54.0\ \mathrm{mA}}$

(c) $Z_2 = \frac{\Delta V_{2,\mathrm{rms}}}{I_{2,\mathrm{rms}}} = \frac{10.0\times 10^3\ \mathrm{V}}{0.054\ \mathrm{A}} = \boxed{185\ \mathrm{k}\Omega}$

33.47 (a) $R = (4.50\times 10^{-4}\ \Omega/\mathrm{m})(6.44\times 10^5\ \mathrm{m}) = 290\ \Omega$ and $I_{\mathrm{rms}} = \frac{\mathcal{P}}{\Delta V_{\mathrm{rms}}} = \frac{5.00\times 10^6\ \mathrm{W}}{5.00\times 10^5\ \mathrm{V}} = 10.0\ \mathrm{A}$

$\mathcal{P}_{\mathrm{loss}} = I_{\mathrm{rms}}^2 R = (10.0\ \mathrm{A})^2(290\ \Omega) = \boxed{29.0\ \mathrm{kW}}$

(b) $\frac{\mathcal{P}_{\mathrm{loss}}}{\mathcal{P}} = \frac{2.90\times 10^4}{5.00\times 10^6} = \boxed{5.80\times 10^{-3}}$

(c) It is impossible to transmit so much power at such low voltage. Maximum power transfer occurs when load resistance equals the line resistance of 290 Ω, and is

$\frac{(4.50\times 10^3\ \mathrm{V})^2}{2\cdot 2(290\ \Omega)} = 17.5\ \mathrm{kW}$, far below the required 5000 kW

33.48 For the filter circuit, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{X_C}{\sqrt{R^2 + X_C^2}}$

(a) At $f = 600\text{ Hz}$, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(600\text{ Hz})\left(8.00\times10^{-9}\text{ F}\right)} = 3.32\times10^{4}\ \Omega$

and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{3.32\times10^{4}\ \Omega}{\sqrt{(90.0\ \Omega)^2 + \left(3.32\times10^{4}\ \Omega\right)^2}} \approx \boxed{1.00}$

(b) At $f = 600\text{ kHz}$, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi\left(600\times10^{3}\text{ Hz}\right)\left(8.00\times10^{-9}\text{ F}\right)} = 33.2\ \Omega$

and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{33.2\ \Omega}{\sqrt{(90.0\ \Omega)^2 + (33.2\ \Omega)^2}} = \boxed{0.346}$

33.49 For this RC high-pass filter, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$

(a) When $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = 0.500$,

then $\dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + X_C^2}} = 0.500$ or $X_C = 0.866\ \Omega$

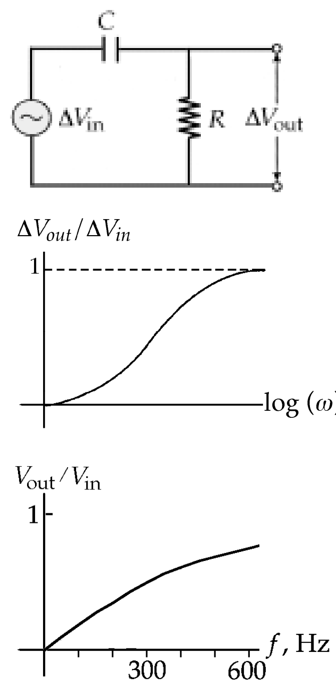
If this occurs at $f = 300\text{ Hz}$, the capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(300\text{ Hz})(0.866\ \Omega)} = 6.13\times10^{-4}\text{ F} = \boxed{613\ \mu\text{F}}$$

(b) With this capacitance and a frequency of 600 Hz,

$$X_C = \frac{1}{2\pi(600\text{ Hz})\left(6.13\times10^{-4}\text{ F}\right)} = 0.433\ \Omega$$

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$$



(a)

(b)

(c)

Figures for Goal Solution

Goal Solution

The RC high-pass filter shown in Figure 33.22 has a resistance $R = 0.500\ \Omega$. (a) What capacitance gives an output signal that has one-half the amplitude of a 300-Hz input signal? (b) What is the gain ($\Delta V_{out} / \Delta V_{in}$) for a 600-Hz signal?

G: It is difficult to estimate the capacitance required without actually calculating it, but we might expect a typical value in the μF to pF range. The nature of a high-pass filter is to yield a larger gain at higher frequencies, so if this circuit is designed to have a gain of 0.5 at 300 Hz, then it should have a higher gain at 600 Hz. We might guess it is near 1.0 based on Figure (b) above.

O: The output voltage of this circuit is taken across the resistor, but the input sees the impedance of the resistor and the capacitor. Therefore, the gain will be the ratio of the resistance to the impedance.

A: $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$$

(a) When $\Delta V_{out} / \Delta V_{in} = 0.500$

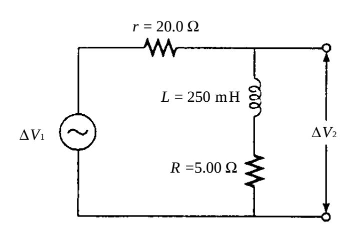
solving for C gives $$C = \frac{1}{\omega R\sqrt{\left(\frac{\Delta V_{\text{in}}}{\Delta V_{\text{out}}}\right)^2 - 1}} = \frac{1}{(2\pi)(300\text{ Hz})(0.500\ \Omega)\sqrt{(2.00)^2 - 1}} = 613\ \mu\text{F}$$

(b) At 600 Hz, we have $\omega = (2\pi\text{ rad})(600\text{ s}^{-1})$

so $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + \left(\frac{1}{(1200\pi\text{ rad / s})(613\ \mu\text{F})}\right)^2}} = 0.756$$

L: The capacitance value seems reasonable, but the gain is considerably less than we expected. Based on our calculation, we can modify the graph in Figure (b) to more transparently represent the characteristics of this high-pass filter, now shown in Figure (c). If this were an audio filter, it would reduce low frequency "humming" sounds while allowing high pitch sounds to pass through. A low pass filter would be needed to reduce high frequency "static" noise.

33.50 $\Delta V_1 = I\sqrt{(r+R)^2 + X_L^2}$, and $\Delta V_2 = I\sqrt{R^2 + X_L^2}$

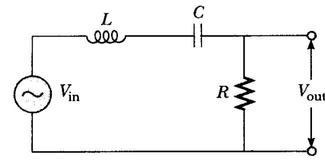


Thus, when $\Delta V_1 = 2\Delta V_2$ $$(r+R)^2 + X_L^2 = 4\left(R^2 + X_L^2\right)$$

or $$(25.0\ \Omega)^2 + X_L^2 = 4(5.00\ \Omega)^2 + 4X_L^2$$

which gives $$X_L = 2\pi f(0.250\text{ H}) = \sqrt{\frac{625 - 100}{3}}\ \Omega \quad \text{and} \quad f = \boxed{8.42\text{ Hz}}$$

***33.51** $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$



(a) At 200 Hz: $\dfrac{1}{4} = \dfrac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[400\pi L - \dfrac{1}{400\pi C}\right]^2}$

At 4000 Hz: $(8.00\ \Omega)^2 + \left[8000\pi L - \dfrac{1}{8000\pi C}\right]^2 = 4(8.00\ \Omega)^2$

At the low frequency, $X_L - X_C < 0$. This reduces to $400\pi L - \dfrac{1}{400\pi C} = -13.9\ \Omega$ [1]

For the high frequency half-voltage point, $8000\pi L - \dfrac{1}{8000\pi C} = +13.9\ \Omega$ [2]

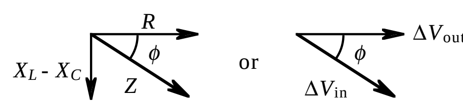
Solving Equations (1) and (2) simultaneously gives $C = \boxed{54.6\ \mu\text{F}}$ and $L = \boxed{580\ \mu\text{H}}$

(b) When $X_L = X_C$, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \left(\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}}\right)_{\max} = \boxed{1.00}$

(c) $X_L = X_C$ requires $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(5.80\times 10^{-4}\ \text{H})(5.46\times 10^{-5}\ \text{F})}} = \boxed{894\ \text{Hz}}$

(d) At 200 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_C > X_L$,

so the phasor diagram is as shown:

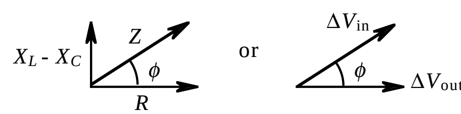


$\phi = -\cos^{-1}\left(\dfrac{R}{Z}\right) = -\cos^{-1}\left(\dfrac{1}{2}\right)$ so $\boxed{\Delta V_{\text{out}} \text{ leads } \Delta V_{\text{in}} \text{ by } 60.0°}$

At f_0, $X_L = X_C$ so $\boxed{\Delta V_{\text{out}} \text{ and } \Delta V_{\text{in}} \text{ have a phase difference of } 0°}$

At 4000 Hz, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_L - X_C > 0$

Thus, $\phi = \cos^{-1}\left(\dfrac{1}{2}\right) = 60.0°$



or $\boxed{\Delta V_{\text{out}} \text{ lags } \Delta V_{\text{in}} \text{ by } 60.0°}$

(e) At 200 Hz and at 4 kHz, $P = \dfrac{(\Delta V_{\text{out,rms}})^2}{R} = \dfrac{\left(\frac{1}{2}\Delta V_{\text{in,rms}}\right)^2}{R} = \dfrac{\frac{1}{2}\left(\frac{1}{2}\Delta V_{\text{in,max}}\right)^2}{R} = \dfrac{(10.0\ \text{V})^2}{8(8.00\ \Omega)} = \boxed{1.56\ \text{W}}$

At f_0, $P = \dfrac{(\Delta V_{\text{out,rms}})^2}{R} = \dfrac{(\Delta V_{\text{in,rms}})^2}{R} = \dfrac{\frac{1}{2}(\Delta V_{\text{in,max}})^2}{R} = \dfrac{(10.0\ \text{V})^2}{2(8.00\ \Omega)} = \boxed{6.25\ \text{W}}$

(f) We take: $Q = \dfrac{\omega_0 L}{R} = \dfrac{2\pi f_0 L}{R} = \dfrac{2\pi(894\ \text{Hz})(5.80\times 10^{-4}\ \text{H})}{8.00\ \Omega} = \boxed{0.408}$

33.52 For a high-pass filter,

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

$$\frac{(\Delta V_{\text{out}})_1}{(\Delta V_{\text{in}})_1} = \frac{R}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}} \quad \text{and} \quad \frac{(\Delta V_{\text{out}})_2}{(\Delta V_{\text{in}})_2} = \frac{R}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

Now $(\Delta V_{\text{in}})_2 = (\Delta V_{\text{out}})_1$ so

$$\frac{(\Delta V_{\text{out}})_2}{(\Delta V_{\text{in}})_1} = \frac{R^2}{R^2 + \left(\dfrac{1}{\omega C}\right)^2} = \boxed{\frac{1}{1 + \left(\dfrac{1}{\omega RC}\right)^2}}$$

33.53 Rewrite the circuit in terms of impedance as shown in Fig. (b).

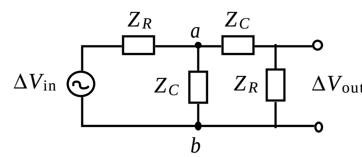


Figure (a)

Find: $$\Delta V_{\text{out}} = \frac{Z_R}{Z_R + Z_C}\Delta V_{ab} \qquad [1]$$

From Figure (c), $$\Delta V_{ab} = \frac{Z_C \parallel (Z_R + Z_C)}{Z_R + Z_C \parallel (Z_R + Z_C)}\Delta V_{\text{in}}$$

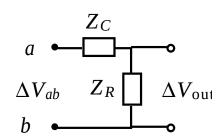


Figure (b)

So Eq. [1] becomes $$\Delta V_{\text{out}} = \frac{Z_R\left[Z_C \parallel (Z_R + Z_C)\right]}{(Z_R + Z_C)\left[Z_R + Z_C \parallel (Z_R + Z_C)\right]}\Delta V_{\text{in}}$$

or $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{Z_R\left[\dfrac{1}{Z_C} + \dfrac{1}{Z_R + Z_C}\right]^{-1}}{(Z_R + Z_C)\left[Z_R + \left(\dfrac{1}{Z_C} + \dfrac{1}{Z_R + Z_C}\right)^{-1}\right]}$$

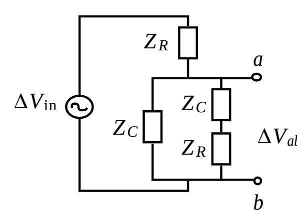
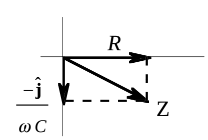


Figure (c)

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{Z_R Z_C}{Z_C(Z_C + Z_R) + Z_R(Z_R + 2Z_C)} = \frac{Z_R}{3Z_R + Z_C + (Z_R)^2 / Z_C}$$

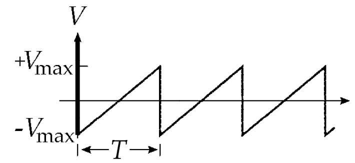
Now, $Z_R = R$ and $Z_C = \dfrac{-\text{j}}{\omega C}$ where $\text{j} = \sqrt{-1}$



$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{3R - \left(\dfrac{1}{\omega C}\right)\text{j} + R^2\omega C\,\text{j}}$$ where we used $\dfrac{1}{\text{j}} = -\text{j}$.

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{3R - \left(\dfrac{1}{\omega C} - R^2\omega C\right)\text{j}} = \frac{R}{\sqrt{(3R)^2 + \left(\dfrac{1}{\omega C} - R^2\omega C\right)^2}} = \frac{1.00\times10^3}{\sqrt{\left(3.00\times10^3\right)^2 + (1592 - 628)^2}} = \boxed{0.317}$$

33.54 The equation for $\Delta v(t)$ during the first period (using $y = mx + b$) is:



$$\Delta v(t) = \frac{2(\Delta V_{\max})t}{T} - \Delta V_{\max}$$

$$\left[(\Delta v)^2\right]_{\text{ave}} = \frac{1}{T}\int_0^T \left[\Delta v(t)\right]^2 dt = \frac{(\Delta V_{\max})^2}{T}\int_0^T \left[\frac{2}{T}t - 1\right]^2 dt$$

$$\left[(\Delta v)^2\right]_{\text{ave}} = \frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{\left[2t/T - 1\right]^3}{3}\Bigg|_{t=0}^{t=T} = \frac{(\Delta V_{\max})^2}{6}\left[(+1)^3 - (-1)^3\right] = \frac{(\Delta V_{\max})^2}{3}$$

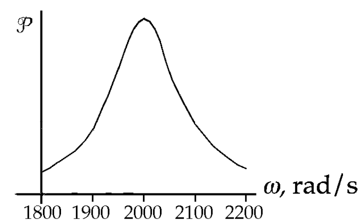
$$\Delta V_{\text{rms}} = \sqrt{\left[(\Delta v)^2\right]_{\text{ave}}} = \sqrt{\frac{(\Delta V_{\max})^2}{3}} = \boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$

33.55 $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.0500\text{ H})(5.00\times 10^{-6}\text{ F})}} = 2000\text{ s}^{-1}$$

so the operating frequency of the circuit is $\omega = \frac{\omega_0}{2} = 1000\text{ s}^{-1}$

Using Equation 33.35, $$\mathcal{P} = \frac{(\Delta V_{\text{rms}})^2 R\omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_0^2\right)^2}$$

$$\mathcal{P} = \frac{(400)^2(8.00)(1000)^2}{(8.00)^2(1000)^2 + (0.0500)^2\left[(1.00 - 4.00)\times 10^6\right]^2} = \boxed{56.7\text{ W}}$$



$(Q \approx 12.5)$

Figure for Goal Solution

Goal Solution

A series *RLC* circuit consists of an 8.00-Ω resistor, a 5.00-μF capacitor, and a 50.0-mH inductor. A variable frequency source applies an emf of 400 V (rms) across the combination. Determine the power delivered to the circuit when the frequency is equal to one half the resonance frequency.

G: Maximum power is delivered at the resonance frequency, and the power delivered at other frequencies depends on the quality factor, Q. For the relatively small resistance in this circuit, we could expect a high $Q = \omega_0 L/R$. So at half the resonant frequency, the power should be a small fraction of the maximum power, $\mathcal{P}_{av,\max} = \Delta V_{rms}^2/R = (400\text{ V})^2/8\ \Omega = 20\text{ kW}$.

O: We must first calculate the resonance frequency in order to find half this frequency. Then the power delivered by the source must equal the power taken out by the resistor. This power can be found from $\mathcal{P}_{av} = I_{rms}^2 R$ where $I_{rms} = \Delta V_{rms} / Z$.

A: The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.0500\text{ H})\left(5.00\times10^{-6}\text{ F}\right)}} = 318\text{ Hz}$

The operating frequency is $f = f_0 / 2 = 159\text{ Hz}$. We can calculate the impedance at this frequency:

$X_L = 2\pi f L = 2\pi(159\text{ Hz})(0.0500\text{ H}) = 50.0\ \Omega$ and $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(159\text{ Hz})\left(5.00\times10^{-6}\text{ F}\right)} = 200\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{8.00^2 + (50.0 - 200)^2}\ \Omega = 150\ \Omega$$

So, $I_{rms} = \dfrac{\Delta V_{rms}}{Z} = \dfrac{400\text{ V}}{150\ \Omega} = 2.66\text{ A}$

The power delivered by the source is the power dissipated by the resistor:

$$\mathcal{P}_{av} = I_{rms}{}^2 R = (2.66\text{ A})^2(8.00\ \Omega) = 56.7\text{ W}$$

L: This power is only about 0.3% of the 20 kW peak power delivered at the resonance frequency. The significant reduction in power for frequencies away from resonance is a consequence of the relatively high Q-factor of about 12.5 for this circuit. A high Q is beneficial if, for example, you want to listen to your favorite radio station that broadcasts at 101.5 MHz, and you do not want to receive the signal from another local station that broadcasts at 101.9 MHz.

33.56 The resistance of the circuit is $R = \dfrac{\Delta V}{I} = \dfrac{12.0\text{ V}}{0.630\text{ A}} = 19.0\ \Omega$

The impedance of the circuit is $Z = \dfrac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{24.0\text{ V}}{0.570\text{ A}} = 42.1\ \Omega$

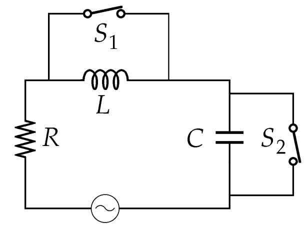
$$Z^2 = R^2 + \omega^2 L^2$$

$$L = \frac{1}{\omega}\sqrt{Z^2 - R^2} = \frac{1}{377}\sqrt{(42.1)^2 - (19.0)^2} = \boxed{99.6\text{ mH}}$$

33.57 (a) When ωL is very large, the bottom branch carries negligible current. Also, $1/\omega C$ will be negligible compared to $200\ \Omega$ and $45.0\text{ V}/200\ \Omega = \boxed{225\text{ mA}}$ flows in the power supply and the top branch.

(b) Now $1/\omega C \to \infty$ and $\omega L \to 0$ so the generator and bottom branch carry $\boxed{450\text{ mA}}$

33.58 (a) With both switches closed, the current goes only through generator and resistor.



$$\boxed{i(t) = \frac{\Delta V_{\max}}{R} \cos \omega t}$$

(b) $$\boxed{\mathcal{P} = \frac{1}{2} \frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i(t) = \frac{\Delta V_{\max}}{\sqrt{R^2 + \omega^2 L^2}} \cos[\omega t + \text{Arctan}(\omega L / R)]}$$

(d) For $$0 = \phi = \text{Arctan}\left(\frac{\omega_0 L - \frac{1}{\omega_0 C}}{R}\right)$$

We require $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $$\boxed{C = \frac{1}{\omega_0^{\,2} L}}$$

(e) At this resonance frequency, $Z = \boxed{R}$

(f) $U = \frac{1}{2} C (\Delta V_C)^2 = \frac{1}{2} C I^2 X_C^{\,2}$

$$U_{\max} = \tfrac{1}{2} C I_{\max}^2 X_C^{\,2} = \tfrac{1}{2} C \frac{(\Delta V_{\max})^2}{R^2} \frac{1}{\omega_0^{\,2} C^2} = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

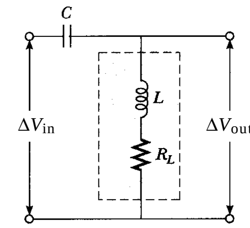
(g) $$U_{\max} = \tfrac{1}{2} L I_{\max}^2 = \boxed{\tfrac{1}{2} L \frac{(\Delta V_{\max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$

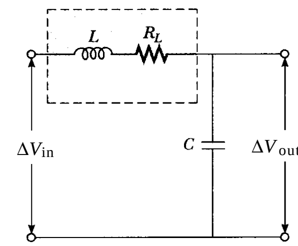
So $$\phi = \text{Arctan}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \text{Arctan}\left(\frac{2\sqrt{\frac{L}{C}} - \frac{1}{2}\sqrt{\frac{L}{C}}}{R}\right) = \boxed{\text{Arctan}\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}$$

(i) Now $\omega L = \dfrac{1}{2} \dfrac{1}{\omega C}$ $$\omega = \boxed{\frac{1}{\sqrt{2LC}}} = \frac{\omega_0}{\sqrt{2}}$$

33.59 (a) As shown in part (b), circuit (a) is a high-pass filter and circuit (b) is a low-pass filter.



Circuit (a)



Circuit (b)

(b) For circuit (a), $$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{\sqrt{R_L^2 + X_L^2}}{\sqrt{R_L^2 + (X_L - X_C)^2}} = \boxed{\frac{\sqrt{R_L^2 + (\omega L)^2}}{\sqrt{R_L^2 + (\omega L - 1/\omega C)^2}}}$$

As $\omega \to 0$, $\frac{\Delta V_{out}}{\Delta V_{in}} \approx \omega R_L C \approx 0$

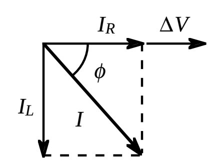
As $\omega \to \infty$, $\frac{\Delta V_{out}}{\Delta V_{in}} \approx 1$ (high-pass filter)

For circuit (b), $$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{X_C}{\sqrt{R_L^2 + (X_L - X_C)^2}} = \boxed{\frac{1/\omega C}{\sqrt{R_L^2 + (\omega L - 1/\omega C)^2}}}$$

As $\omega \to 0$, $\frac{\Delta V_{out}}{\Delta V_{in}} \approx 1$

As $\omega \to \infty$, $\frac{\Delta V_{out}}{\Delta V_{in}} \approx \frac{1}{\omega^2 LC} \approx 0$ (low-pass filter)

33.60 (a) $$I_{R,\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{100\text{ V}}{80.0\ \Omega} = \boxed{1.25\text{ A}}$$



(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram at the right.

$$I_{L,\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{100\text{ V}}{2\pi(60.0\text{ s}^{-1})(0.200\text{ H})} = 1.33\text{ A}$$

Thus, the phase angle is: $$\phi = \tan^{-1}\left(\frac{I_{L,\text{rms}}}{I_{R,\text{rms}}}\right) = \tan^{-1}\left(\frac{1.33\text{ A}}{1.25\text{ A}}\right) = \boxed{46.7°}$$

***33.61** Suppose each of the 20 000 people uses an average power of 500 W. (This means 12 kWh per day, or $36 per 30 days at 10¢ per kWh). Suppose the transmission line is at 20 kV. Then

$$I_{\text{rms}} = \frac{P}{\Delta V_{\text{rms}}} = \frac{(20\,000)(500\text{ W})}{20\,000\text{ V}} \quad \boxed{\sim 10^3\text{ A}}$$

If the transmission line had been at 200 kV, the current would be only $\boxed{\sim 10^2\text{ A}}$.

33.62 $L = 2.00\text{ H},\ C = 10.0\times 10^{-6}\text{ F},\ R = 10.0\ \Omega,\ \Delta v(t) = (100\sin\omega t)$

(a) The resonant frequency ω_0 produces the maximum current and thus the maximum power dissipation in the resistor.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00)(10.0\times 10^{-6})}} = \boxed{224\text{ rad/s}}$$

(b) $$\mathcal{P} = \frac{(\Delta V_{\max})^2}{2R} = \frac{(100)^2}{2(10.0)} = \boxed{500\text{ W}}$$

(c) $$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$ and $$(I_{\text{rms}})_{\max} = \frac{\Delta V_{\text{rms}}}{R}$$

$$I_{\text{rms}}^2 R = \frac{1}{2}\left(I_{\text{rms}}^2\right)_{\max} R$$ or $$\frac{(\Delta V_{\text{rms}})^2}{Z^2}R = \frac{1}{2}\frac{(\Delta V_{\text{rms}})^2}{R^2}R$$

This occurs where $Z^2 = 2R^2$: $$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\omega^4 L^2 C^2 - 2L\omega^2 C - R^2\omega^2 C^2 + 1 = 0$$ or $$L^2C^2\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0$$

$$\left[(2.00)^2(10.0\times 10^{-6})^2\right]\omega^4 - \left[2(2.00)(10.0\times 10^{-6}) + (10.0)^2(10.0\times 10^{-6})^2\right]\omega^2 + 1 = 0$$

Solving this quadratic equation, we find that $\omega^2 = 51\,130,\quad 48\,894$

$$\omega_1 = \sqrt{48\,894} = \boxed{221\text{ rad/s}}$$ and $$\omega_2 = \sqrt{51\,130} = \boxed{226\text{ rad/s}}$$

33.63 $R = 200\ \Omega,\ L = 663\text{ mH},\ C = 26.5\ \mu\text{F},\ \omega = 377\text{ s}^{-1},\ \Delta V_{\max} = 50.0\text{ V}$

$$\omega L = 250\ \Omega,\quad \left(\frac{1}{\omega C}\right) = 100\ \Omega,\quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 250\ \Omega$$

(a) $$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{50.0\text{ V}}{250\ \Omega} = \boxed{0.200\text{ A}}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \boxed{36.8^\circ}$$ (ΔV leads I)

(b) $\Delta V_{R,\max} = I_{\max}R = \boxed{40.0\text{ V}}$ at $\boxed{\phi = 0^\circ}$

(c) $\Delta V_{C,\max} = \dfrac{I_{\max}}{\omega C} = \boxed{20.0\text{ V}}$ at $\boxed{\phi = -90.0^\circ}$ (I leads ΔV)

(d) $\Delta V_{L,\max} = I_{\max}\omega L = \boxed{50.0\text{ V}}$ at $\boxed{\phi = +90.0^\circ}$ (ΔV leads I)

***33.64** $P = I_{rms}^2 R = \left(\frac{\Delta V_{rms}}{Z}\right)^2 R$, so $250\text{ W} = \frac{(120\text{ V})^2}{Z^2}(40.0\ \Omega)$: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$250 = \frac{(120)^2(40.0)}{(40.0)^2 + \left[2\pi f(0.185) - \frac{1}{2\pi f(65.0\times 10^{-6})}\right]^2} \quad \text{and} \quad 250 = \frac{576\,000 f^2}{1600 f^2 + (1.1624 f^2 - 2448.5)^2}$$

$$1 = \frac{2304 f^2}{1600 f^2 + 1.3511 f^4 - 5692.3 f^2 + 5\,995\,300} \quad \text{so} \quad 1.3511 f^4 - 6396.3 f^2 + 5\,995\,300 = 0$$

$$f^2 = \frac{6396.3 \pm \sqrt{(6396.3)^2 - 4(1.3511)(5\,995\,300)}}{2(1.3511)} = 3446.5 \text{ or } 1287.4$$

$f =$ $\boxed{58.7\text{ Hz or } 35.9\text{ Hz}}$

33.65 (a) From Equation 33.39, $\frac{N_1}{N_2} = \frac{\Delta V_1}{\Delta V_2}$

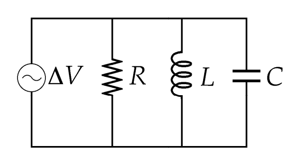
Let output impedance $Z_1 = \frac{\Delta V_1}{I_1}$ and the input impedance $Z_2 = \frac{\Delta V_2}{I_2}$

so that $\frac{N_1}{N_2} = \frac{Z_1 I_1}{Z_2 I_2}$ But from Eq. 33.40, $\frac{I_1}{I_2} = \frac{\Delta V_2}{\Delta V_1} = \frac{N_2}{N_1}$

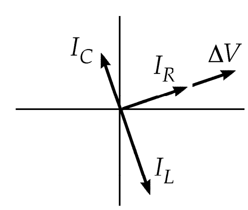
So, combining with the previous result we have $\boxed{\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}}$

(b) $\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{8000}{8.00}} = \boxed{31.6}$

33.66 $I_R = \frac{\Delta V_{rms}}{R}$; $I_L = \frac{\Delta V_{rms}}{\omega_L}$; $I_C = \frac{\Delta V_{rms}}{(\omega C)^{-1}}$

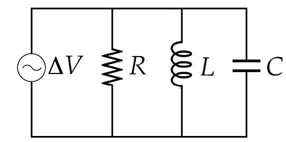


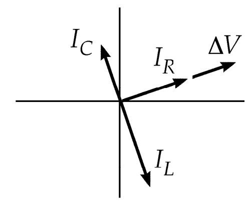
(a) $I_{rms} = \sqrt{I_R^2 + (I_C - I_L)^2} = \boxed{\Delta V_{rms}\sqrt{\left(\frac{1}{R^2}\right) + \left(\omega C - \frac{1}{\omega L}\right)^2}}$



(b) $\tan\phi = \frac{I_C - I_L}{I_R} = \Delta V_{rms}\left[\frac{1}{X_C} - \frac{1}{X_L}\right]\left(\frac{1}{\Delta V_{rms} / R}\right)$

$\boxed{\tan\phi = R\left[\frac{1}{X_C} - \frac{1}{X_L}\right]}$





33.67 (a) $I_{\text{rms}} = \Delta V_{\text{rms}}\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$

$\Delta V_{\text{rms}} \to (\Delta V_{\text{rms}})_{\max}$ when $\omega C = \dfrac{1}{\omega L}$

$f = \dfrac{1}{2\pi\sqrt{LC}}$

$f = \dfrac{1}{2\pi\sqrt{200\times 10^{-3}\text{ H})(0.150\times 10^{-6}\text{ F})}} = \boxed{919\text{ Hz}}$

(b) $I_R = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{120\text{ V}}{80.0\ \Omega} = \boxed{1.50\text{ A}}$

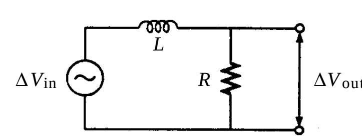
$I_L = \dfrac{\Delta V_{\text{rms}}}{\omega L} = \dfrac{120\text{ V}}{(374\text{ s}^{-1})(0.200\text{ H})} = \boxed{1.60\text{ A}}$

$I_C = \Delta V_{\text{rms}}(\omega C) = (120\text{ V})(374\text{ s}^{-1})(0.150\times 10^{-6}\text{ F}) = \boxed{6.73\text{ mA}}$

(c) $I_{\text{rms}} = \sqrt{I_R^2 + (I_C - I_L)^2} = \sqrt{(1.50)^2 + (0.00673 - 1.60)^2} = \boxed{2.19\text{ A}}$

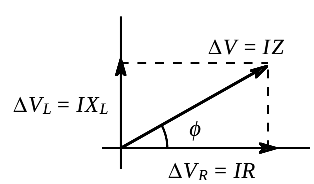
(d) $\phi = \tan^{-1}\left[\dfrac{I_C - I_L}{I_R}\right] = \tan^{-1}\left[\dfrac{0.00673 - 1.60}{1.50}\right] = \boxed{-46.7°}$

The $\boxed{\text{current is lagging the voltage}}$.



33.68 (a) $\tan\phi = \dfrac{\Delta V_L}{\Delta V_R} = \dfrac{I(\omega L)}{IR} = \dfrac{\omega L}{R}$

Thus, $R = \dfrac{\omega L}{\tan\phi} = \dfrac{(200\text{ s}^{-1})(0.500\text{ H})}{\tan(30.0°)} = \boxed{173\ \Omega}$



(b) $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{\Delta V_R}{\Delta V_{\text{in}}} = \cos\phi$

$\Delta V_{\text{out}} = (\Delta V_{\text{in}})\cos\phi = (10.0\text{ V})\cos 30.0° = \boxed{8.66\text{ V}}$

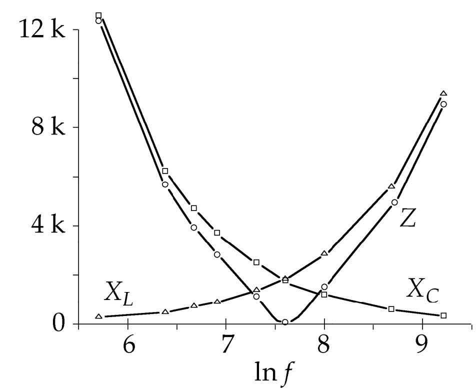
33.69 (a) $X_L = X_C = 1884\ \Omega$ when $f = 2000$ Hz

$$L = \frac{X_L}{2\pi f} = \frac{1884\ \Omega}{4000\pi\ \text{rad/s}} = 0.150\ \text{H} \quad \text{and} \quad C = \frac{1}{(2\pi f)X_C} = \frac{1}{(4000\pi\ \text{rad/s})(1884\ \Omega)} = 42.2\ \text{nF}$$

$$X_L = 2\pi f(0.150\ \text{H}) \quad X_C = \frac{1}{(2\pi f)(4.22\times10^{-8}\ \text{F})} \quad Z = \sqrt{(40.0\ \Omega)^2 + (X_L - X_C)^2}$$

f (Hz)	X_L (Ω)	X_C (Ω)	Z (Ω)
300	283	12600	12300
600	565	6280	5720
800	754	4710	3960
1000	942	3770	2830
1500	1410	2510	1100
2000	1880	1880	40
3000	2830	1260	1570
4000	3770	942	2830
6000	5650	628	5020
10000	9420	377	9040

(b)



33.70 $$\omega_0 = \frac{1}{\sqrt{LC}} = 1.00\times10^6\ \text{rad/s}$$

For each angular frequency, we find

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

then $I = (1.00\ \text{V})/Z$

and $P = I^2(1.00\ \Omega)$

ω/ω_0	ωL (Ω)	$\frac{1}{\omega C}$ (Ω)	Z (Ω)	$P = I^2R$ (W)
0.9990	999.0	1001.0	2.24	0.19984
0.9991	999.1	1000.9	2.06	0.23569
0.9993	999.3	1000.7	1.72	0.33768
0.9995	999.5	1000.5	1.41	0.49987
0.9997	999.7	1000.3	1.17	0.73524
0.9999	999.9	1000.1	1.02	0.96153
1.0000	1000	1000.0	1.00	1.00000
1.0001	1000.1	999.9	1.02	0.96154
1.0003	1000.3	999.7	1.17	0.73535
1.0005	1000.5	999.5	1.41	0.50012
1.0007	1000.7	999.3	1.72	0.33799
1.0009	1000.9	999.1	2.06	0.23601
1.0010	1001	999.0	2.24	0.20016

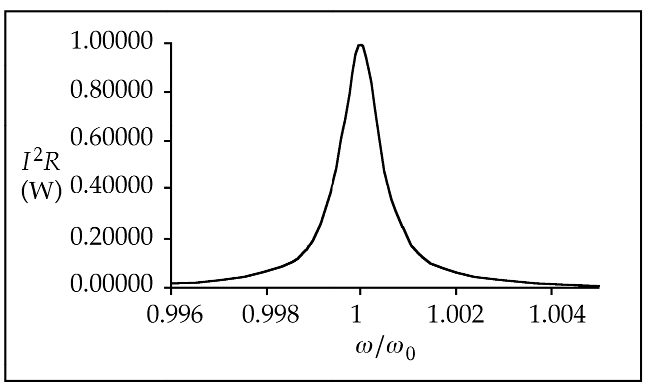
The full width at half maximum is:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{(1.0005 - 0.9995)\omega_0}{2\pi}$$

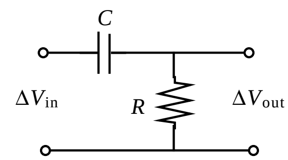
$$\Delta f = \frac{1.00\times10^3\ \text{s}^{-1}}{2\pi} = 159\ \text{Hz}$$

while

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi(1.00\times10^{-3}\ \text{H})} = 159\ \text{Hz}$$



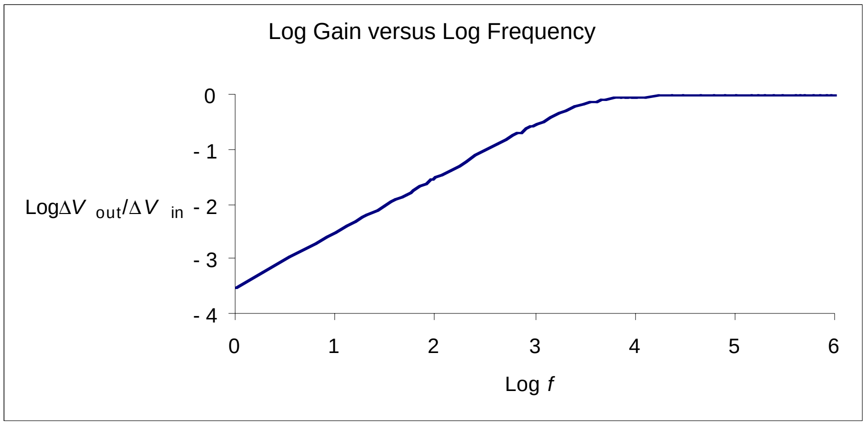
33.71 $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{R}{\sqrt{R^2 + (1/2\pi f C)^2}}$$



(a) $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1}{2} \quad \text{when} \quad \frac{1}{\omega C} = R\sqrt{3}$$

Hence, $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi RC\sqrt{3}} = \boxed{1.84 \text{ kHz}}$

(b)



Chapter 34 Solutions

34.1 Since the light from this star travels at 3.00×10^8 m/s, the last bit of light will hit the Earth in $\dfrac{6.44\times 10^{18}\text{ m}}{3.00\times 10^8\text{ m/s}} = 2.15 \times 10^{10}$ s = 680 years. Therefore, it will disappear from the sky in the year 1999 + 680 = $\boxed{2.68 \times 10^3 \text{ A.D.}}$

34.2

$$v = \frac{1}{\sqrt{k\mu_0 \epsilon_0}} = \frac{1}{\sqrt{1.78}}c = 0.750c = \boxed{2.25\times 10^8\text{ m/s}}$$

34.3 $\dfrac{E}{B} = c$ or $\dfrac{220}{B} = 3.00\times 10^8$; so $B = 7.33\times 10^{-7}\text{ T} = \boxed{733\text{ nT}}$

34.4 $\dfrac{E_{\max}}{B_{\max}} = v$ is the generalized version of Equation 34.13.

$$B_{\max} = \frac{E_{\max}}{v} = \frac{7.60\times 10^{-3}\text{ V/m}}{(2/3)(3.00\times 10^8\text{ m/s})}\left(\frac{\text{N}\cdot\text{m}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right) = 3.80\times 10^{-11}\text{ T} = \boxed{38.0\text{ pT}}$$

34.5 (a) $f\lambda = c$ or $f(50.0\text{ m}) = 3.00\times 10^8$ m/s so $\boxed{f = 6.00\times 10^6\text{ Hz} = 6.00\text{ MHz}}$

(b) $\dfrac{E}{B} = c$ or $\dfrac{22.0}{B_{\max}} = 3.00\times 10^8$ so $\mathbf{B}_{\max} = \boxed{(73.3\text{ nT})(-\mathbf{k})}$

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{50.0} = 0.126\text{ m}^{-1}$ and $\omega = 2\pi f = 2\pi(6.00\times 10^6\text{ s}^{-1}) = 3.77\times 10^7$ rad/s

$$\mathbf{B} = \mathbf{B}_{\max}\cos(kx - \omega t) = \boxed{(73.3\text{ nT})\cos(0.126x - 3.77\times 10^7 t)(-\mathbf{k})}$$

34.6 $\omega = 2\pi f = 6.00\pi\times 10^9\text{ s}^{-1} = 1.88\times 10^{10}\text{ s}^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{6.00\pi\times 10^9\text{ s}^{-1}}{3.00\times 10^8\text{ m/s}} = 20.0\pi = 62.8\text{ m}^{-1} \qquad B_{\max} = \frac{E}{c} = \frac{300\text{ V/m}}{3.00\times 10^8\text{ m/s}} = 1.00\ \mu\text{T}$$

$$\boxed{E = \left(300\ \frac{\text{V}}{\text{m}}\right)\cos\left(62.8x - 1.88\times 10^{10}t\right)} \qquad \boxed{B = (1.00\ \mu\text{T})\cos(62.8x - 1.88\times 10^{10}t)}$$

34.7 (a) $B = \frac{E}{c} = \frac{100\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = 3.33 \times 10^{-7}\ \text{T} = \boxed{0.333\ \mu\text{T}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.00 \times 10^7\ \text{m}^{-1}} = \boxed{0.628\ \mu\text{m}}$

(c) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{6.28 \times 10^{-7}\ \text{m}} = \boxed{4.77 \times 10^{14}\ \text{Hz}}$

34.8 $E = E_{\max} \cos(kx - \omega t)$

$\frac{\partial E}{\partial x} = -E_{\max} \sin(kx - \omega t)(k)$ $\qquad \frac{\partial E}{\partial t} = -E_{\max} \sin(kx - \omega t)(-\omega)$

$\frac{\partial^2 E}{\partial x^2} = -E_{\max} \cos(kx - \omega t)(k^2)$ $\qquad \frac{\partial^2 E}{\partial t^2} = -E_{\max} \cos(kx - \omega t)(-\omega)^2$

We must show: $\frac{\partial E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

That is, $-\left(k^2\right) E_{\max} \cos\left(kx - \omega t\right) = -\mu_0 \epsilon_0 (-\omega)^2 E_{\max} \cos\left(kx - \omega t\right)$

But this is true, because $\frac{k^2}{\omega^2} = \left(\frac{1}{f\lambda}\right)^2 = \frac{1}{c^2} = \mu_0 \epsilon_0$

The proof for the wave of magnetic field is precisely similar.

***34.9** In the fundamental mode, there is a single loop in the standing wave between the plates. Therefore, the distance between the plates is equal to half a wavelength.

$\lambda = 2L = 2(2.00\ \text{m}) = 4.00\ \text{m}$

Thus, $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{4.00\ \text{m}} = 7.50 \times 10^7\ \text{Hz} = \boxed{75.0\ \text{MHz}}$

***34.10** $d_{\text{A to A}} = 6\ \text{cm} \pm 5\% = \frac{\lambda}{2}$

$\lambda = 12\ \text{cm} \pm 5\%$

$$v = \lambda f = (0.12 \text{ m} \pm 5\%)(2.45 \times 10^9 \text{ s}^{-1}) = \boxed{2.9 \times 10^8 \text{ m/s} \pm 5\%}$$

34.11 $S = I = \dfrac{U}{At} = \dfrac{Uc}{V} = uc$

$$\frac{\text{Energy}}{\text{Unit Volume}} = u = \frac{I}{c} = \frac{1000 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = \boxed{3.33 \ \mu\text{J/m}^3}$$

34.12 $S_{\text{av}} = \dfrac{\overline{\mathcal{P}}}{4\pi r^2} = \dfrac{4.00 \times 10^3 \text{ W}}{4\pi(4.00 \times 1609 \text{ m})^2} = 7.68 \ \mu\text{W/m}^2$

$$E_{\max} = \sqrt{2\mu_0 c S_{av}} = 0.0761 \text{ V/m}$$

$$\Delta V_{\max} = E_{\max} \cdot L = (76.1 \text{ mV/m})(0.650 \text{ m}) = \boxed{49.5 \text{ mV (amplitude)}} \quad \text{or} \quad 35.0 \text{ mV (rms)}$$

34.13 $r = (5.00 \text{ mi})(1609 \text{ m/mi}) = 8.04 \times 10^3 \text{ m}$

$$S = \frac{\overline{\mathcal{P}}}{4\pi r^2} = \frac{250 \times 10^3 \text{ W}}{4\pi(8.04 \times 10^3 \text{ m})^2} = \boxed{307 \ \mu\text{W/m}^2}$$

Goal Solution

What is the average magnitude of the Poynting vector 5.00 miles from a radio transmitter broadcasting isotropically with an average power of 250 kW?

G: As the distance from the source is increased, the power per unit area will decrease, so at a distance of 5 miles from the source, the power per unit area will be a small fraction of the Poynting vector near the source.

O: The Poynting vector is the power per unit area, where A is the surface area of a sphere with a 5-mile radius.

A: The Poynting vector is $S_{av} = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\pi r^2}$

In meters, $r = (5.00 \text{ mi})(1609 \text{ m / mi}) = 8045 \text{ m}$

and the magnitude is $S = \frac{250 \times 10^3 \text{ W}}{(4\pi)(8045)^2} = 3.07 \times 10^{-4} \text{ W / m}^2$

L: The magnitude of the Poynting vector ten meters from the source is 199 W/m^2, on the order of a million times larger than it is 5 miles away! It is surprising to realize how little power is actually received by a radio (at the 5-mile distance, the signal would only be about 30 nW, assuming a receiving area of about 1 cm^2).

34.14 $I = \frac{100 \text{ W}}{4\pi(1.00 \text{ m})^2} = 7.96 \text{ W/m}^2$

$u = \frac{I}{c} = 2.65 \times 10^{-8} \text{ J/m}^3 = 26.5 \text{ nJ/m}^3$

(a) $u_E = \frac{1}{2} u = \boxed{13.3 \text{ nJ/m}^3}$

(b) $u_B = \frac{1}{2} u = \boxed{13.3 \text{ nJ/m}^3}$

(c) $I = \boxed{7.96 \text{ W/m}^2}$

34.15 Power output = (power input)(efficiency)

Thus, Power input = $\frac{\text{power out}}{\text{eff}} = \frac{1.00 \times 10^6 \text{ W}}{0.300} = 3.33 \times 10^6 \text{ W}$

and $A = \frac{\mathcal{P}}{I} = \frac{3.33 \times 10^6 \text{ W}}{1.00 \times 10^3 \text{ W/m}^2} = \boxed{3.33 \times 10^3 \text{ m}^2}$

***34.16** $I = \dfrac{B_{max}^2 c}{2\mu_0} = \dfrac{P}{4\pi r^2}$

$$B_{max} = \sqrt{\left(\frac{P}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\frac{(10.0\times 10^3)(2)(4\pi\times 10^{-7})}{4\pi(5.00\times 10^3)^2(3.00\times 10^8)}} = \boxed{5.16\times 10^{-10}\ \text{T}}$$

Since the magnetic field of the Earth is approximately 5×10^{-5} T, the Earth's field is some 100,000 times stronger.

34.17 (a) $P = I^2R = 150$ W; $A = 2\pi rL = 2\pi(0.900 \times 10^{-3}\ \text{m})(0.0800\ \text{m}) = 4.52 \times 10^{-4}\ \text{m}^2$

$S = \dfrac{P}{A} = \boxed{332\ \text{kW/m}^2}$ (points radially inward)

(b) $B = \mu_0 \dfrac{I}{2\pi r} = \dfrac{\mu_0(1.00)}{2\pi(0.900\times 10^{-3})} = \boxed{222\ \mu\text{T}}$

$E = \dfrac{\Delta V}{\Delta x} = \dfrac{IR}{L} = \dfrac{150\ \text{V}}{0.0800\ \text{m}} = \boxed{1.88\ \text{kV/m}}$

Note: $S = \dfrac{EB}{\mu_0} = 332\ \text{kW/m}^2$

34.18 (a) $\mathbf{E}\cdot\mathbf{B} = (80.0\mathbf{i} + 32.0\mathbf{j} - 64.0\mathbf{k})(\text{N/C})\cdot(0.200\mathbf{i} + 0.0800\mathbf{j} + 0.290\mathbf{k})\mu\text{T}$

$$\mathbf{E}\cdot\mathbf{B} = (16.0 + 2.56 - 18.56)\text{N}^2\cdot\text{s/C}^2\cdot\text{m} = \boxed{0}$$

(b) $$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{(80.0\mathbf{i} + 32.0\mathbf{j} - 64.0\mathbf{k})(\text{N/C})\times(0.200\mathbf{i} + 0.0800\mathbf{j} + 0.290\mathbf{k})\mu\text{T}}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}$$

$$\mathbf{S} = \frac{(6.40\mathbf{k} - 23.2\mathbf{j} - 6.40\mathbf{k} + 9.28\mathbf{i} - 12.8\mathbf{j} + 5.12\mathbf{i})10^{-6}\ \text{W/m}^2}{4\pi\times10^{-7}}$$

$$\mathbf{S} = \boxed{(11.5\mathbf{i} - 28.6\mathbf{j})\ \text{W/m}^2} = 30.9\ \text{W/m}^2 \text{ at } -68.2° \text{ from the } +x \text{ axis}$$

34.19 We call the current I_{rms} and the intensity I. The power radiated at this frequency is

$$\mathcal{P} = (0.0100)(\Delta V_{\text{rms}})I_{\text{rms}} = \frac{0.0100(\Delta V_{\text{rms}})^2}{R} = 1.31\ \text{W}$$

If it is isotropic, the intensity one meter away is

$$I = \frac{\mathcal{P}}{A} = \frac{1.31\ \text{W}}{4\pi(1.00\ \text{m})^2} = 0.104\ \text{W/m}^2 = S_{\text{av}} = \frac{c}{2\mu_0}B_{\max}^2$$

$$B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.104\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}}} = \boxed{29.5\ \text{nT}}$$

***34.20** (a) $$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}\times100\% = \left(\frac{700\ \text{W}}{1400\ \text{W}}\right)\times100\% = \boxed{50.0\%}$$

(b) $$S_{\text{av}} = \frac{\mathcal{P}}{A} = \frac{700\ \text{W}}{(0.0683\ \text{m})(0.0381\ \text{m})} = 2.69\times10^5\ \text{W/m}^2$$

$$\mathbf{S}_{\text{av}} = \boxed{269\ \text{kW/m}^2 \text{ toward the oven chamber}}$$

(c) $$S_{\text{av}} = \frac{E_{\max}^2}{2\mu_0 c}$$

$$E_{\max} = \sqrt{2\left(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)\left(2.69\times10^5\ \frac{\text{W}}{\text{m}^2}\right)} = 1.42\times10^4\ \frac{\text{V}}{\text{m}} = \boxed{14.2\ \text{kV/m}}$$

34.21 (a) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{7.00 \times 10^5 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.33 \text{ mT}}$

(b) $I = \dfrac{E_{max}^2}{2\mu_0 c} = \dfrac{(7.00 \times 10^5)^2}{2(4\pi \times 10^{-7})(3.00 \times 10^8)} = \boxed{650 \text{ MW/m}^2}$

(c) $I = \dfrac{\mathcal{P}}{A}$: $\mathcal{P} = IA = (6.50 \times 10^8 \text{ W/m}^2)\dfrac{\pi}{4}(1.00 \times 10^{-3} \text{ m})^2 = \boxed{510 \text{ W}}$

34.22 Power $= SA = \dfrac{E_{max}^2}{2\mu_0 c}(4\pi r^2)$; solving for r, $r = \sqrt{\dfrac{\mathcal{P}\mu_0 c}{E_{max}^2 2\pi}} = \sqrt{\dfrac{(100 \text{ W})\mu_0 c}{2\pi(15.0 \text{ V/m})^2}} = \boxed{5.16 \text{ m}}$

34.23 (a) $I = \dfrac{(10.0 \times 10^{-3})\text{W}}{\pi(0.800 \times 10^{-3} \text{ m})^2} = \boxed{4.97 \text{ kW/m}^2}$

(b) $u_{av} = \dfrac{I}{c} = \dfrac{4.97 \times 10^3 \text{ J/m}^2 \cdot \text{s}}{3.00 \times 10^8 \text{ m/s}} = \boxed{16.6 \ \mu\text{J/m}^3}$

34.24 (a) $E = cB = (3.00 \times 10^8 \text{ m/s})(1.80 \times 10^{-6} \text{ T}) = \boxed{540 \text{ V/m}}$

(b) $u_{av} = \dfrac{B^2}{\mu_0} = \dfrac{(1.80 \times 10^{-6})^2}{4\pi \times 10^{-7}} = \boxed{2.58 \ \mu\text{J/m}^3}$

(c) $S_{av} = cu_{av} = (3.00 \times 10^8)(2.58 \times 10^{-6}) = \boxed{773 \text{ W/m}^2}$

(d) This is $\boxed{\text{77.3\% of the flux in Example 34.5}}$. It may be cloudy, or the Sun may be setting.

34.25 For complete absorption, $P = \dfrac{S}{c} = \dfrac{25.0}{3.00 \times 10^8} = \boxed{83.3 \text{ nPa}}$

***34.26** (a) $\mathcal{P} = (S_{av})(A) = (6.00 \text{ W/m}^2)(40.0 \times 10^{-4} \text{ m}^2) = 2.40 \times 10^{-2} \text{ J/s}$

In one second, the total energy U impinging on the mirror is 2.40×10^{-2} J. The momentum p transferred each second for total reflection is

$$p = \frac{2U}{c} = \frac{2(2.40 \times 10^{-2} \text{ J})}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.60 \times 10^{-10} \ \frac{\text{kg} \cdot \text{m}}{\text{s}}} \quad \text{(each second)}$$

(b) $F = \dfrac{dp}{dt} = \dfrac{1.60 \times 10^{-10} \text{ kg} \cdot \text{m/s}}{1 \text{ s}} = \boxed{1.60 \times 10^{-10} \text{ N}}$

34.27 (a) The radiation pressure is $\dfrac{(2)(1340\text{ W / m}^2)}{3.00\times10^8\text{ m / s}^2} = 8.93\times10^{-6}\text{ N / m}^2$

Multiplying by the total area, $A = 6.00\times10^5\text{ m}^2$ gives: $F = \boxed{5.36\text{ N}}$

(b) The acceleration is: $a = \dfrac{F}{m} = \dfrac{5.36\text{ N}}{6000\text{ kg}} = \boxed{8.93\times10^{-4}\text{ m/s}^2}$

(c) It will take a time t where: $d = \frac{1}{2}at^2$

or $t = \sqrt{\dfrac{2d}{a}} = \sqrt{\dfrac{2\left(3.84\times10^8\text{ m}\right)}{\left(8.93\times10^{-4}\text{ m / s}^2\right)}} = 9.27\times10^5\text{ s} = \boxed{10.7\text{ days}}$

34.28 The pressure P upon the mirror is $P = \dfrac{2S_{\text{av}}}{c}$

where A is the cross-sectional area of the beam and $S_{\text{av}} = \dfrac{\mathcal{P}}{A}$

The force on the mirror is then $F = PA = \dfrac{2}{c}\left(\dfrac{\mathcal{P}}{A}\right)A = \dfrac{2\mathcal{P}}{c}$

Therefore, $F = \dfrac{2(100\times10^{-3})}{(3\times10^8)} = \boxed{6.67\times10^{-10}\text{ N}}$

34.29 $I = \dfrac{\mathcal{P}}{\pi r^2} = \dfrac{E_{\max}^2}{2\mu_0 c}$

(a) $E_{\max} = \sqrt{\dfrac{\mathcal{P}(2\mu_0 c)}{\pi r^2}} = \boxed{1.90\text{ kN/C}}$

(b) $\dfrac{15\times10^{-3}\text{ J/s}}{3.00\times10^8\text{ m/s}}(1.00\text{ m}) = \boxed{50.0\text{ pJ}}$

(c) $p = \dfrac{U}{c} = \dfrac{5\times10^{-11}}{3.00\times10^8} = \boxed{1.67\times10^{-19}\text{ kg}\cdot\text{m/s}}$

34.30 (a) If $\mathcal{P}_S$ is the total power radiated by the Sun, and r_E and r_M are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{\mathcal{P}_S}{4\pi r_E^2} \quad \text{and} \quad I_M = \frac{\mathcal{P}_S}{4\pi r_M^2}$$

Thus, $$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1340\ \text{W/m}^2\right)\left(\frac{1.496\times10^{11}\ \text{m}}{2.28\times10^{11}\ \text{m}}\right)^2 = \boxed{577\ \text{W/m}^2}$$

(b) Mars intercepts the power falling on its circular face:

$$\mathcal{P}_M = I_M\left(\pi R_M^2\right) = \left(577\ \text{W/m}^2\right)\pi\left(3.37\times10^6\ \text{m}\right)^2 = \boxed{2.06\times10^{16}\ \text{W}}$$

(c) If Mars behaves as a perfect absorber, it feels pressure $P = \frac{S_M}{c} = \frac{I_M}{c}$

and force $F = PA = \frac{I_M}{c}\left(\pi R_M^2\right) = \frac{\mathcal{P}_M}{c} = \frac{2.06\times10^{16}\ \text{W}}{3.00\times10^8\ \text{m/s}} = \boxed{6.87\times10^7\ \text{N}}$

(d) The attractive gravitational force exerted on Mars by the Sun is

$$F_g = \frac{GM_SM_M}{r_M^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(6.42\times10^{23}\ \text{kg}\right)}{\left(2.28\times10^{11}\ \text{m}\right)^2} = 1.64\times10^{21}\ \text{N}$$

which is $\boxed{\sim10^{13}\ \text{times stronger}}$ than the repulsive force of (c).

34.31 (a) The total energy absorbed by the surface is

$$U = \left(\tfrac{1}{2}I\right)At = \left[\tfrac{1}{2}\left(750\ \frac{\text{W}}{\text{m}^2}\right)\right]\left(0.500\times1.00\ \text{m}^2\right)\left(60.0\ \text{s}\right) = \boxed{11.3\ \text{kJ}}$$

(b) The total energy incident on the surface in this time is $2U = 22.5$ kJ, with $U = 11.3$ kJ being absorbed and $U = 11.3$ kJ being reflected. The total momentum transferred to the surface is

$$p = \left(\text{momentum from absorption}\right) + \left(\text{momentum from reflection}\right)$$

$$p = \left(\frac{U}{c}\right) + \left(\frac{2U}{c}\right) = \frac{3U}{c} = \frac{3\left(11.3\times10^3\ \text{J}\right)}{3.00\times10^8\ \text{m/s}} = \boxed{1.13\times10^{-4}\ \text{kg}\cdot\text{m/s}}$$

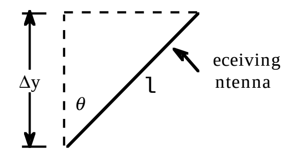
34.32 $$S_{\text{av}} = \frac{\mu_0 J_{\max}^2 c}{8} \quad \text{or} \quad 570 = \frac{(4\pi\times10^{-7})J_{\max}^2(3.00\times10^8)}{8} \quad \text{so} \quad \boxed{J_{\max} = 3.48\ \text{A/m}^2}$$

34.33 (a) $\mathcal{P} = S_{av}A = \left(\frac{\mu_0 J_{max}^2 c}{8}\right)A$

$$\mathcal{P} = \left(\frac{4\pi\times10^{-7}(10.0)^2(3.00\times10^8)}{8}\right)(1.20\times0.400) = \boxed{2.26\text{ kW}}$$

(b) $S_{av} = \frac{\mu_0 J_{max}^2 c}{8} = \frac{(4\pi\times10^{-7}(10.0)^2(3.00\times10^8)}{8} = \boxed{4.71\text{ kW/m}^2}$

***34.34** $\mathcal{P} = \frac{(\Delta V)^2}{R}$ or $\mathcal{P} \propto (\Delta V)^2$



$\Delta V = (-)E_y \cdot \Delta y = E_y \cdot \ell\cos\theta$

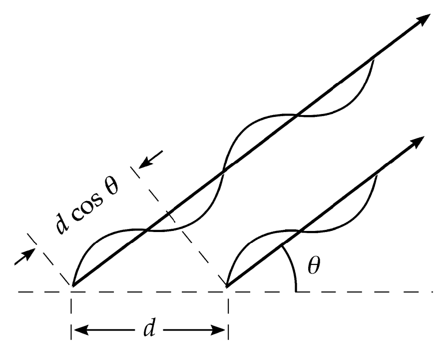
$\Delta V \propto \cos\theta$ so $\mathcal{P} \propto \cos^2\theta$

(a) $\theta = 15.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(15.0°) = 0.933\mathcal{P}_{max} = \boxed{93.3\%}$

(b) $\theta = 45.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(45.0°) = 0.500\mathcal{P}_{max} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(90.0°) = \boxed{0}$

34.35 (a) Constructive interference occurs when $d\cos\theta = n\lambda$ for some integer n.



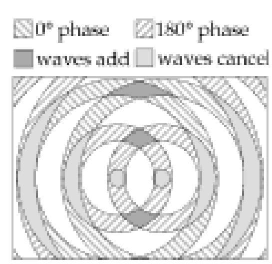
$$\cos\theta = n\frac{\lambda}{d} = n\left(\frac{\lambda}{\lambda/2}\right) = 2n \quad n = 0, \pm1, \pm2, \ldots$$

$\therefore$ $\boxed{\text{strong signal @ } \theta = \cos^{-1} 0 = 90°, 270°}$

(b) Destructive interference occurs when

$d\cos\theta = \left(\frac{2n+1}{2}\right)\lambda$: $\cos\theta = 2n + 1$

$\therefore$ $\boxed{\text{weak signal @ } \theta = \cos^{-1}(\pm1) = 0°, 180°}$



Goal Solution

Two radio-transmitting antennas are separated by half the broadcast wavelength and are driven in phase with each other. In which directions are (a) the strongest and (b) the weakest signals radiated?

G: The strength of the radiated signal will be a function of the location around the two antennas and will depend on the interference of the waves.

O: A diagram helps to visualize this situation. The two antennas are driven in phase, which means that they both create maximum electric field strength at the same time, as shown in the diagram. The radio EM waves travel radially outwards from the antennas, and the received signal will be the vector sum of the two waves.

A: (a) Along the perpendicular bisector of the line joining the antennas, the distance is the same to both transmitting antennas. The transmitters oscillate in phase, so along this line the two signals will be received in phase, constructively interfering to produce a maximum signal strength that is twice the amplitude of one transmitter.

(b) Along the extended line joining the sources, the wave from the more distant antenna must travel one-half wavelength farther, so the waves are received 180° out of phase. They interfere destructively to produce the weakest signal with zero amplitude.

L: Radio stations may use an antenna array to direct the radiated signal toward a highly-populated region and reduce the signal strength delivered to a sparsely-populated area.

34.36 $\lambda = \frac{c}{f} = 536 \text{ m}$ so $h = \frac{\lambda}{4} = \boxed{134 \text{ m}}$

$\lambda = \frac{c}{f} = 188 \text{ m}$ so $h = \frac{\lambda}{4} = \boxed{46.9 \text{ m}}$

34.37 For the proton: $\Sigma F = ma \Rightarrow qvB \sin 90.0° = mv^2/R$

The period and frequency of the proton's circular motion are therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} = \frac{2\pi(1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ T})} = 1.87 \times 10^{-7} \text{ s} \qquad f = 5.34 \times 10^6 \text{ Hz}.$$

The charge will radiate at this same frequency, with $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.34 \times 10^6 \text{ Hz}} = \boxed{56.2 \text{ m}}$

34.38 For the proton, $\Sigma F = ma$ yields $qvB \sin 90.0° = \frac{mv^2}{R}$

The period of the proton's circular motion is therefore: $T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$

The frequency of the proton's motion is $f = 1 / T$

The charge will radiate electromagnetic waves at this frequency, with $\lambda = \frac{c}{f} = cT = \boxed{\frac{2\pi mc}{qB}}$

***34.39** From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

Frequency f	Wavelength, $\lambda = c/f$	Classification
$2 \text{ Hz} = 2 \times 10^0 \text{ Hz}$	150 Mm	Radio
$2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$	150 km	Radio
$2 \text{ MHz} = 2 \times 10^6 \text{ Hz}$	150 m	Radio
$2 \text{ GHz} = 2 \times 10^9 \text{ Hz}$	15 cm	Microwave
$2 \text{ THz} = 2 \times 10^{12} \text{ Hz}$	150 μm	Infrared
$2 \text{ PHz} = 2 \times 10^{15} \text{ Hz}$	150 nm	Ultraviolet
$2 \text{ EHz} = 2 \times 10^{18} \text{ Hz}$	150 pm	x-ray
$2 \text{ ZHz} = 2 \times 10^{21} \text{ Hz}$	150 fm	Gamma ray
$2 \text{ YHz} = 2 \times 10^{24} \text{ Hz}$	150 am	Gamma Ray

Wavelength, λ	Frequency $f = c/\lambda$	Classification
$2 \text{ km} = 2 \times 10^3 \text{ m}$	$1.5 \times 10^5 \text{ Hz}$	Radio
$2 \text{ m} = 2 \times 10^0 \text{ m}$	$1.5 \times 10^8 \text{ Hz}$	Radio
$2 \text{ mm} = 2 \times 10^{-3} \text{ m}$	$1.5 \times 10^{11} \text{ Hz}$	Microwave
$2 \text{ } \mu\text{m} = 2 \times 10^{-6} \text{ m}$	$1.5 \times 10^{14} \text{ Hz}$	Infrared
$2 \text{ nm} = 2 \times 10^{-9} \text{ m}$	$1.5 \times 10^{17} \text{ Hz}$	Ultraviolet/x-ray
$2 \text{ pm} = 2 \times 10^{-12} \text{ m}$	$1.5 \times 10^{20} \text{ Hz}$	x-ray/Gamma ray
$2 \text{ fm} = 2 \times 10^{-15} \text{ m}$	$1.5 \times 10^{23} \text{ Hz}$	Gamma ray
$2 \text{ am} = 2 \times 10^{-18} \text{ m}$	$1.5 \times 10^{26} \text{ Hz}$	Gamma ray

***34.40** (a) $f = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8 \text{ m/s}}{1.7 \text{ m}}$ $\boxed{\sim 10^8 \text{ Hz}}$ $\boxed{\text{radio wave}}$

(b) 1000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick

$$f = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{-5} \text{ m}} \; \boxed{\sim 10^{13} \text{ Hz}} \; \boxed{\text{infrared}}$$

***34.41** $f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^8 \text{ m/s}}{5.50 \times 10^{-7} \text{ m}} = \boxed{5.45 \times 10^{14} \text{ Hz}}$

34.42 (a) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1150 \times 10^3/\text{s}} = 261 \text{ m}$ so $\dfrac{180 \text{ m}}{261 \text{ m}} = \boxed{0.690 \text{ wavelengths}}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{98.1 \times 10^6/\text{s}} = 3.06 \text{ m}$ so $\dfrac{180 \text{ m}}{3.06 \text{ m}} = \boxed{58.9 \text{ wavelengths}}$

34.43 (a) $f\lambda = c$ gives $(5.00 \times 10^{19}\text{ Hz})\lambda = 3.00 \times 10^{8}\text{ m/s}$: $\boxed{\lambda = 6.00 \times 10^{-12}\text{ m} = 6.00\text{ pm}}$

(b) $f\lambda = c$ gives $(4.00 \times 10^{9}\text{ Hz})\lambda = 3.00 \times 10^{8}\text{ m/s}$: $\boxed{\lambda = 0.075\text{ m} = 7.50\text{ cm}}$

***34.44** Time to reach object $= \frac{1}{2}$(total time of flight) $= \frac{1}{2}(4.00 \times 10^{-4}\text{ s}) = 2.00 \times 10^{-4}\text{ s}$

Thus, $d = vt = (3.00 \times 10^{8}\text{ m/s})(2.00 \times 10^{-4}\text{ s}) = 6.00 \times 10^{4}\text{ m} = \boxed{60.0\text{ km}}$

34.45 The time for the radio signal to travel 100 km is: $t_r = \dfrac{100 \times 10^{3}\text{ m}}{3.00 \times 10^{8}\text{ m/s}} = 3.33 \times 10^{-4}\text{ s}$

The sound wave to travel 3.00 m across the room in: $t_s = \dfrac{3.00\text{ m}}{343\text{ m/s}} = 8.75 \times 10^{-3}\text{ s}$

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the newsroom by a total time difference of

$\Delta t = 8.75 \times 10^{-3}\text{ s} - 3.33 \times 10^{-4}\text{ s} = 8.41 \times 10^{-3}\text{ s}$

***34.46** The wavelength of an ELF wave of frequency 75.0 Hz is $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{75.0\text{ Hz}} = 4.00 \times 10^{6}\text{ m}$

The length of a quarter-wavelength antenna would be $L = 1.00 \times 10^{6}\text{ m} = \boxed{1.00 \times 10^{3}\text{ km}}$

or $L = (1000\text{ km})\left(\dfrac{0.621\text{ mi}}{1.00\text{ km}}\right) = \boxed{621\text{ mi}}$

Thus, while the project may be theoretically possible, it is not very practical.

34.47 (a) For the AM band, $\lambda_{\text{max}} = \dfrac{c}{f_{\text{min}}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{540 \times 10^{3}\text{ Hz}} = \boxed{556\text{ m}}$

$$\lambda_{\text{min}} = \frac{c}{f_{\text{max}}} = \frac{3.00 \times 10^{8}\text{ m/s}}{1600 \times 10^{3}\text{ Hz}} = \boxed{187\text{ m}}$$

(b) For the FM band, $\lambda_{\text{max}} = \dfrac{c}{f_{\text{min}}} = \dfrac{3.00 \times 10^{8}\text{ m/s}}{88.0 \times 10^{6}\text{ Hz}} = \boxed{3.41\text{ m}}$

$$\lambda_{\text{min}} = \frac{c}{f_{\text{max}}} = \frac{3.00 \times 10^{8}\text{ m/s}}{108 \times 10^{6}\text{ Hz}} = \boxed{2.78\text{ m}}$$

34.48 CH_4: $f_{min} = 66$ MHz $\qquad \lambda_{max} = \boxed{4.55 \text{ m}}$

$f_{max} = 72$ MHz $\qquad \lambda_{min} = \boxed{4.17 \text{ m}}$

CH_6: $f_{min} = 82$ MHz $\qquad \lambda_{max} = \boxed{3.66 \text{ m}}$

$f_{max} = 88$ MHz $\qquad \lambda_{min} = \boxed{3.41 \text{ m}}$

CH_8: $f_{min} = 180$ MHz $\qquad \lambda_{max} = \boxed{1.67 \text{ m}}$

$f_{max} = 186$ MHz $\qquad \lambda_{min} = \boxed{1.61 \text{ m}}$

34.49 (a) $\mathcal{P} = SA = (1340 \text{ W/m}^2)4\pi(1.496 \times 10^{11} \text{ m})^2 = \boxed{3.77 \times 10^{26} \text{ W}}$

(b) $S = \dfrac{cB_{max}^2}{2\mu_0}$ so $B_{max} = \sqrt{\dfrac{2\mu_0 S}{c}} = \sqrt{\dfrac{2(4\pi \times 10^{-7} \text{ N/A}^2)(1340 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}}} = \boxed{3.35\ \mu\text{T}}$

$S = \dfrac{E_{max}^2}{2\mu_0 c}$ so $E_{max} = \sqrt{2\mu_0 cS} = \sqrt{2\left(4\pi \times 10^{-7}\right)\left(3.00 \times 10^8\right)(1340)} = \boxed{1.01 \text{ kV/m}}$

***34.50** Suppose you cover a 1.7 m-by-0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is

$(1.7 \text{ m})(0.3 \text{ m})(\cos 30°) = 0.4 \text{ m}^2$

Now $I = \dfrac{\mathcal{P}}{A} = \dfrac{E}{At}$: $\qquad E = IAt = 1340 \dfrac{\text{W}}{\text{m}^2}(0.6)(0.5)(0.4 \text{ m}^2)$ 3600 s $\boxed{\sim 10^6 \text{ J}}$

34.51 (a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(BA\cos\theta) = -A\dfrac{d}{dt}(B_{max}\cos\omega t\cos\theta) = AB_{max}\,\omega(\sin\omega t\cos\theta)$

$\mathcal{E}(t) = 2\pi f B_{max} A \sin 2\pi f t \cos\theta = 2\pi^2 r^2 f B_{max} \cos\theta \sin 2\pi f t$

Thus, $\boxed{\mathcal{E}_{max} = 2\pi^2 r^2 f B_{max}\cos\theta}$, where θ is the angle between the magnetic field and the normal to the loop.

(b) If **E** is vertical, then **B** is horizontal, so the $\boxed{\text{plane of the loop should be vertical}}$ and the $\boxed{\text{plane should contain the line of sight to the transmitter}}$.

34.52 (a) $F_{grav} = \dfrac{GM_s m}{R^2} = \left(\dfrac{GM_s}{R^2}\right)\rho(4/3)\pi r^3$

where M_s = mass of Sun, r = radius of particle and R = distance from Sun to particle.

Since $F_{\text{rad}} = \dfrac{S\pi r^2}{c}$, $\quad \dfrac{F_{rad}}{F_{grav}} = \left(\dfrac{1}{r}\right)\left(\dfrac{3SR^2}{4cGM_s\rho}\right) \propto \dfrac{1}{r}$

(b) From the result found in part (a), when $F_{\text{grav}} = F_{\text{rad}}$, we have $\quad r = \dfrac{3SR^2}{4cGM_s\,\rho}$

$$r = \frac{3\left(214\text{ W/m}^2\right)\left(3.75\times10^{11}\text{ m}\right)^2}{4\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(1500\text{ kg/m}^3\right)\left(3.00\times10^{8}\text{ m/s}\right)} = \boxed{3.78\times10^{-7}\text{ m}}$$

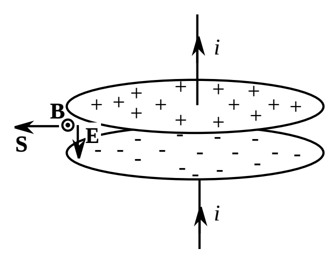
34.53 (a) $B_{\max} = \dfrac{E_{\max}}{c} = \boxed{6.67\times10^{-16}\text{ T}}$

(b) $S_{\text{av}} = \dfrac{E_{\max}^2}{2\mu_0 c} = \boxed{5.31\times10^{-17}\text{ W/m}^2}$

(c) $\mathcal{P} = S_{\text{av}} A = \boxed{1.67\times10^{-14}\text{ W}}$

(d) $F = PA = \left(\dfrac{S_{av}}{c}\right)A = \boxed{5.56\times10^{-23}\text{ N}}$ (≈ weight of 3000 H atoms!)

***34.54** (a) The electric field between the plates is $E = \Delta V/\ell$, directed downward in the figure. The magnetic field between the plate's edges is $B = \mu_0 i/2\pi r$ counterclockwise.



The Poynting vector is: $\quad \mathbf{S} = \dfrac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \boxed{\dfrac{(\Delta V)i}{2\pi r\ell}\text{ (radially outward)}}$

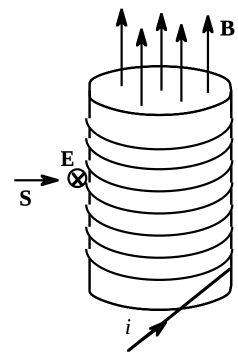
(b) The lateral surface area surrounding the electric field volume is

$A = 2\pi r\ell$, so the power output is $\mathcal{P} = SA = \left(\dfrac{(\Delta V)i}{2\pi r\ell}\right)(2\pi r\ell) = \boxed{(\Delta V)i}$

(c) As the capacitor charges, the polarity of the plates and hence the direction of the electric field is unchanged. Reversing the current reverses the direction of the magnetic field, and therefore the Poynting vector.

The Poynting vector is now directed radially inward.

***34.55** (a) The magnetic field in the enclosed volume is directed upward, with magnitude $B = \mu_0 ni$ and increasing at the rate $\frac{dB}{dt} = \mu_0 n \frac{di}{dt}$. The changing magnetic field induces an electric field around any circle of radius r, according to Faraday's Law:



$$E(2\pi r) = -\mu_0 n\left(\frac{di}{dt}\right)\left(\pi r^2\right) \qquad E = -\frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)$$

or $$\mathbf{E} = \frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right) \text{ (clockwise)}$$

Then, $$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = \frac{1}{\mu_0}\left[\frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)\right](\mu_0 ni) \text{ inward,}$$

or the Poynting vector is $$\mathbf{S} = \boxed{\frac{\mu_0 n^2 ri}{2}\left(\frac{di}{dt}\right) \text{ (radially inward)}}$$

(b) The power flowing into the volume is $\mathcal{P} = SA_{\text{lat}}$ where A_{lat} is the lateral area perpendicular to $\mathbf{S}$. Therefore,

$$\mathcal{P} = \left[\frac{\mu_0 n^2 ri}{2}\left(\frac{di}{dt}\right)\right](2\pi r\ell) = \boxed{\mu_0 \pi n^2 r^2 \ell i\left(\frac{di}{dt}\right)}$$

(c) Taking A_{cross} to be the cross-sectional area perpendicular to $\mathbf{B}$, the induced voltage between the ends of the inductor, which has $N = n\ell$ turns, is

$$\Delta V = |\mathcal{E}| = N\left(\frac{dB}{dt}\right)A_{\text{cross}} = n\ell\left(\mu_0 n\frac{di}{dt}\right)\left(\pi r^2\right) = \mu_0 \pi n^2 r^2 \ell\left(\frac{di}{dt}\right)$$

and it is observed that $\boxed{\mathcal{P} = (\Delta V)i}$

***34.56** (a) The power incident on the mirror is: $$\mathcal{P}_I = IA = \left(1340\ \frac{\text{W}}{\text{m}^2}\right)\left[\pi(100\text{ m})^2\right] = 4.21\times10^7\text{ W}$$

The power reflected through the atmosphere is $\mathcal{P}_R = 0.746\left(4.21\times10^7\text{ W}\right) = \boxed{3.14\times10^7\text{ W}}$

(b) $$S = \frac{\mathcal{P}_R}{A} = \frac{3.14\times10^7\text{ W}}{\pi\left(4.00\times10^3\text{ m}\right)^2} = \boxed{0.625\text{ W/m}^2}$$

(c) Noon sunshine in Saint Petersburg produces this power-per-area on a horizontal surface:

$$\mathcal{P}_N = 0.746\left(1340\text{ W / m}^2\right)\sin 7.00° = 122\text{ W / m}^2$$

The radiation intensity received from the mirror is

$$\left(\frac{0.625\ \text{W/m}^2}{122\ \text{W/m}^2}\right)100\% = \boxed{0.513\%}$$ of that from the noon Sun in January.

34.57 $u = \frac{1}{2}\epsilon_0 E_{max}^2$ (Equation 34.21)

$$E_{max} = \sqrt{\frac{2u}{\epsilon_0}} = \boxed{95.1\ \text{mV/m}}$$

***34.58** The area over which we model the antenna as radiating is the lateral surface of a cylinder,

$$A = 2\pi r \ell = 2\pi\left(4.00\times10^{-2}\ \text{m}\right)(0.100\ \text{m}) = 2.51\times10^{-2}\ \text{m}^2$$

(a) The intensity is then: $S = \frac{\mathcal{P}}{A} = \frac{0.600\ \text{W}}{2.51\times10^{-2}\ \text{m}^2} = \boxed{23.9\ \text{W/m}^2}$

(b) The standard is: $0.570\ \frac{\text{mW}}{\text{cm}^2} = 0.570\left(\frac{\text{mW}}{\text{cm}^2}\right)\left(\frac{1.00\times10^{-3}\ \text{W}}{1.00\ \text{mW}}\right)\left(\frac{1.00\times10^4\ \text{cm}^2}{1.00\ \text{m}^2}\right) = 5.70\ \frac{\text{W}}{\text{m}^2}$

While it is on, the telephone is over the standard by $\frac{23.9\ \text{W/m}^2}{5.70\ \text{W/m}^2} = \boxed{4.19\ \text{times}}$

34.59 (a) $B_{max} = \frac{E_{max}}{c} = \frac{175\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = \boxed{5.83\times10^{-7}\ \text{T}}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.0150\ \text{m})} = \boxed{419\ \text{rad/m}}$$

$$\omega = kc = \boxed{1.26\times10^{11}\ \text{rad/s}}$$

Since **S** is along x, and **E** is along y, **B** must be in $\boxed{\text{the } z \text{ direction}}$. (That is $\mathbf{S} \propto \mathbf{E}\times\mathbf{B}$.)

(b) $S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \boxed{40.6\ \text{W/m}^2}$

(c) $P_r = \frac{2S}{c} = \boxed{2.71\times10^{-7}\ \text{N/m}^2}$

(d) $a = \frac{\Sigma F}{m} = \frac{PA}{m} = \frac{\left(2.71\times10^{-7}\ \text{N/m}^2\right)\left(0.750\ \text{m}^2\right)}{0.500\ \text{kg}} = \boxed{4.06\times10^{-7}\ \text{m/s}^2}$

***34.60** (a) At steady-state, $\mathcal{P}_{in} = \mathcal{P}_{out}$ and the power radiated out is $\mathcal{P}_{out} = e\sigma A T^4$.

Thus, $$0.900\left(1000\ \frac{\text{W}}{\text{m}^2}\right)A = (0.700)\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)AT^4$$

or $$T = \left[\frac{900\ \text{W/m}^2}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{388\ \text{K}} = 115°\text{C}$$

(b) The box of horizontal area A, presents projected area $A\sin 50.0°$ perpendicular to the sunlight. Then by the same reasoning,

$$0.900\left(1000\ \frac{\text{W}}{\text{m}^2}\right)A\sin 50.0° = (0.700)\left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)AT^4$$

or $$T = \left[\frac{\left(900\ \text{W/m}^2\right)\sin 50.0°}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{363\ \text{K}} = 90.0\ °\text{C}$$

34.61 (a) $P = \dfrac{F}{A} = \dfrac{I}{c}$

$$F = \frac{IA}{c} = \frac{\mathcal{P}}{c} = \frac{100\ \text{J/s}}{3.00\times10^8\ \text{m/s}} = 3.33\times10^{-7}\ \text{N} = (110\ \text{kg})a$$

$a = 3.03\times10^{-9}\ \text{m/s}^2$ and $x = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2x}{a}} = 8.12\times10^4\ \text{s} = \boxed{22.6\ \text{h}}$$

(b) $0 = (107\ \text{kg})v - (3.00\ \text{kg})(12.0\ \text{m/s} - v) = (107\ \text{kg})v - 36.0\ \text{kg}\cdot\text{m/s} + (3.00\ \text{kg})v$

$$v = \frac{36.0}{110} = 0.327\ \text{m/s}$$

$t = \boxed{30.6\ \text{s}}$

Goal Solution

An astronaut, stranded in space 10.0 m from his spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Since he has a 100-W light source that forms a directed beam, he decides to use the beam as a photon rocket to propel himself continuously toward the spacecraft. (a) Calculate how long it takes him to reach the spacecraft by this method. (b) Suppose, instead, he decides to throw the light source away in a direction opposite the spacecraft. If the mass of the light source has a mass of 3.00 kg and, after being thrown, moves at 12.0 m/s **relative to the recoiling astronaut**, how long does it take for the astronaut to reach the spacecraft?

G: Based on our everyday experience, the force exerted by photons is too small to feel, so it may take a very long time (maybe days!) for the astronaut to travel 10 m with his "photon rocket." Using the momentum of the thrown light seems like a better solution, but it will still take a while (maybe a few minutes) for the astronaut to reach the spacecraft because his mass is so much larger than the mass of the light source.

O: In part (a), the radiation pressure can be used to find the force that accelerates the astronaut toward the spacecraft. In part (b), the principle of conservation of momentum can be applied to find the time required to travel the 10 m.

A: (a) Light exerts on the astronaut a pressure $P = F/A = S/c$, and a force of

$$F = \frac{SA}{c} = \frac{\mathcal{P}}{c} = \frac{100 \text{ J/s}}{3.00 \times 10^{8} \text{ m/s}} = 3.33 \times 10^{-7} \text{ N}$$

By Newton's 2nd law,

$$a = \frac{F}{m} = \frac{3.33 \times 10^{-7} \text{ N}}{110 \text{ kg}} = 3.03 \times 10^{-9} \text{ m/s}^2$$

This acceleration is constant, so the distance traveled is $x = \frac{1}{2}at^2$, and the amount of time it travels is

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10.0 \text{ m})}{3.03 \times 10^{-9} \text{ m/s}^2}} = 8.12 \times 10^{4} \text{ s} = 22.6 \text{ h}$$

(b) Because there are no external forces, the momentum of the astronaut before throwing the light is the same as afterwards when the now 107-kg astronaut is moving at speed v towards the spacecraft and the light is moving away from the spacecraft at $(12.0 \text{ m/s} - v)$. Thus, $\mathbf{p}_i = \mathbf{p}_f$ gives

$$0 = (107 \text{ kg})v - (3.00 \text{ kg})(12.0 \text{ m/s} - v)$$

$$0 = (107 \text{ kg})v - (36.0 \text{ kg} \cdot \text{m/s}) + (3.00 \text{ kg})v$$

$$v = \frac{36.0}{110} = 0.327 \text{ m/s}$$

$$t = \frac{x}{v} = \frac{10.0 \text{ m}}{0.327 \text{ m/s}} = 30.6 \text{ s}$$

L: Throwing the light away is certainly a more expedient way to reach the spacecraft, but there is not much chance of retrieving the lamp unless it has a very long cord. How long would the cord need to be, and does its length depend on how hard the astronaut throws the lamp? (You should verify that the minimum cord length is 367 m, independent of the speed that the lamp is thrown.)

34.62 The 38.0% of the intensity $S = 1340\ \frac{\text{W}}{\text{m}^2}$ that is reflected exerts a pressure $P_1 = \frac{2S_r}{c} = \frac{2(0.380)S}{c}$

The absorbed light exerts pressure $P_2 = \frac{S_a}{c} = \frac{(0.620)S}{c}$

Altogether the pressure at the subsolar point on Earth is

(a) $P_{tot} = P_1 + P_2 = \frac{1.38S}{c} = \frac{1.38(1340\ \text{W/m}^2)}{3.00 \times 10^8\ \text{m/s}} = \boxed{6.16 \times 10^{-6}\ \text{Pa}}$

(b) $\frac{P_a}{P_{tot}} = \frac{1.01 \times 10^5\ \text{N/m}^2}{6.16 \times 10^{-6}\ \text{N/m}^2} = \boxed{1.64 \times 10^{10}\ \text{times smaller than atmospheric pressure}}$

34.63 Think of light going up and being absorbed by the bead which presents a face area πr_b^2 .

The light pressure is $P = \frac{S}{c} = \frac{I}{c}$.

(a) $F_l = \frac{I\pi r_b^2}{c} = mg = \rho \frac{4}{3} \pi r_b^3\ g$ and $I = \frac{4\rho gc}{3}\left(\frac{3m}{4\pi\rho}\right)^{1/3} = \boxed{8.32 \times 10^7\ \text{W/m}^2}$

(b) $\mathcal{P} = IA = (8.32 \times 10^7\ \text{W/m}^2)\pi(2.00 \times 10^{-3}\ \text{m})^2 = \boxed{1.05\ \text{kW}}$

34.64 Think of light going up and being absorbed by the bead which presents face area πr_b^2 .

If we take the bead to be perfectly absorbing, the light pressure is $P = \frac{S_{av}}{c} = \frac{I}{c} = \frac{F_l}{A}$

(a) $F_l = F_g$ so $I = \frac{F_l c}{A} = \frac{F_g c}{A} = \frac{mgc}{\pi r_b^2}$

From the definition of density, $\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r_b^3}$ so $\frac{1}{r_b} = \left(\frac{4}{3}\pi\rho/m\right)^{1/3}$

Substituting for r_b, $I = \frac{mgc}{\pi}\left(\frac{4\pi\rho}{3m}\right)^{2/3} = gc\left(\frac{4\rho}{3}\right)^{2/3}\left(\frac{m}{\pi}\right)^{1/3} = \boxed{\frac{4\rho gc}{3}\left(\frac{3m}{4\pi\rho}\right)^{1/3}}$

(b) $\mathcal{P} = IA = \boxed{\frac{\pi r^2 4\rho gc}{3}\left(\frac{3m}{4\pi\rho}\right)^{1/3}}$

34.65 The mirror intercepts power

$$\mathcal{P} = I_1 A_1 = (1.00\times10^{3}\ \mathrm{W/m^2})\pi(0.500\ \mathrm{m})^2 = 785\ \mathrm{W}$$

In the image,

(a) $$I_2 = \frac{\mathcal{P}}{A_2} = \frac{785\ \mathrm{W}}{\pi(0.0200\ \mathrm{m})^2} = \boxed{625\ \mathrm{kW/m^2}}$$

(b) $$I_2 = \frac{E_{\max}^2}{2\mu_0 c} \quad \text{so} \quad E_{\max} = (2\mu_0 c\, I_2)^{1/2} = [2(4\pi\times10^{-7})(3.00\times10^{8})(6.25\times10^{5})]^{1/2} = \boxed{21.7\ \mathrm{kN/C}}$$

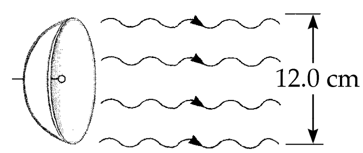
$$B_{\max} = \frac{E_{\max}}{c} = \boxed{72.4\ \mu\mathrm{T}}$$

(c) $$0.400\,\mathcal{P}\,t = mc\,\Delta T$$

$$0.400(785\ \mathrm{W})t = (1.00\ \mathrm{kg})\left(4186\ \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{C}^\circ}\right)(100^\circ\mathrm{C} - 20.0^\circ\mathrm{C})$$

$$t = \frac{3.35\times10^{5}\ \mathrm{J}}{314\ \mathrm{W}} = 1.07\times10^{3}\ \mathrm{s} = \boxed{17.8\ \mathrm{min}}$$

34.66 (a) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \mathrm{m/s}}{20.0\times10^{9}\ \mathrm{s^{-1}}} = \boxed{1.50\ \mathrm{cm}}$$



(b) $$U = \mathcal{P}(\Delta t) = \left(25.0\times10^{3}\ \frac{\mathrm{J}}{\mathrm{s}}\right)\left(1.00\times10^{-9}\ \mathrm{s}\right) = 25.0\times10^{-6}\ \mathrm{J} = \boxed{25.0\ \mu\mathrm{J}}$$

(c) $$u_{av} = \frac{U}{V} = \frac{U}{(\pi r^2)\ell} = \frac{U}{(\pi r^2)c(\Delta t)} = \frac{25.0\times10^{-6}\ \mathrm{J}}{\pi(0.0600\ \mathrm{m})^2\left(3.00\times10^{8}\ \mathrm{m/s}\right)\left(1.00\times10^{-9}\ \mathrm{s}\right)}$$

$$u_{av} = 7.37\times10^{-3}\ \mathrm{J/m^3} = \boxed{7.37\ \mathrm{mJ/m^3}}$$

(d) $$E_{\max} = \sqrt{\frac{2u_{av}}{\epsilon_0}} = \sqrt{\frac{2\left(7.37\times10^{-3}\ \mathrm{J/m^3}\right)}{8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}}} = 4.08\times10^{4}\ \mathrm{V\ m} = \boxed{40.8\ \mathrm{kV/m}}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{4.08\times10^{4}\ \mathrm{V/m}}{3.00\times10^{8}\ \mathrm{m/s}} = 1.36\times10^{-4}\ \mathrm{T} = \boxed{136\ \mu\mathrm{T}}$$

(e) $$F = PA = \left(\frac{S}{c}\right)A = \left(\frac{c\,u_{av}}{c}\right)A = u_{av}A = \left(7.37\times10^{-3}\ \frac{\mathrm{J}}{\mathrm{m^3}}\right)\pi(0.0600\ \mathrm{m})^2 = 8.33\times10^{-5}\ \mathrm{N} = \boxed{83.3\ \mu\mathrm{N}}$$

34.67 (a) On the right side of the equation, $\dfrac{C^2(m/s^2)^2}{(C^2/N\cdot m^2)(m/s)^3} = \dfrac{N\cdot m^2\cdot C^2\cdot m^2\cdot s^3}{C^2\cdot s^4\cdot m^3} = \dfrac{N\cdot m}{s} = \dfrac{J}{s} = W$

(b) $F = ma = qE$ or $a = \dfrac{qE}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(100\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = \boxed{1.76\times10^{13}\text{ m/s}^2}$

The radiated power is then: $\mathcal{P} = \dfrac{q^2a^2}{6\pi\epsilon_0c^3} = \dfrac{(1.60\times10^{-19})^2(1.76\times10^{13})^2}{6\pi(8.85\times10^{-12})(3.00\times10^8)^3} = \boxed{1.75\times10^{-27}\text{ W}}$

(c) $F = ma_r = m\left(\dfrac{v^2}{r}\right) = qvB$ so $v = \dfrac{qBr}{m}$

The proton accelerates at $a = \dfrac{v^2}{r} = \dfrac{q^2B^2r}{m^2} = \dfrac{(1.60\times10^{-19})^2(0.350)^2(0.500)}{(1.67\times10^{-27})^2} = 5.62\times10^{14}\text{ m/s}^2$

The proton then radiates $\mathcal{P} = \dfrac{q^2a^2}{6\pi\epsilon_0c^3} = \dfrac{(1.60\times10^{-19})^2(5.62\times10^{14})^2}{6\pi(8.85\times10^{-12})(3.00\times10^8)^3} = \boxed{1.80\times10^{-24}\text{ W}}$

34.68 $P = \dfrac{S}{c} = \dfrac{\text{Power}}{Ac} = \dfrac{\mathcal{P}}{2\pi r l c} = \dfrac{60.0\text{ W}}{2\pi(0.0500\text{ m})(1.00\text{ m})(3.00\times10^8\text{ m \ s})} = \boxed{6.37\times10^{-7}\text{ Pa}}$

34.69 $F = PA = \dfrac{SA}{c} = \dfrac{(\mathcal{P}/A)A}{c} = \dfrac{\mathcal{P}}{c}$, $\tau = F\left(\dfrac{l}{2}\right) = \dfrac{\mathcal{P}l}{2c}$, and $\tau = \kappa\theta$

Therefore, $\theta = \dfrac{\mathcal{P}l}{2c\kappa} = \dfrac{(3.00\times10^{-3})(0.0600)}{2(3.00\times10^8)(1.00\times10^{-11})} = \boxed{3.00\times10^{-2}\text{ deg}}$

***34.70** We take R to be the planet's distance from its star. The planet, of radius r, presents a $\boxed{\text{projected area } \pi r^2}$ perpendicular to the starlight. $\boxed{\text{It radiates over area } 4\pi r^2.}$

At steady-state, $\mathcal{P}_{in} = \mathcal{P}_{out}$: $eI_{\text{in}}(\pi r^2) = e\sigma(4\pi r^2)T^4$

$e\left(\dfrac{6.00\times10^{23}\text{ W}}{4\pi R^2}\right)(\pi r^2) = e\sigma(4\pi r^2)T^4$ so that $6.00\times10^{23}\text{ W} = 16\pi\sigma R^2T^4$

$$R = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi\sigma T^4}} = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(310\text{ K})^4}} = \boxed{4.77\times10^9\text{ m} = 4.77\text{ Gm}}$$

34.71 The light intensity is $I = S_{av} = \dfrac{E^2}{2\mu_0 c}$

The light pressure is $P = \dfrac{S}{c} = \dfrac{E^2}{2\mu_0 c^2} = \frac{1}{2}\epsilon_0 E^2$

For the asteroid, $PA = ma$ and $\boxed{a = \dfrac{\epsilon_0 E^2 A}{2m}}$

34.72 $f = 90.0\text{ MHz}$, $E_{max} = 2.00\times10^{-3}\text{ V/m} = 200\text{ mV/m}$

(a) $\lambda = \dfrac{c}{f} = \boxed{3.33\text{ m}}$

$T = \dfrac{1}{f} = 1.11\times10^{-8}\text{ s} = \boxed{11.1\text{ ns}}$

$B_{max} = \dfrac{E_{max}}{c} = 6.67\times10^{-12}\text{ T} = \boxed{6.67\text{ pT}}$

(b) $\boxed{\mathbf{E} = (2.00\text{ mV/m})\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)\mathbf{j}}$ $\boxed{\mathbf{B} = (6.67\text{ pT})\mathbf{k}\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)}$

(c) $I = \dfrac{E_{max}^2}{2\mu_0 c} = \dfrac{(2.00\times10^{-3})^2}{2(4\pi\times10^{-7})(3.00\times10^8)} = \boxed{5.31\times10^{-9}\text{ W/m}^2}$

(d) $I = cu_{av}$ so $u_{av} = \boxed{1.77\times10^{-17}\text{ J/m}^3}$

(e) $P = \dfrac{2I}{c} = \dfrac{(2)(5.31\times10^{-9})}{3.00\times10^8} = \boxed{3.54\times10^{-17}\text{ Pa}}$

Chapter 35 Solutons

35.1 The Moon's radius is 1.74×10^6 m and the Earth's radius is 6.37×10^6 m. The total distance traveled by the light is:

$$d = 2(3.84 \times 10^8 \text{ m} - 1.74 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m}) = 7.52 \times 10^8 \text{ m}$$

This takes 2.51 s, so $v = \dfrac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = 2.995 \times 10^8 \text{ m/s} = \boxed{299.5 \text{ Mm/s}}$

35.2 $\Delta x = ct$; $\quad c = \dfrac{\Delta x}{t} = \dfrac{2(1.50 \times 10^8 \text{ km})(1000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = 2.27 \times 10^8 \text{ m/s} = \boxed{227 \text{ Mm/s}}$

35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = 2\ell/c$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right) \quad \text{so} \quad \omega = \frac{c\theta}{2\ell} = \frac{(2.998 \times 10^8)[2\pi/(720)]}{2(11.45 \times 10^3)} = \boxed{114 \text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

35.4 (a) For the light beam to make it through both slots, the time for the light to travel the distance d must equal the time for the disk to rotate through the angle θ, if c is the speed of light,

$$\frac{d}{c} = \frac{\theta}{\omega}, \quad \text{so} \quad \boxed{c = \frac{d\omega}{\theta}}$$

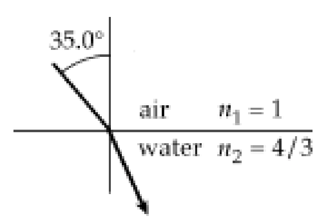
(b) We are given that

$$d = 2.50 \text{ m}, \quad \theta = \frac{1.00°}{60.0}\left(\frac{\pi \text{ rad}}{180°}\right) = 2.91 \times 10^{-4} \text{ rad}, \quad \omega = 5555 \frac{\text{rev}}{\text{s}}\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = 3.49 \times 10^4 \text{ rad/s}$$

$$c = \frac{d\omega}{\theta} = \frac{(2.50 \text{ m})(3.49 \times 10^4 \text{ rad/s})}{2.91 \times 10^{-4} \text{ rad}} = 3.00 \times 10^8 \text{ m/s} = \boxed{300 \text{ Mm/s}}$$

35.5 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$

$\theta_2 = \boxed{25.5°}$ $\qquad \lambda_2 = \dfrac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$



35.6 (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{6.328\times10^{-7}\ \text{m}} = \boxed{4.74\times10^{14}\ \text{Hz}}$

(b) $\lambda_{\text{glass}} = \dfrac{\lambda_{\text{air}}}{n} = \dfrac{632.8\ \text{nm}}{1.50} = \boxed{422\ \text{nm}}$

(c) $v_{\text{glass}} = \dfrac{c_{\text{air}}}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.50} = 2.00\times10^{8}\ \text{m/s} = \boxed{200\ \text{Mm/s}}$

35.7 $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\sin\theta_1 = 1.333 \sin 45.0°$

$\sin\theta_1 = (1.33)(0.707) = 0.943$

$\theta_1 = 70.5° \rightarrow \boxed{19.5° \text{ above the horizon}}$

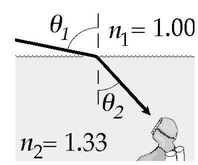


Figure for Goal Solution

Goal Solution

An underwater scuba diver sees the Sun at an apparent angle of 45.0° from the vertical. What is the actual direction of the Sun?

G: The sunlight refracts as it enters the water from the air. Because the water has a higher index of refraction, the light slows down and bends toward the vertical line that is normal to the interface. Therefore, the elevation angle of the Sun above the water will be less than 45° as shown in the diagram to the right, even though it appears to the diver that the sun is 45° above the horizon.

O: We can use Snell's law of refraction to find the precise angle of incidence.

A: Snell's law is: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

which gives $\sin\theta_1 = 1.333 \sin 45.0°$

$\sin\theta_1 = (1.333)(0.707) = 0.943$

The sunlight is at $\theta_1 = 70.5°$ to the vertical, so the Sun is 19.5° above the horizon.

L: The calculated result agrees with our prediction. When applying Snell's law, it is easy to mix up the index values and to confuse angles-with-the-normal and angles-with-the-surface. Making a sketch and a prediction as we did here helps avoid careless mistakes.

***35.8** (a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = n \sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{6.328\times10^{-7} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$ in air and in syrup.

(d) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.52} = 1.98\times10^8 \text{ m/s} = \boxed{198 \text{ Mm/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{1.98\times10^8 \text{ m/s}}{4.74\times10^{14} \text{ /s}} = \boxed{417 \text{ nm}}$

35.9 (a) Flint Glass: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.66} = 1.81\times10^8 \text{ m/s} = \boxed{181 \text{ Mm/s}}$

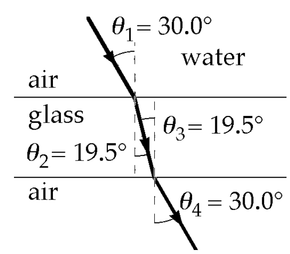
(b) Water: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.333} = 2.25\times10^8 \text{ m/s} = \boxed{225 \text{ Mm/s}}$

(c) Cubic Zirconia: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{2.20} = 1.36\times10^8 \text{ m/s} = \boxed{136 \text{ Mm/s}}$

35.10 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $1.333 \sin 37.0° = n_2 \sin 25.0°$

$n_2 = 1.90 = \dfrac{c}{v}$; $v = \dfrac{c}{1.90} = 1.58\times10^8 \text{ m/s} = \boxed{158 \text{ Mm/s}}$

35.11 $n_1 \sin\theta_1 = n_2 \sin\theta_2$; $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$\theta_2 = \sin^{-1}\left\{\dfrac{(1.00)(\sin 30°)}{1.50}\right\} = \boxed{19.5°}$

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals. So, $\theta_3 = \theta_2 = \boxed{19.5°}$.

$1.50 \sin\theta_3 = (1.00)\sin\theta_4$

$\theta_4 = \boxed{30.0°}$

35.12 (a) Water $\lambda = \frac{\lambda_0}{n} = \frac{436\text{ nm}}{1.333} = \boxed{327\text{ nm}}$

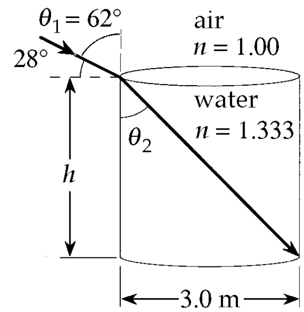
(b) Glass $\lambda = \frac{\lambda_0}{n} = \frac{436\text{ nm}}{1.52} = \boxed{287\text{ nm}}$

***35.13** $\sin\theta_1 = n_w \sin\theta_2$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

$$\theta_2 = \sin^{-1} 0.662 = 41.5°$$

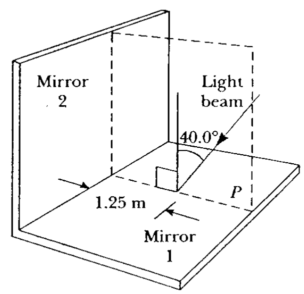
$$h = \frac{d}{\tan\theta_2} = \frac{3.00\text{ m}}{\tan 41.5°} = \boxed{3.39\text{ m}}$$

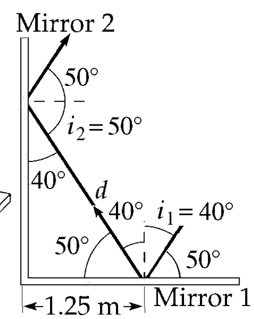


35.14 (a) From geometry, 1.25 m = $d \sin 40.0°$

so $d = \boxed{1.94\text{ m}}$

(b) $\boxed{50.0° \text{ above horizontal}}$, or parallel to the incident ray





***35.15** The incident light reaches the left-hand mirror at distance

(1.00 m) tan 5.00° = 0.0875 m

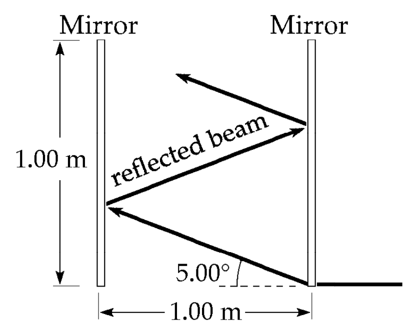
above its bottom edge. The reflected light first reaches the right-hand mirror at height

2(0.0875 m) = 0.175 m

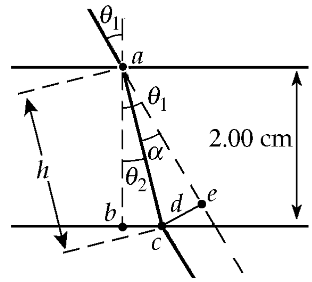
It bounces between the mirrors with this distance between points of contact with either.

Since $\frac{1.00\text{ m}}{0.175\text{ m}} = 5.72$, the light reflects

$\boxed{\text{five times from the right-hand mirror and six times from the left.}}$



***35.16** At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $1.00 \sin 30.0° = 1.50 \sin\theta_2$



$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{(2.00\text{ cm})}{h} \quad \text{or} \quad h = \frac{(2.00\text{ cm})}{\cos 19.5°} = 2.12\text{ cm}$$

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

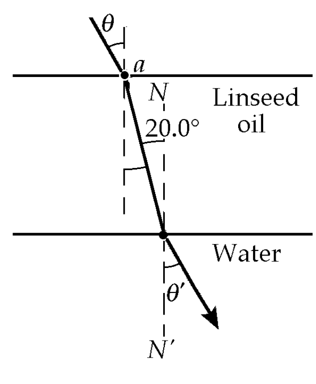
The offset distance comes from $\sin\alpha = \dfrac{d}{h}$: $d = (2.21\text{ cm})\sin 10.5° = \boxed{0.388\text{ cm}}$

***35.17** The distance, h, traveled by the light is $h = \dfrac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$

The speed of light in the material is $v = \dfrac{c}{n} = \dfrac{3.00\times10^8\text{ m/s}}{1.50} = 2.00\times10^8\text{ m/s}$

Therefore, $t = \dfrac{h}{v} = \dfrac{2.12\times10^{-2}\text{ m}}{2.00\times10^8\text{ m/s}} = 1.06\times10^{-10}\text{ s} = \boxed{106\text{ ps}}$

***35.18** Applying Snell's law at the air-oil interface,



$n_{air}\sin\theta = n_{oil}\sin 20.0°$ yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$n_w \sin\theta' = n_{oil}\sin 20.0°$ yields $\boxed{\theta' = 22.3°}$

***35.19** time difference = (time for light to travel 6.20 m in ice) – (time to travel 6.20 m in air)

$$\Delta t = \frac{6.20\text{ m}}{v_{ice}} - \frac{6.20\text{ m}}{c} \quad \text{but} \quad v = \frac{c}{n}$$

$$\Delta t = (6.20\text{ m})\left(\frac{1.309}{c} - \frac{1}{c}\right) = \frac{(6.20\text{ m})}{c}(0.309) = 6.39\times10^{-9}\text{ s} = \boxed{6.39\text{ ns}}$$

***35.20** Consider glass with an index of refraction of 1.5, which is 3 mm thick The speed of light in the glass is

$$\frac{3\times10^{8}\ \text{m/s}}{1.5}=2\times10^{8}\ \text{m/s}$$

The extra travel time is $\frac{3\times10^{-3}\ \text{m}}{2\times10^{8}\ \text{m/s}}-\frac{3\times10^{-3}\ \text{m}}{3\times10^{8}\ \text{m/s}}$ $\boxed{\sim10^{-11}\ \text{s}}$

For light of wavelength 600 nm in vacuum and wavelength $\frac{600\ \text{nm}}{1.5}=400\ \text{nm}$ in glass,

the extra optical path, in wavelengths, is $\frac{3\times10^{-3}\ \text{m}}{4\times10^{-7}\ \text{m}}-\frac{3\times10^{-3}\ \text{m}}{6\times10^{-7}\ \text{m}}$ $\boxed{\sim10^{3}\ \text{wavelengths}}$

***35.21** Refraction proceeds according to $(1.00)\sin\theta_1=(1.66)\sin\theta_2$ (1)

(a) For the normal component of velocity to be constant, $v_1\cos\theta_1=v_2\cos\theta_2$

or $(c)\cos\theta_1=(c/1.66)\cos\theta_2$ (2)

We multiply Equations (1) and (2), obtaining: $\sin\theta_1\cos\theta_1=\sin\theta_2\cos\theta_2$

or $\sin2\theta_1=\sin2\theta_2$

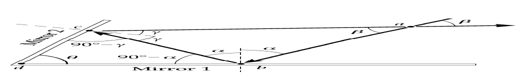
The solution $\theta_1=\theta_2=0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1=180°-2\theta_2$ or $\theta_2=90.0°-\theta_1$. Then Equation (1) becomes:

$\sin\theta_1=1.66\cos\theta_1$, or $\tan\theta_1=1.66$

which yields $\theta_1=\boxed{58.9°}$

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass, so that component cannot remain constant, or will remain constant only in the trivial case $\boxed{\theta_1=\theta_2=0}$

35.22 See the sketch showing the path of the light ray. α and γ are angles of incidence at mirrors 1 and 2.



For triangle abca, $2\alpha+2\gamma+\beta=180°$

or $\beta=180°-2(\alpha+\gamma)$ (1)

Now for triangle bcdb,

$$(90.0°-\alpha)+(90.0°-\gamma)+\theta=180°$$

or $\theta=\alpha+\gamma$ (2)

Substituting Equation (2) into Equation (1) gives $\boxed{\beta = 180° - 2\theta}$

Note: From Equation (2), $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < \theta$. For $\alpha > \theta$, γ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

35.23 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \frac{h}{c}\left(\frac{n+1.000}{2}\right)$$

$$t = \frac{20.0 \times 10^3 \text{ m}}{3.00 \times 10^8 \text{ m/s}}\left(\frac{1.005 + 1.000}{2}\right) = \boxed{66.8 \ \mu\text{s}}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$\left(\frac{n + 1.000}{2}\right) = 1.0025$ times larger or $\boxed{0.250\% \text{ longer}}$.

35.24 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface. Then,

$$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time required to traverse the atmosphere is

$$t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be h/c. Thus, the time in the presence of an atmosphere is

$$\boxed{\left(\frac{n + 1.000}{2}\right) \text{ times larger}}$$

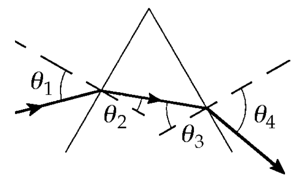
35.25 From Fig. 35.20 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm

Then $(1.00)\sin\theta = 1.470\sin\theta_v$ and $(1.00)\sin\theta = 1.458\sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

35.26 $n_1\sin\theta_1 = n_2\sin\theta_2$ so $\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$



$$\theta_2 = \sin^{-1}\left(\frac{(1.00)(\sin 30.0°)}{1.50}\right) = \boxed{19.5°}$$

$$\theta_3 = \left(\left[(90.0° - 19.5°) + 60.0°\right] - 180°\right) + 90.0° = \boxed{40.5°}$$

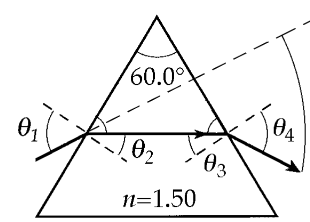
$n_3\sin\theta_3 = n_4\sin\theta_4$ so $\theta_4 = \sin^{-1}\left(\frac{n_3\sin\theta_3}{n_4}\right) = \sin^{-1}\left(\frac{(1.50)(\sin 40.5°)}{1.00}\right) = \boxed{77.1°}$

35.27 Taking Φ to be the apex angle and δ_{min} to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left(\frac{\Phi + \delta_{min}}{2}\right)}{\sin(\Phi/2)}$$

Solving for δ_{min}, $\delta_{min} = 2\sin^{-1}\left(n\sin\frac{\Phi}{2}\right) - \Phi = 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° = \boxed{86.8°}$

35.28 $n(700\text{ nm}) = 1.458$



(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2$; $\theta_2 = \boxed{41.5°}$

(b) Let $\theta_3 + \beta = 90.0°$, $\theta_2 + \alpha = 90.0°$; then $\alpha + \beta + 60.0° = 180°$

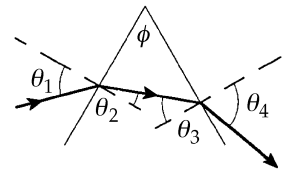
So $60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$

(c) $1.458\sin 18.5° = 1.00\sin\theta_4$ $\theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + [\beta - (90.0° - \theta_4)]$

$$\gamma = 75.0° - 41.5° + (90.0° - 18.5°) - (90.0° - 27.6°) = \boxed{42.6°}$$

35.29 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$



Using the figure to the right, $(\theta_2)_{\text{violet}} = \sin^{-1}\left(\dfrac{\sin\ 50.0°}{1.66}\right) = 27.48°$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin\ 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray, $\theta_3' = 60.0° - \theta_2$ and $\sin\theta_4 = n\sin\theta_3$

$$(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\ \sin\ 32.52°] = 63.17°$$

$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\ \sin\ 31.78°] = 58.56°$$

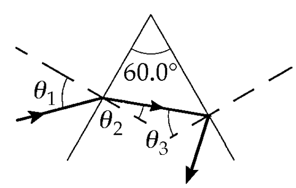
The dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$

35.30 $n = \dfrac{\sin\left(\dfrac{\Phi + \delta_{\min}}{2}\right)}{\sin(\Phi/2)}$

For small Φ, $\delta_{\min} \approx \Phi$ so $\dfrac{\Phi + \delta_{\min}}{2}$ is also a small angle. Then, using the small angle approximation ($\sin\theta \approx \theta$ when $\theta << 1$ rad), we have:

$n \approx \dfrac{(\Phi + \delta_{\min})/2}{\Phi/2} = \dfrac{\Phi + \delta_{\min}}{\Phi}$ or $\boxed{\delta_{\min} \approx (n-1)\Phi}$ where Φ is in radians.

35.31 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$



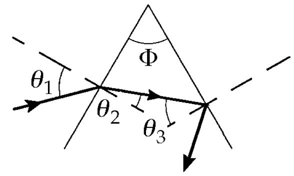
The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\dfrac{1.00}{1.50}\right) = 41.8°$.

But, $\theta_2 = 60.0° - \theta_3$. Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$), it is necessary that $\theta_2 > 18.2°$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

$$\sin\theta_1 > (1.50)\sin(18.2°) = 0.468, \text{ or } \theta_1 > \boxed{27.9°}$$

35.32 At the first refraction, $(1.00)\sin\theta_1 = n\sin\theta_2$. The critical angle at the second surface is given by



$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}(1.00/n)$

But $(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$, which gives $\theta_2 = \Phi - \theta_3$.

Thus, to have $\theta_3 < \sin^{-1}(1.00/n)$ and avoid total internal reflection at the second surface, it is necessary that $\theta_2 > \Phi - \sin^{-1}(1.00/n)$. Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

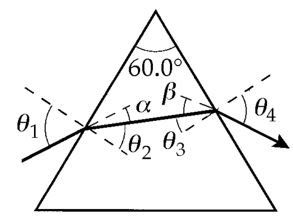
$$\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right] \quad \text{or} \quad \theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}$$

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)}$

35.33 $n = \dfrac{\sin(\delta+\phi)}{\sin(\phi/2)}$ so $1.544\sin\left(\frac{1}{2}\phi\right) = \sin\left(5° + \frac{1}{2}\phi\right) = \cos\left(\frac{1}{2}\phi\right)\sin 5° + \sin\left(\frac{1}{2}\phi\right)\cos 5°$

$\tan\left(\frac{1}{2}\phi\right) = \dfrac{\sin 5°}{1.544 - \cos 5°}$ and $\phi = \boxed{18.1°}$

***35.34** Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$



so $\theta_3 = \Phi - \theta_2$

At the first surface is $\alpha = \theta_1 - \theta_2$

At exit, the deviation is $\beta = \theta_4 - \theta_3$

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\dfrac{\sin 48.6°}{1.50}\right) = 30.0°$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}[1.50\sin(30.0°)] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \dfrac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$

(d) At entry: $\sin\theta_2 = \dfrac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$

35.35 $n \sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\dfrac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\dfrac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\dfrac{1}{1.309}\right) = \boxed{49.8°}$

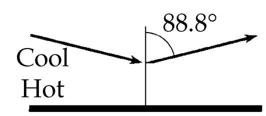
35.36 $\sin\theta_c = \dfrac{n_2}{n_1}$; $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

(a) Diamond: $\theta_c = \sin^{-1}\left(\dfrac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) Flint glass: $\theta_c = \sin^{-1}\left(\dfrac{1.333}{1.66}\right) = \boxed{53.4°}$

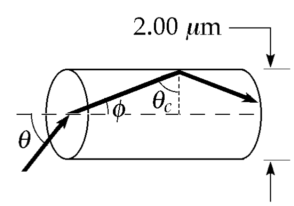
(c) Ice: Since $n_2 > n_1$, $\boxed{\text{there is no critical angle}}$.

35.37 $\sin\theta_c = \dfrac{n_2}{n_1}$ (Equation 35.10)



$n_2 = n_1 \sin 88.8° = (1.0003)(0.9998) = \boxed{1.000\ 08}$

***35.38** $\sin\theta_c = \dfrac{n_{air}}{n_{pipe}} = \dfrac{1.00}{1.36} = 0.735$ $\theta_c = 47.3°$



Geometry shows that the angle of refraction at the end is

$$\theta_r = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$$

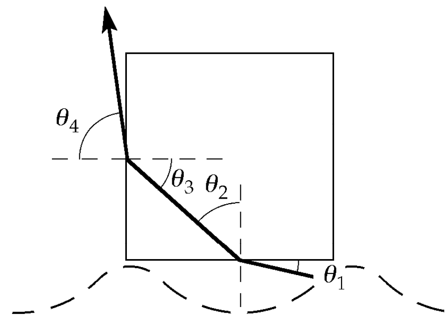
Then, Snell's law at the end, $1.00 \sin\theta = 1.36 \sin 42.7°$

gives $\boxed{\theta = 67.2°}$

35.39 For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$(1.50)\sin\theta_1 = (1.33)(1.00)$ or $\theta_1 = \boxed{62.4°}$

35.40 To avoid internal reflection and come out through the vertical face, light inside the cube must have



$\theta_3 < \sin^{-1}(1/n)$

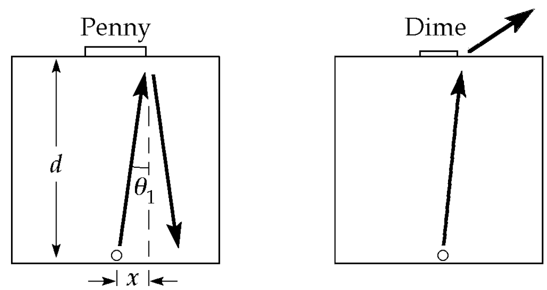
So $\theta_2 > 90.0° - \sin^{-1}(1/n)$

But $\theta_1 < 90.0°$ and $n \sin\theta_2 < 1$

In the critical case, $\sin^{-1}(1/n) = 90.0° - \sin^{-1}(1/n)$

$1/n = \sin 45.0°$ $\boxed{n = 1.41}$

35.41 From Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$



At the extreme angle of viewing, $\theta_2 = 90.0°$

$(1.59)(\sin\theta_1) = (1.00)\cdot\sin 90.0°$

So $\theta_1 = 39.0°$

Therefore, the depth of the air bubble is

$$\frac{r_d}{\tan\theta_1} < d < \frac{r_p}{\tan\theta_1}$$

or $\boxed{1.08 \text{ cm} < d < 1.17 \text{ cm}}$

***35.42** (a) $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1850 \text{ m/s}}{343 \text{ m/s}} \quad \text{so} \quad \theta_c = \sin^{-1} 0.185 = \boxed{10.7°}$$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

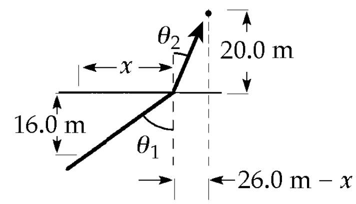
***35.43** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \leq \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from both the sides of the slab and from facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$, since

$$\theta_c = \sin^{-1}\left(\frac{1.50}{2.12}\right) = 45.0°.$$

***35.44** Assume the lifeguard's path makes angle θ_1 with the north-south normal to the shoreline, and angle θ_2 with this normal in the water. By Fermat's principle, his path should follow the law of refraction:



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{7.00\text{ m/s}}{1.40\text{ m/s}} = 5.00 \qquad \text{or} \qquad \theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{5}\right)$$

The lifeguard on land travels eastward a distance $x = (16.0\text{ m})\tan\theta_1$. Then in the water, he travels $26.0\text{ m} - x = (20.0\text{ m})\tan\theta_2$ further east. Thus, $26.0\text{ m} = (16.0\text{ m})\tan\theta_1 + (20.0\text{ m})\tan\theta_2$

or $$26.0\text{ m} = (16.0\text{ m})\tan\theta_1 + (20.0\text{ m})\tan\left[\sin^{-1}\left(\frac{\sin\theta_1}{5}\right)\right]$$

We home in on the solution as follows:

θ_1 (deg)	50.0	60.0	54.0	54.8	54.81
right-hand side	22.2 m	31.2 m	25.3 m	25.99 m	26.003 m

The lifeguard should start running at $\boxed{54.8° \text{ east of north}}$.

***35.45** Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$

or $1.56 \sin\theta_2 = \sin\theta_1$

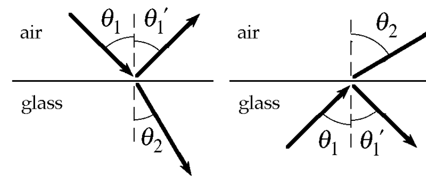
But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin\theta_2 = \sin 2\theta_2$

We now use the double-angle trig identity suggested. $1.56 \sin\theta_2 = 2\sin\theta_2 \cos\theta_2$

or $\cos\theta_2 = \frac{1.56}{2} = 0.780$

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$

***35.46** (a) $\theta_1' = \theta_1 = \boxed{30.0°}$



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$(1.00)\sin 30.0° = 1.55\ \sin\theta_2$$

$$\theta_2 = \boxed{18.8°}$$

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c) and (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

35.47 For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$

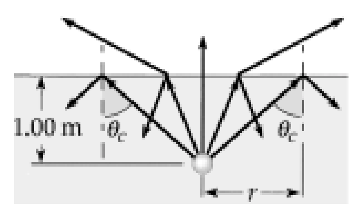


Figure for Goal Solution

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}$$

Goal Solution

A small underwater pool light is 1.00 m below the surface. The light emerging from the water forms a circle on the water's surface. What is the diameter of this circle?

G: Only the light that is directed upwards and hits the water's surface at less than the critical angle will be transmitted to the air so that someone outside can see it. The light that hits the surface farther from the center at an angle greater than θ_c will be totally reflected within the water, unable to be seen from the outside. From the diagram above, the diameter of this circle of light appears to be about 2 m.

O: We can apply Snell's law to find the critical angle, and the diameter can then be found from the geometry.

A: The critical angle is found when the refracted ray just grazes the surface (θ_2 = 90°). The index of refraction of water is $n_2 = 1.33$, and $n_1 = 1.00$ for air, so

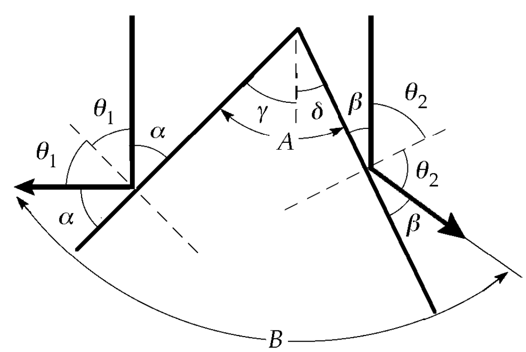
$n_1 \sin\theta_c = n_2 \sin 90°$ gives $\theta_c = \sin^{-1}\left(\frac{1}{1.333}\right) = \sin^{-1}(0.750) = 48.6°$

The radius then satisfies $\tan\theta_c = \frac{r}{(1.00 \text{ m})}$

So the diameter is $d = 2r = 2(1.00 \text{ m})\tan 48.6° = 2.27 \text{ m}$

L: Only the light rays within a 97.2° cone above the lamp escape the water and can be seen by an outside observer (Note: this angle does not depend on the depth of the light source). The path of a light ray is always reversible, so if a person were located beneath the water, they could see the whole hemisphere above the water surface within this cone; this is a good experiment to try the next time you go swimming!

***35.48** Call θ_1 the angle of incidence and of reflection on the left face and θ_2 those angles on the right face. Let α represent the complement of θ_1 and β be the complement of θ_2. Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles. We have



$$A = \gamma + \delta = \alpha + \beta$$

and $B = \alpha + A + \beta = \alpha + \beta + A = \boxed{2A}$

***35.49** (a) We see the Sun swinging around a circle in the extended plane of our parallel of latitude. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = 7.27\times10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})\left(7.27\times10^{-5} \text{ rad/s}\right) = 1.72\times10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide: $v = r\omega = 2(0.174 \text{ mm/s}) = \boxed{0.345 \text{ mm/s}}$

(c) and (d)

As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at } 50.0°}$.

***35.50** By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

With $v = \dfrac{c}{n}$, $\dfrac{c}{v_1}\sin\theta_1 = \dfrac{c}{v_2}\sin\theta_2$ or $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$

This is also true for sound. Here, $\dfrac{\sin 12.0°}{340 \text{ m/s}} = \dfrac{\sin\theta_2}{1510 \text{ m/s}}$

$$\theta_2 = \arcsin(4.44 \sin 12.0°) = \boxed{67.4°}$$

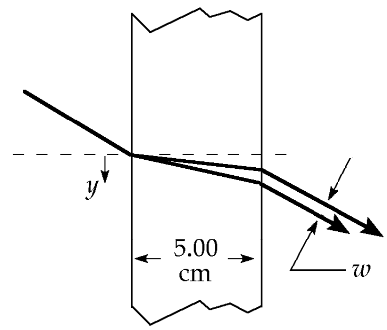
***35.51** (a) $$n = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{\left(61.15 \dfrac{\text{km}}{\text{hr}}\right)\left(\dfrac{1.00 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{1.00\times10^3 \text{ m}}{1.00 \text{ km}}\right)} = \boxed{1.76\times10^7}$$

(b) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ so $\left(1.76\times10^7\right)\sin\theta_1 = (1.00)\sin 90.0°$

$$\theta_1 = \boxed{3.25\times10^{-6} \text{ degree}}$$

This problem is misleading. The speed of energy transport is slow, but the speed of the wavefront advance is normally fast. The condensate's index of refraction is not far from unity.

***35.52** Violet light:



$$(1.00)\sin 25.0° = 1.689 \sin\theta_2 \;\Rightarrow\; \theta_2 = 14.490°$$

$$y_v = (5.00 \text{ cm})\tan\theta_2 = (5.00 \text{ cm})\tan 14.490° = 1.2622 \text{ cm}$$

Red Light:

$$(1.00)\sin 25.0° = 1.642 \sin\theta_2 \;\Rightarrow\; \theta_2 = 14.915°$$

$$y_R = (5.00 \text{ cm})\tan 14.915° = 1.3318 \text{ cm}$$

The emergent beams are both at 25.0° from the normal. Thus,

$w = \Delta y \cos 25.0°$ where $\Delta y = 1.3318 \text{ cm} - 1.2622 \text{ cm} = 0.0396 \text{ cm}$

$$w = (0.396 \text{ mm})\cos 25.0° = \boxed{0.359 \text{ mm}}$$

35.53 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b) below, by just the certain special raindrops at 40.0° to 42.0° from the hiker's shadow, and reach the hiker as the rainbow.

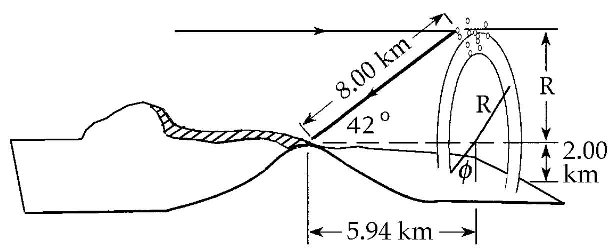


Figure (a)

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

$$R = (8.00 \text{ km})(\sin 42.0°) = 5.35 \text{ km}$$

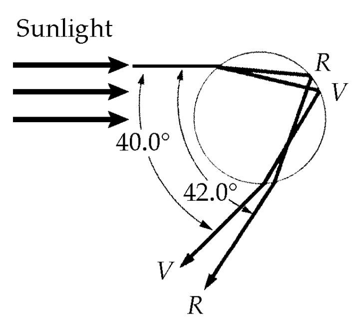


Figure (b)

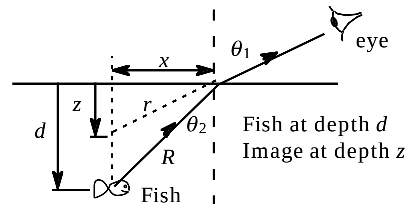
Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00 \text{ km}}{R} = \frac{2.00 \text{ km}}{5.35 \text{ km}} = 0.374 \quad \text{or} \quad \phi = 68.1°$$

The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$, so the visible bow is

$$\frac{224°}{360°} = \boxed{62.2\% \text{ of a circle}}$$

35.54 From Snell's law, $(1.00)\sin\theta_1 = \frac{4}{3}\sin\theta_2$



$$x = R\sin\theta_2 = r\sin\theta_1$$

so $$\frac{r}{R} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{3}{4}$$

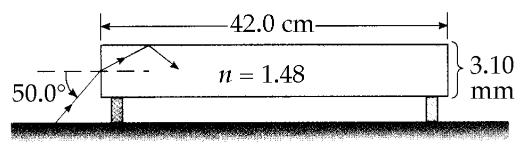
$$\frac{\text{apparent depth}}{\text{actual depth}} = \frac{z}{d} = \frac{r\cos\theta_1}{R\cos\theta_2} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\sin^2\theta_2}}$$

But $$\sin^2\theta_2 = \left(\frac{3}{4}\sin\theta_1\right)^2 = \frac{9}{16}\left(1-\cos^2\theta_1\right)$$

So $$\frac{z}{d} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{1-\frac{9}{16}+\frac{9}{16}\cos^2\theta_1}} = \frac{3}{4}\frac{\cos\theta_1}{\sqrt{\frac{7+9\cos^2\theta_1}{16}}} \quad \text{or} \quad \boxed{z = \frac{3d\cos\theta_1}{\sqrt{7+9\cos^2\theta_1}}}$$

35.55 As the beam enters the slab, $(1.00)\sin 50.0° = (1.48)\sin\theta_2$ giving $\theta_2 = 31.2°$. The beam then strikes the top of the slab at $x_1 = 1.55\text{ mm}/\tan(31.2°)$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional $420\text{ mm} - x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm} - x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan(31.2°)}{3.10\text{ mm}/\tan(31.2°)} = 81.5$$



or 81 reflections since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

***35.56** (a) $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.52-1.00}{1.52+1.00}\right]^2 = \boxed{0.0426}$$

(b) If medium 1 is glass and medium 2 is air, $$\frac{S_1'}{S_1} = \left[\frac{n_2-n_1}{n_2+n_1}\right]^2 = \left[\frac{1.00-1.52}{1.00+1.52}\right]^2 = 0.0426;$$

There is $\boxed{\text{no difference}}$

(c) $$\frac{S_1'}{S_1} = \left[\frac{1.76\times10^7-1.00}{1.76\times10^7+1.00}\right]^2 = \left[\frac{1.76\times10^7+1.00-2.00}{1.76\times10^7+1.00}\right]^2$$

$$\frac{S_1'}{S_1} = \left[1.00-\frac{2.00}{1.76\times10^7+1.00}\right]^2 \approx 1.00-2\left(\frac{2.00}{1.76\times10^7+1.00}\right) = 1.00-2.27\times10^{-7} \quad \text{or} \quad \boxed{100\%}$$

This suggests he appearance would be $\boxed{\text{very shiny, reflecting practically all incident light}}$. See, however, the note concluding the solution to problem 35.51.

***35.57** (a) With $n_1 = 1$ and $n_2 = n$, the reflected fractional intensity is $\dfrac{S_1'}{S_1} = \left(\dfrac{n-1}{n+1}\right)^2$.

The remaining intensity must be transmitted:

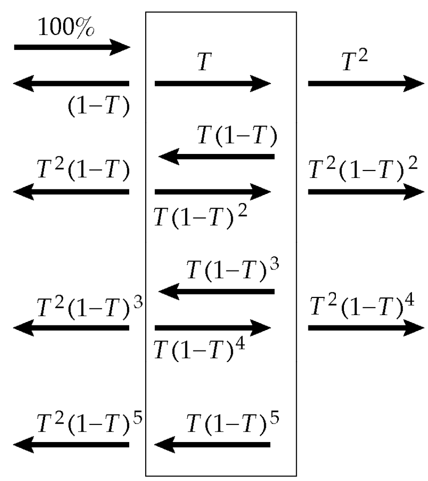
$$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{n^2 + 2n + 1 - n^2 + 2n - 1}{(n+1)^2} = \boxed{\frac{4n}{(n+1)^2}}$$

(b) At entry, $\dfrac{S_2}{S_1} = 1 - \left(\dfrac{n-1}{n+1}\right)^2 = \dfrac{4(2.419)}{(2.419+1)^2} = 0.828$

At exit, $\dfrac{S_3}{S_2} = 0.828$

Overall, $\dfrac{S_3}{S_1} = \left(\dfrac{S_3}{S_2}\right)\left(\dfrac{S_2}{S_1}\right) = (0.828)^2 = 0.685$ or $\boxed{68.5\%}$

***35.58** Define $T = \dfrac{4n}{(n+1)^2}$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in problem 57.



As shown in the figure, the total amount transmitted is

$$T^2 + T^2(1-T)^2 + T^2(1-T)^4 + T^2(1-T)^6 + \ldots + T^2(1-T)^{2n} + \ldots$$

We have $1 - T = 1 - 0.828 = 0.172$ so the total transmission is

$$(0.828)^2\left[1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots\right]$$

To sum this series, define $F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$.
Note that $(0.172)^2 F = (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots$, and
$1 + (0.172)^2 F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \ldots = F$.

Then, $1 = F - (0.172)^2 F$ or $F = \dfrac{1}{1-(0.172)^2}$

The overall transmission is then $\dfrac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$

35.59 $n \sin 42.0° = \sin 90.0°$ so $n = \dfrac{1}{\sin 42.0°} = 1.49$

$\sin\theta_1 = n \sin 18.0°$ and $\sin\theta_1 = \dfrac{\sin 18.0°}{\sin 42.0°}$

$\theta_1 = \boxed{27.5°}$

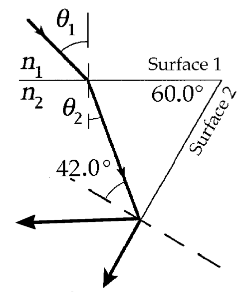


Figure for Goal Solution

<u>**Goal Solution**</u>

The light beam shown in Figure P35.59 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

G: From the diagram it appears that the angle of incidence is about 40°.

O: We can find θ_1 by applying Snell's law at the first interface where the light is refracted. At surface 2, knowing that the 42.0° angle of reflection is the critical angle, we can work backwards to find θ_1.

A: Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio n_2/n_1:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°. Thus,

$$(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$$

Therefore, $\theta_2 = 18.0°$

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

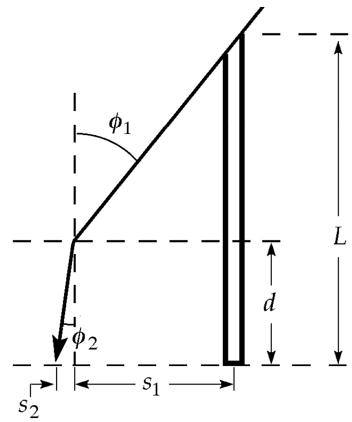
$$\sin\theta_1 = (n_2/n_1)\sin\theta_2 = (1.49)\sin 18.0°$$

$$\theta_1 = 27.5°$$

L: The result is a bit less than the 40.0° we expected, but this is probably because the figure is not drawn to scale. This problem was a bit tricky because it required four key concepts (refraction, reflection, critical angle, and geometry) in order to find the solution. One practical extension of this problem is to consider what would happen to the exiting light if the angle of incidence were varied slightly. Would all the light still be reflected off surface 2, or would some light be refracted and pass through this second surface?

35.60 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance

$$s_1 = \frac{(L-d)}{\tan\theta} \text{ from the pole,}$$

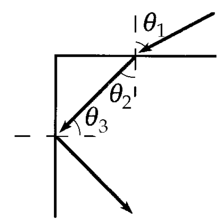


and has an angle of refraction ϕ_2 from $(1.00)\sin\phi_1 = n\sin\phi_2$. Then $s_2 = d\tan\phi_2$ and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right) = \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

35.61 (a) For polystyrene *surrounded by air*, internal reflection requires



$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from the geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = (1.49)\sin 47.8° = 1.10$

This has no solution. Therefore, total internal reflection $\boxed{\text{always happens}}$.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$ since the beam is initially traveling in a medium of lower index of refraction.

***35.62** $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1\sin\theta_1 = n_2\sin\theta_2$ with $n_1 = 1,\ n_2 = \frac{4}{3}$

Thus, $\theta_1 = \sin^{-1}(n_2\sin\theta_2) = \sin^{-1}[n_2\sin(\theta_1 - 10.0°)]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

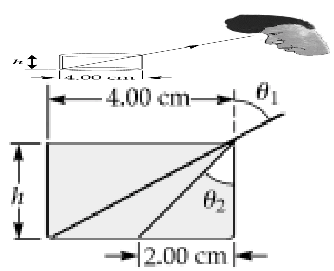
We are given that $$\sin\theta_1 = \frac{4}{3}\sin(\theta_1 - 10.0°)$$

This is the sine of a difference, so $$\frac{3}{4}\sin\theta_1 = \sin\theta_1\cos 10.0° - \cos\theta_1\sin 10.0°$$

Rearranging, $$\sin 10.0°\cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$$

$$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1 \quad \text{and} \quad \theta_1 = \tan^{-1} 0.740 = \boxed{36.5°}$$

35.63 $\tan\theta_1 = \dfrac{4.00\text{ cm}}{h}$ and $\tan\theta_2 = \dfrac{2.00\text{ cm}}{h}$



$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{(1-\sin^2\theta_1)} = 4.00\frac{\sin^2\theta_2}{(1-\sin^2\theta_2)} \qquad (1)$$

Snell's law in this case is: $n_1\sin\theta_1 = n_2\sin\theta_2$

$$\sin\theta_1 = 1.333\ \sin\theta_2$$

Squaring both sides, $$\sin^2\theta_1 = 1.777\ \sin^2\theta_2 \qquad (2)$$

Substituting (2) into (1), $$\frac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\frac{\sin^2\theta_2}{1-\sin^2\theta_2}$$

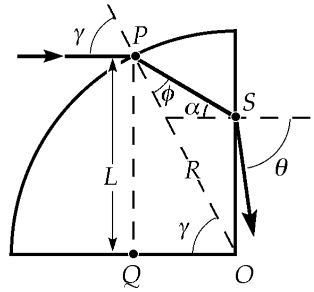
Defining $x = \sin^2\theta$, $$\frac{0.444}{(1-1.777x)} = \frac{1}{(1-x)}$$

Solving for x, $$0.444 - 0.444x = 1 - 1.777x \quad \text{and} \quad x = 0.417$$

From x we can solve for θ_2: $$\theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$$

Thus, the height is $$h = \frac{(2.00\text{ cm})}{\tan\theta_2} = \frac{(2.00\text{ cm})}{\tan(40.2°)} = \boxed{2.37\text{ cm}}$$

35.64 Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ: $\sin\gamma = L/R$.



Also, $\cos\gamma = \sqrt{1-\sin^2\gamma} = \dfrac{\sqrt{R^2-L^2}}{R}$

Applying Snell's law at point P, $(1.00)\sin\gamma = n\sin\phi$

Thus, $\sin\phi = \dfrac{\sin\gamma}{n} = \dfrac{L}{nR}$

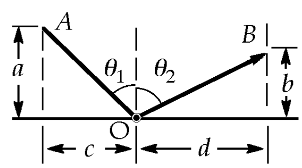
and $\cos\phi = \sqrt{1-\sin^2\phi} = \dfrac{\sqrt{n^2R^2-L^2}}{nR}$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S gives $(1.00)\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$, or

$$\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right) \quad \text{and} \quad \boxed{\theta = \sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]}$$

35.65 To derive the law of *reflection*, locate point O so that the time of travel from point A to point B will be minimum.



The *total* light path is $L = a\sec\theta_1 + b\sec\theta_2$

The time of travel is $t = \left(\dfrac{1}{v}\right)(a\sec\theta_1 + b\sec\theta_2)$

If point O is displaced by dx, then

$$dt = \left(\frac{1}{v}\right)(a\sec\theta_1\tan\theta_1\,d\theta_1 + b\sec\theta_2\tan\theta_2\,d\theta_2) = 0 \qquad (1)$$

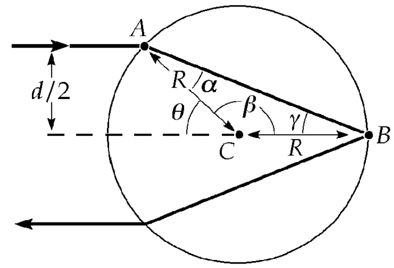
(since for minimum time $dt = 0$).

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $$a\sec^2\theta_1\,d\theta_1 + b\sec^2\theta_2\,d\theta_2 = 0 \qquad (2)$$

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$

35.66 As shown in the sketch, the angle of incidence at point A is:



$$\theta = \sin^{-1}\left[\frac{(d/2)}{R}\right] = \sin^{-1}\left[\frac{1.00\text{ m}}{2.00\text{ m}}\right] = 30.0°$$

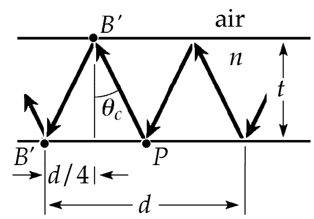
If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the center line CB of the cylinder . In the isosceles triangle ABC, $\gamma = \alpha$ and $\beta = 180° - \theta$. Therefore, $\alpha + \beta + \gamma = 180°$ becomes

$$2\alpha + 180° - \theta = 180° \quad \text{or} \quad \alpha = \frac{\theta}{2} = 15.0°$$

Then, applying Snell's law at point A, $n \sin\alpha = (1.00)\sin\theta$

or $$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

35.67 (a) At the boundary of the air and glass, the critical angle is given by



$$\sin\theta_c = \frac{1}{n}$$

Consider the critical ray PBB': $\tan\theta_c = \frac{d/4}{t}$ or $\frac{\sin\theta_c}{\cos\theta_c} = \frac{d}{4t}$

Squaring the last equation gives: $\frac{\sin^2\theta_c}{\cos^2\theta_c} = \frac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\frac{d}{4t}\right)^2$

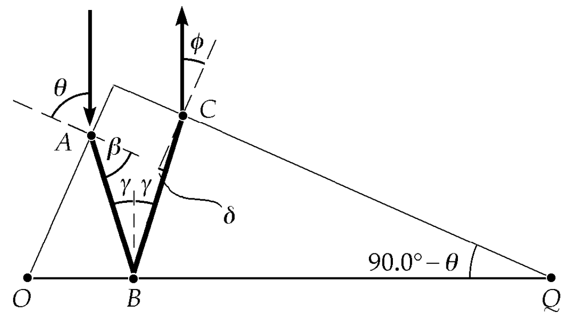
Since $\sin\theta_c = \frac{1}{n}$, this becomes $\frac{1}{n^2-1} = \left(\frac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+(4t/d)^2}}$

(b) Solving for d, $d = \frac{4t}{\sqrt{n^2-1}}$

Thus, if $n = 1.52$ and $t = 0.600$ cm, $d = \frac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

35.68 From the sketch, observe that the angle of incidence at point A is the same as the prism angle θ at point O. Given that $\theta = 60.0°$, application of Snell's law at point A gives



$1.50 \sin\beta = 1.00 \sin 60.0°$ or $\beta = 35.3°$

From triangle AOB, we calculate the angle of incidence (and reflection) at point B.

$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180°$ so $\gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$

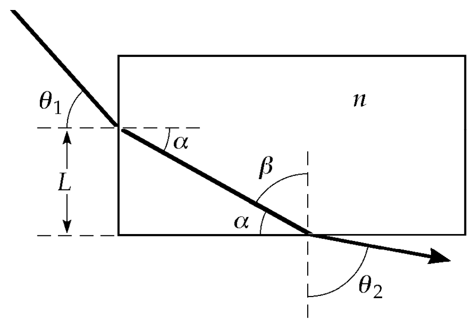
Now, using triangle BCQ: $(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$

Thus the angle of incidence at point C is $\delta = (90.0° - \theta) - \gamma = 30.0° - 24.7° = 5.30°$

Finally, Snell's law applied at point C gives $1.00 \sin\phi = 1.50 \sin 5.30°$

or $\phi = \sin^{-1}(1.50 \sin 5.30°) = \boxed{7.96°}$

35.69 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$, Snell's law at the first surface gives



$n \sin\alpha = (1.00)\sin 45.0°$ (1)

Observe that the angle of incidence at the second surface is $\beta = 90.0° - \alpha$. Thus, Snell's law at the second surface yields

$n \sin\beta = n \sin(90.0° - \alpha) = (1.00)\sin 76.0°$, or

$n \cos\alpha = \sin 76.0°$ (2)

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$ or $\alpha = 36.1°$

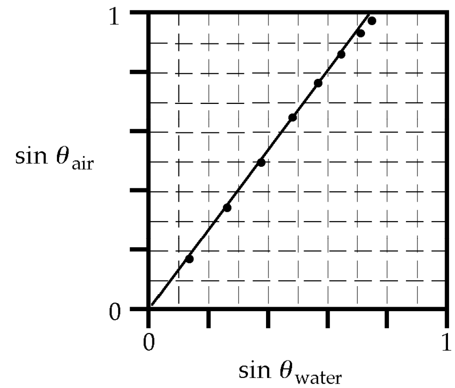
Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = L/\sin\alpha$. Also, the speed of light in the plastic is $v = c/n$, so the time required to travel through the plastic is

$$t = \frac{d}{v} = \frac{nL}{c \sin\alpha} = \frac{(1.20)(0.500 \text{ m})}{(3.00\times10^8 \text{ m/s})\sin 36.1°} = 3.40\times10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}$$

35.70

$\sin\theta_1$	$\sin\theta_2$	$\sin\theta_1 / \sin\theta_2$
0.174	0.131	1.3304
0.342	0.261	1.3129
0.500	0.379	1.3177
0.643	0.480	1.3385
0.766	0.576	1.3289
0.866	0.647	1.3390
0.940	0.711	1.3220
0.985	0.740	1.3315



The straightness of the graph line demonstrates Snell's proportionality. The slope of the line is $\bar{n} = 1.3276 \pm 0.01$

and $n =$ $\boxed{1.328 \pm 0.8\%}$

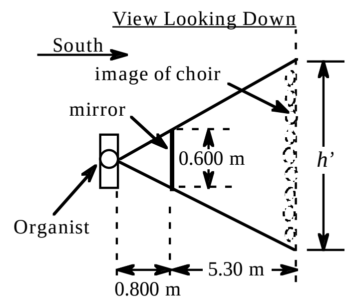
Chapter 36 Solutions

***36.1** I stand 40 cm from my bathroom mirror. I scatter light which travels to the mirror and back to me in time

$$\frac{0.8 \text{ m}}{3 \times 10^8 \text{ m/s}} \boxed{\sim 10^{-9} \text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

***36.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.



The image of the choir is $0.800 \text{ m} + 5.30 \text{ m} = 6.10 \text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}} \quad \text{or} \quad h' = (0.600 \text{ m})\left(\frac{6.10 \text{ m}}{0.800 \text{ m}}\right) = \boxed{4.58 \text{ m}}$$

36.3 The flatness of the mirror is described by $R = \infty$, $f = \infty$, and $1/f = 0$. By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \quad \text{or} \quad q = -p$$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

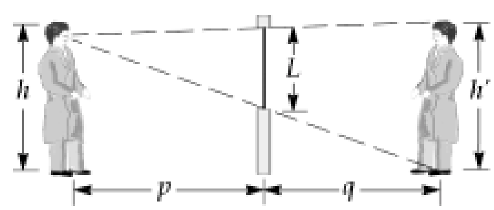


Figure for Goal Solution

$$M = \frac{-q}{p} = 1 = \frac{h'}{h} \quad \text{so} \quad h' = h = 70.0"$$

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0" high}}$.

Goal Solution

Determine the minimum height of a vertical flat mirror in which a person 5'10" in height can see his or her full image. (A ray diagram would be helpful.)

G: A diagram with the optical rays that create the image of the person is shown above. From this diagram, it appears that the mirror only needs to be about half the height of the person.

O: The required height of the mirror can be found from the mirror equation, where this flat mirror is described by

$$R = \infty,\ f = \infty, \text{ and } 1/f = 0.$$

A: The general mirror equation is $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$, so with $f = \infty$, $q = -p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ in.

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of the image, as shown. From the geometry of the similar triangles, we see that the length of the mirror must be:

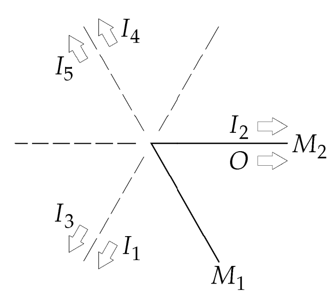
$$L = h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2} = \frac{70.0 \text{ in}}{2} = 35.0 \text{ in}.$$

Thus, the mirror must be at least 35.0 in high.

L: Our result agrees with our prediction from the ray diagram. Evidently, a full-length mirror only needs to be a half-height mirror! On a practical note, the vertical positioning of such a mirror is also important for the person to be able to view his or her full image. To allow for some variation in positioning and viewing by persons of different heights, most full-length mirrors are about 5' in length.

36.4 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O,

and (O, I_3, I_4) and (I_1, I_2, I_5) forming the vertices of equilateral triangles.



***36.5** (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft}}$ from the person.

***36.6** For a concave mirror, both R and f are positive. We also know that $f = \frac{R}{2} = 10.0 \text{ cm}$

(a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{3}{40.0 \text{ cm}}$, and $\boxed{q = 13.3 \text{ cm}}$

$$M = \frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$$

The image is 13.3 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{1}{20.0 \text{ cm}}$, and $\boxed{q = 20.0 \text{ cm}}$

$$M = \frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$$

The image is 20.0 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$.

(c) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = 0$ Thus, q = infinity.

$\boxed{\text{No image is formed.}}$ The rays are reflected parallel to each other.

***36.7** $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{0.275 \text{ m}} - \frac{1}{10.0 \text{ m}}$ gives $\boxed{q = -0.267 \text{ m}}$

Thus, the image is $\boxed{\text{virtual}}$.

$$M = \frac{-q}{p} = -\frac{-0.267}{10.0 \text{ m}} = \boxed{0.0267}$$

Thus, the image is $\boxed{\text{upright}}$ $(+M)$ and $\boxed{\text{diminished}}$ $(|M| < 1)$

***36.8** With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f = \frac{R}{2} = 1.25 \text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{so} \quad \frac{1}{2.00 \text{ m}} + \frac{1}{q} = \frac{1}{1.25 \text{ m}} \quad q = \boxed{3.33 \text{ m}}$$

36.9 (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(30.0 \text{ cm})} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(30.0 \text{ cm})} = -0.0833 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{(30.0 \text{ cm})} = \boxed{0.400}$$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{(60.0 \text{ cm})} + \frac{1}{q} = -\frac{2}{(40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{(40.0 \text{ cm})} - \frac{1}{(60.0 \text{ cm})} = -0.0666 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-15.0 \text{ cm}}$$

$$M = -\frac{q}{p} = -\frac{(-15.0 \text{ cm})}{(60.0 \text{ cm})} = \boxed{0.250}$$

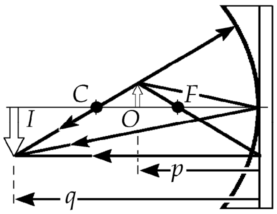
(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

36.10 (a) $M = -\frac{q}{p}$. For a real image, $q > 0$ so in this case $M = -4.00$

$q = -pM = 120$ cm and from $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$$R = \frac{2pq}{(p+q)} = \frac{2(30.0 \text{ cm})(120 \text{ cm})}{(150 \text{ cm})} = \boxed{48.0 \text{ cm}}$$

(b)



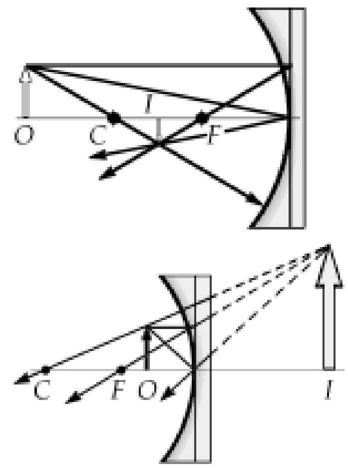
36.11 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(90.0\text{ cm})}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(45.0\text{ cm})}{(90.0\text{ cm})}=\boxed{-0.500}$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{(60.0\text{ cm})}-\frac{1}{(20.0\text{ cm})}$,

$q=\boxed{-60.0\text{ cm}}$ and $M=-\frac{q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$

(c) The image in (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.15(a) and 36.15(b), respectively.



(a)

(b)

Figures for Goal Solution

Goal Solution

A concave mirror has a radius of curvature of 60.0 cm. Calculate the image position and magnification of an object placed in front of the mirror (a) at a distance of 90.0 cm and (b) at a distance of 20.0 cm. (c) In each case, draw ray diagrams to obtain the image characteristics.

G: It is always a good idea to first draw a ray diagram for any optics problem. This gives a qualitative sense of how the image appears relative to the object. From the ray diagrams above, we see that when the object is 90 cm from the mirror, the image will be real, inverted, diminished, and located about 45 cm in front of the mirror, midway between the center of curvature and the focal point. When the object is 20 cm from the mirror, the image is be virtual, upright, magnified, and located about 50 cm behind the mirror.

O: The mirror equation can be used to find precise quantitative values.

A: (a) The mirror equation is applied using the sign conventions listed in the text.

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{90.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \qquad \text{so} \qquad q = 45.0\text{ cm (real, in front of the mirror)}$$

$$M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=-0.500\text{ (inverted)}$$

(b) $$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{or} \quad \frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{2}{60.0\text{ cm}} \qquad \text{so} \qquad q = -60.0\text{ cm (virtual, behind the mirror)}$$

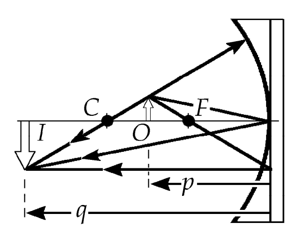
$$M=-\frac{q}{p}=-\frac{-60.0\text{ cm}}{20.0\text{ cm}}=3.00\text{ (upright)}$$

L: The calculated image characteristics agree well with our predictions. It is easy to miss a minus sign or to make a computational mistake when using the mirror-lens equation, so the qualitative values obtained from the ray diagrams are useful for a check on the reasonableness of the calculated values.

36.12 For a concave mirror, R and f are positive. Also, for an erect image, M is positive. Therefore, $M=-\frac{q}{p}=4$ and $q=-4p$.

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}=\frac{1}{p}-\frac{1}{4p}=\frac{3}{4p}; \qquad \text{from which, } p=\boxed{30.0\text{ cm}}$$

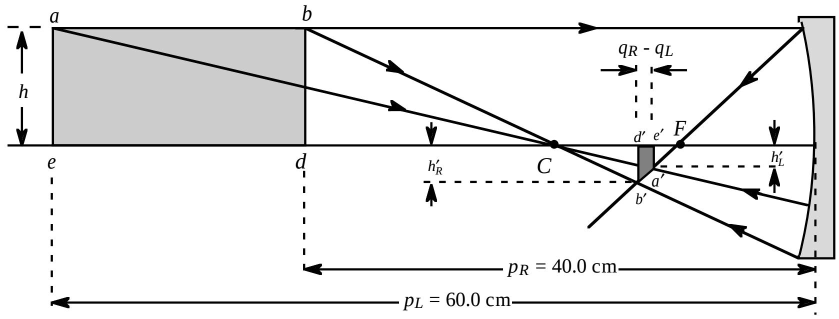
36.13 (a) $q=(p+5.00\text{ m})$ and, since the image must be real, $M=-\frac{q}{p}=-5$ or $q=5p$. Therefore, $p+5.00=5p$ or $p=1.25$ m and $q=6.25$ m.



From $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$, $R=\frac{2pq}{(q+p)}=\frac{2(1.25)(6.25)}{(6.25+1.25)}=\boxed{2.08\text{ m (concave)}}$

(b) From part (a), $p = 1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

36.14 (a) The image is the trapezoid $a'b'd'e'$ as shown in the ray diagram.



(b) To find the area of the trapezoid, the image distances, q_R and q_L, along with the heights h'_R and h'_L, must be determined. The mirror equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text{becomes} \quad \frac{1}{40.0\text{ cm}}+\frac{1}{q_R}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_R = 13.3\text{ cm}$$

$$h_R = hM_R = h\left(\frac{-q_R}{p_R}\right) = (10.0\text{ cm})\left(\frac{-13.3\text{ cm}}{40.0\text{ cm}}\right) = -3.33\text{ cm}$$

Also $$\frac{1}{60.0\text{ cm}}+\frac{1}{q_L}=\frac{2}{20.0\text{ cm}} \quad \text{or} \quad q_L = 12.0\text{ cm}$$

$$h_L = hM_L = (10.0\text{ cm})\left(\frac{-12.0\text{ cm}}{60.0\text{ cm}}\right) = -2.00\text{ cm}$$

The area of the trapezoid is the sum of the area of a square plus the area of a triangle:

$$A_t = A_1 + A_2 = (q_R - q_L)h_L + \frac{1}{2}(q_R - q_L)(h_R - h_L) = \boxed{3.56\text{ cm}^2}$$

36.15 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image (q = – 10.0 cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R = |R|$, $q = -30.0$ cm) $$\frac{1}{p}-\frac{1}{30.0}=\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{30.0\text{ cm}-p}{(30.0\text{ cm})p} \qquad [1]$$

(convex side: $R = -|R|$, $q = -10.0$ cm) $$\frac{1}{p}-\frac{1}{10.0}=-\frac{2}{|R|}, \quad \text{or} \quad \frac{2}{|R|}=\frac{p-10.0\text{ cm}}{(10.0\text{ cm})p} \qquad [2]$$

(a) Equating Equations (1) and (2) gives: $$\frac{30.0\text{ cm}-p}{3.00}=p-10.0\text{ cm or } \quad p = 15.0\text{ cm}$$ Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ($p = 15.0$ cm) in Equation [1] gives:

$$\frac{2}{|R|}=\frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})} \quad \text{or} \quad \frac{2}{|R|}=\frac{1}{30.0\text{ cm}}, \quad \text{and} \quad |R| = 60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0 \text{ cm}}$.

36.16 $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\qquad f=\frac{R}{2} = -1.50 \text{ cm}$

$\boxed{q=-\frac{15.0}{11.0} \text{ cm (behind mirror)}}$ $\qquad M=\frac{-q}{p} = \boxed{\frac{1}{11.0}}$

36.17 (a) The image starts from a point whose height above the mirror vertex is given by

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ $\qquad \frac{1}{3.00 \text{ m}}+\frac{1}{q}=\frac{1}{0.500 \text{ m}}$ Therefore, $\qquad q=0.600 \text{ m}$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1=p_1$. When this is true,

$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500 \text{ m}}=\frac{2}{p_1}$ $\qquad$ or $\qquad p_1=1.00 \text{ m}$.

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2=q_2=0$.

(b) The falling ball passes its real image when it has fallen

$3.00 \text{ m}-1.00 \text{ m}=2.00 \text{ m}=\frac{1}{2}gt^2$, or when $t=\sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{0.639 \text{ s}}$.

The ball reaches its virtual image when it has traversed

$3.00 \text{ m}-0=3.00 \text{ m}=\frac{1}{2}gt^2$, or at $t=\sqrt{\frac{2(3.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{0.782 \text{ s}}$.

36.18 When $R \to \infty$, Equation 36.8 for a spherical surface becomes $q=-p(n_2/n_1)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate:

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00 \text{ cm})=-6.41 \text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass or 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41 \text{ cm}) = -13.84 \text{ cm} \quad \text{or} \quad 13.84 \text{ cm below the water surface.}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0 \text{ cm}) = -9.02 \text{ cm}, \text{ or } 9.02 \text{ cm below the water surface.}$$

Therefore, the apparent thickness of the glass is $\Delta t = 13.84 \text{ cm} - 9.02 \text{ cm} = \boxed{4.82 \text{ cm}}$

36.19 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = 0$ and $R \to \infty$

$$q = -\frac{n_2}{n_1}\, p = -\frac{1}{1.309}(50.0 \text{ cm}) = -38.2 \text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

***36.20** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ so $\frac{1.00}{\infty} + \frac{1.40}{21.0 \text{ mm}} = \frac{1.40 - 1.00}{6.00 \text{ mm}}$ and $0.0667 = 0.0667$

They agree. $\boxed{\text{The image is inverted, real and diminished.}}$

36.21 From Equation 36.8, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$

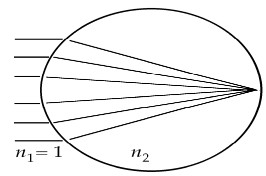
Solve for q to find $q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}$

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $R = -15.0$ cm, and $p = 10.0$ cm,

So $q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57$ cm

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$.

36.22 $p = \infty$ and $q = +2R$



$$\frac{1.00}{p} + \frac{n_2}{q} = \frac{n_2 - 1.00}{R}$$

$0 + \frac{n_2}{2R} = \frac{n_2 - 1.00}{R}$ so $\boxed{n_2 = 2.00}$

36.23 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$ because $\frac{1.00}{p} + \frac{1.50}{q} = \frac{1.50 - 1.00}{6.00\text{ cm}} = \frac{1.00}{12.0\text{ cm}}$

(a) $\frac{1}{20.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{20.0\text{ cm}}\right]} = \boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{10.0\text{ cm}}\right]} = \boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.00\text{ cm}} + \frac{1.50}{q} = \frac{1.00}{12.0\text{ cm}}$ or $q = \frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}} - \frac{1.00}{3.00\text{ cm}}\right]} = \boxed{-6.00\text{ cm}}$

36.24 For a plane surface, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right| = \frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}} = \left|\frac{dq}{dt}\right| = \frac{1.00}{1.33}(2.00\text{ cm/s}) = \boxed{1.50\text{ cm/s}}$$

36.25 $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ $\qquad n_1 = 1.33 \qquad n_2 = 1.00 \qquad p = +10.0 \text{ cm} \qquad R = -15.0 \text{ cm}$

$q = -9.01 \text{ cm}$, or the fish appears to be $\boxed{9.01 \text{ cm inside the bowl}}$

***36.26** Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{2.00 \text{ m}} - \frac{1}{2.50 \text{ cm}}\right) = \frac{0.0500}{\text{cm}} \qquad \text{so} \qquad f = \boxed{20.0 \text{ cm}}$$

36.27 (a) $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (0.440)\left[\frac{1}{(12.0 \text{ cm})} - \frac{1}{(-18.0 \text{ cm})}\right]$: $f = \boxed{16.4 \text{ cm}}$

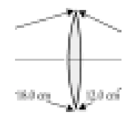


Figure for Goal Solution

(b) $\frac{1}{f} = (0.440)\left[\frac{1}{(18.0 \text{ cm})} - \frac{1}{(-12.0 \text{ cm})}\right]$: $f = \boxed{16.4 \text{ cm}}$

Goal Solution

The left face of a biconvex lens has a radius of curvature of magnitude 12.0 cm, and the right face has a radius of curvature of magnitude 18.0 cm. The index of refraction of the glass is 1.44. (a) Calculate the focal length of the lens. (b) Calculate the focal length if the radii of curvature of the two faces are interchanged.

G: Since this is a biconvex lens, the center is thicker than the edges, and the lens will tend to converge incident light rays. Therefore it has a positive focal length. Exchanging the radii of curvature amounts to turning the lens around so the light enters the opposite side first. However, this does not change the fact that the center of the lens is still thicker than the edges, so we should not expect the focal length of the lens to be different (assuming the thin-lens approximation is valid).

O: The lens makers' equation can be used to find the focal length of this lens.

A: The centers of curvature of the lens surfaces are on opposite sides, so the second surface has a negative radius:

(a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.44 - 1.00)\left(\frac{1}{12.0 \text{ cm}} - \frac{1}{-18.0 \text{ cm}}\right)$ so $f = 16.4 \text{ cm}$

(b) $\frac{1}{f} = (0.440)\left(\frac{1}{18.0 \text{ cm}} - \frac{1}{-12.0 \text{ cm}}\right)$ so $f = 16.4 \text{ cm}$

L: As expected, reversing the orientation of the lens does not change what it does to the light, as long as the lens is relatively thin (variations may be noticed with a thick lens). The fact that light rays can be traced forward or backward through an optical system is sometimes referred to as the **principle of reversibility**. We can see that the focal length of this biconvex lens is about the same magnitude as the average radius of curvature. A few approximations, useful as checks, are that a symmetric biconvex lens with radii of magnitude R will have focal length $f \approx R$; a plano-convex lens with radius R will have $f \approx R/2$; and a symmetric biconcave lens has $f \approx -R$. These approximations apply when the lens has $n \approx 1.5$, which is typical of many types of clear glass and plastic.

***36.28** For a converging lens, f is positive. We use $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$.

(a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{1}{40.0 \text{ cm}}$ $\boxed{q = 40.0 \text{ cm}}$

$M = -\frac{q}{p} = -\frac{40.0}{40.0} = \boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = 0$ $\boxed{q = \text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = -\frac{1}{20.0 \text{ cm}}$ $\boxed{q = -20.0 \text{ cm}}$

$M = \frac{-q}{p} = -\frac{-20.0}{10.0} = \boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$, and 20.0 cm in front of the lens.

***36.29** (a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0 \text{ cm}} - \frac{1}{26.0 \text{ cm}}$ $q = \boxed{650 \text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0 \text{ cm}} - \frac{1}{24.0 \text{ cm}}$ $q = \boxed{-600 \text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

36.30 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{(32.0 \text{ cm})} + \frac{1}{(8.00 \text{ cm})} = \frac{1}{f}$ so $\boxed{f = 6.40 \text{ cm}}$

(b) $M = \frac{-q}{p} = \frac{-(8.00 \text{ cm})}{(32.00 \text{ cm})} = \boxed{-0.250}$

(c) Since $f > 0$, the lens is $\boxed{\text{converging}}$.

36.31 We are looking at an enlarged, upright, virtual image:

$$M = \frac{h'}{h} = 2 = -\frac{q}{p} \qquad \text{so} \qquad p = -\frac{q}{2} = -\frac{-2.84\text{ cm}}{2} = +1.42\text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \text{gives} \qquad \frac{1}{1.42\text{ cm}} + \frac{1}{-2.84\text{ cm}} = \frac{1}{f}$$

$f = \boxed{2.84\text{ cm}}$

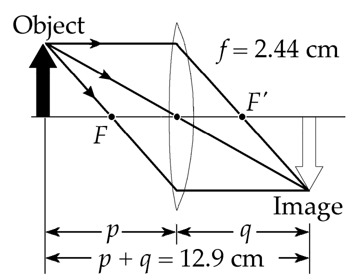
***36.32** To use the lens as a magnifying glass, we form an upright, virtual image:

$$M = +2.00 = \frac{-q}{p} \qquad \text{or} \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

We eliminate $q = -2.00p$::
$$\frac{1}{p} + \frac{1}{-2.00p} = \frac{1}{15.0\text{ cm}} \qquad \text{or} \qquad \frac{-2.00 + 1.00}{-2.00p} = \frac{1}{15.0\text{ cm}}$$

Solving, $p = \boxed{7.50\text{ cm}}$

36.33 (a) Note that $q = 12.9\text{ cm} - p$



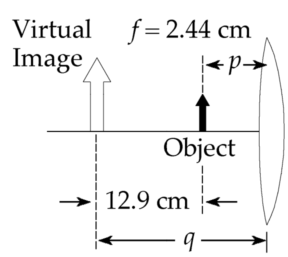
so
$$\frac{1}{p} + \frac{1}{12.9 - p} = \frac{1}{2.44}$$

which yields a quadratic in p: $-p^2 + 12.9p = 31.5$

which has solutions $\boxed{p = 9.63\text{ cm or } p = 3.27\text{ cm}}$

Both solutions are valid.

(b) For a virtual image, $-q = p + 12.9\text{ cm}$



$$\frac{1}{p} - \frac{1}{12.9 + p} = \frac{1}{2.44}$$

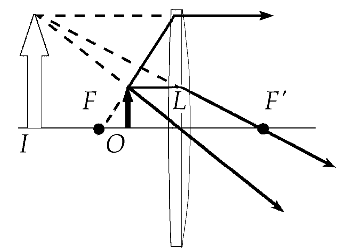
or $p^2 + 12.9p = 31.8$

from which $\boxed{p = 2.10\text{ cm}}$ or $p = -15.0$ cm.

We must have a real object so the -15.0 cm solution must be rejected.

36.34 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $\frac{1}{p} + \frac{1}{-30.0\text{ cm}} = \frac{1}{12.5\text{ cm}}$

$p = 8.82$ cm $M = -\frac{q}{p} = -\frac{(-30.0)}{8.82} = \boxed{3.40,\text{ upright}}$



(b) See the figure to the right.

***36.35** $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $p^{-1} + q^{-1} = \text{constant}$

We may differentiate through with respect to p: $-1p^{-2} - 1q^{-2}\frac{dq}{dp} = 0$

$$\frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2$$

36.36 The image is inverted: $M = \frac{h'}{h} = \frac{-1.80\text{ m}}{0.0240\text{ m}} = -75.0 = \frac{-q}{p}$ $q = 75.0p$

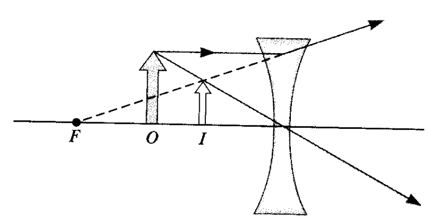
(b) $q + p = 3.00\text{ m} = 75.0p + p$ $p = \boxed{39.5\text{ mm}}$

(a) $q = 2.96$ m $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.0395\text{ m}} + \frac{1}{2.96\text{ m}}$

$f = \boxed{39.0\text{ mm}}$

36.37 (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{(20.0\text{ cm})} + \frac{1}{q} = \frac{1}{(-32.0\text{ cm})}$

so $q = -\left(\frac{1}{20.0} + \frac{1}{32.0}\right)^{-1} = \boxed{-12.3\text{ cm}}$



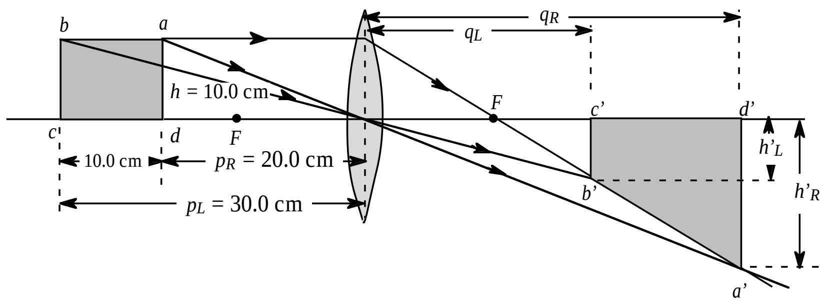
The image is 12.3 cm to the left of the lens.

(b) $M = -\frac{q}{p} = -\frac{(-12.3\text{ cm})}{(20.0\text{ cm})} = \boxed{0.615}$

(c) See the ray diagram above.

36.38 (a) $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.50-1)\left[\frac{1}{15.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$, or $\boxed{f=13.3\text{ cm}}$

(b) Ray Diagram:



(c) To find the area, first find q_R and q_L, along with the heights h'_R and h'_L, using the thin lens equation.

$\frac{1}{p_R}+\frac{1}{q_R}=\frac{1}{f}$ becomes: $\frac{1}{20.0\text{ cm}}+\frac{1}{q_R}=\frac{1}{13.3\text{ cm}}$ or $q_R=40.0\text{ cm}$

$$h'_R=hM_R=h\left(\frac{-q_R}{p_R}\right)=(10.0\text{ cm})(-2.00)=-20.0\text{ cm}$$

$\frac{1}{30.0\text{ cm}}+\frac{1}{q_L}=\frac{1}{13.3\text{ cm}}$: $q_L=24.0\text{ cm}$

$$h'_L=hM_L=(10.0\text{ cm})(-0.800)=-8.00\text{ cm}$$

Thus, the area of the image is: Area = $|q_R-q_L||h'_L|+\frac{1}{2}|q_R-q_L||h'_R-h'_L|=\boxed{224\text{ cm}^2}$

36.39 (a) The distance from the object to the lens is p, so the image distance is $q=5.00\text{ m}-p$.

Thus, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes: $\frac{1}{p}+\frac{1}{5.00\text{ m}-p}=\frac{1}{0.800\text{ m}}$

This reduces to a quadratic equation: $p^2-(5.00\text{ m})p+(4.00\text{ m})=0$

which yields $\boxed{p=4.00\text{ m, or } p=1.00\text{ m}}$.

Thus, there are two possible object distances, both corresponding to real objects.

(b) For $p=4.00$ m: $q=5.00\text{ m}-4.00\text{ m}=1.00\text{ m}$: $M=-\frac{1.00\text{ m}}{4.00\text{ m}}=\boxed{-0.250}$.

For p = 1.00 m: $q=5.00\text{ m}-1.00\text{ m}=4.00\text{ m}$: $M=-\frac{4.00\text{ m}}{1.00\text{ m}}=\boxed{-4.00}$.

Both images are $\boxed{\text{real and inverted}}$, but the magnifications are different, with one being larger than the object and the other smaller.

36.40 (a) The image distance is: $q = d - p$. Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d-p} = \frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + (fd) = 0$

which yields: $\boxed{p = \dfrac{d \pm \sqrt{d^2 - 4fd}}{2} = \left(\dfrac{d}{2}\right) \pm \sqrt{\dfrac{d^2}{4} - fd}}$

Since $f < d/4$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$.

The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$.

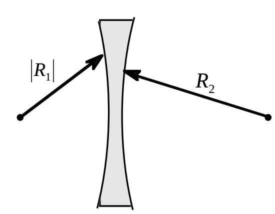
***36.41** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$ becomes $\frac{1}{2000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$ and $q_2 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right)$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}$$

***36.42** (a) The focal length of the lens is given by

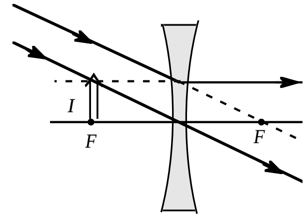


$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

$f = -34.7$ cm

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.

When $p = \infty$, the thin lens equation gives $q = f$. Thus, the violet image of a very distant object is formed at $\boxed{q = -34.7\text{ cm}}$. The image is $\boxed{\text{virtual, upright, and diminished}}$.



(b) The same ray diagram and image characteristics apply for red light. Again, $q = f$, and now

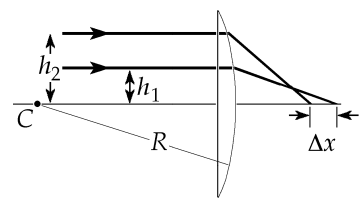
$$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right) \text{ giving } f = \boxed{-36.1\text{ cm}}.$$

36.43 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°$$

Then, $(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$ so $\theta_2 = 2.29°$

The angle this emerging ray makes with the horizontal is



$$\theta_2 - \theta_1 = 0.860°$$

It crosses the axis at a point farther out by f_1 where

$$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500\text{ cm}}{\tan(0.860°)} = 33.3\text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} = 0.00625\text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3\text{ cm} - 0.00625\text{ cm} = 33.3\text{ cm}$$

Now we repeat this calculation for ray h_2: $\theta_1 = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9°$

$$(1.00)\sin\theta_2 = (1.60)\sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_2 - \theta_1)} = \frac{12.0\text{ cm}}{\tan(36.8°)} = 16.0\text{ cm}$$

$$x_2 = (16.0\text{ cm})\left(20.0\text{ cm} - \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2}\right) = 12.0\text{ cm}$$

Now $\Delta x = 33.3\text{ cm} - 12.0\text{ cm} = \boxed{21.3\text{ cm}}$

36.44 For starlight going through Nick's glasses, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$$\frac{1}{\infty}+\frac{1}{(-0.800\text{ m})}=\frac{1}{f}=-1.25\text{ diopters}$$

For a nearby object, $\frac{1}{p}+\frac{1}{(-0.180\text{ m})}=-1.25\text{ m}^{-1}$, so $p=\boxed{23.2\text{ cm}}$

36.45 $P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{\infty}-\frac{1}{0.250\text{ m}}=-4.00\text{ diopters}=\boxed{-4.00\text{ diopters, a diverging lens}}$

36.46 Consider an object at infinity, imaged at the person's far point:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\qquad \frac{1}{\infty}+\frac{1}{q}=-4.00\text{ m}^{-1}\qquad q=-25.0\text{ cm}$$

The person's far point is $25.0\text{ cm}+2.00\text{ cm}=27.0\text{ cm}$ from his eyes. For the contact lenses we want

$$\frac{1}{\infty}+\frac{1}{(-0.270\text{ m})}=\frac{1}{f}=\boxed{-3.70\text{ diopters}}$$

36. 47 First, we use the thin lens equation to find the object distance: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{10.0\text{ cm}}$

Then, $p=7.14\text{ cm}$ and Then, $M=-\frac{q}{p}=-\frac{(-25.0\text{ cm})}{7.14\text{ cm}}=\boxed{3.50}$

36.48 (a) From the thin lens equation: $\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{5.00\text{ cm}}$ or $p=\boxed{4.17\text{ cm}}$

(b) $M=-\frac{q}{p}=1+\frac{25.0\text{ cm}}{f}=1+\frac{25.0\text{ cm}}{5.00\text{ cm}}=\boxed{6.00}$

36.49 Using Equation 36.20, $M=-\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right)=-\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right)=\boxed{-575}$

36.50 $$M = M_1 m_e = M_1\left(\frac{25.0\text{ cm}}{f_e}\right) \Rightarrow f_e = \left(\frac{M_1}{M}\right)(25.0\text{ cm}) = \left(\frac{-12.0}{-140}\right)(25.0\text{ cm}) = \boxed{2.14\text{ cm}}$$

36.51 $f_o = 20.0\text{ m}$ $\quad f_e = 0.0250\text{ m}$

(a) The angular magnification produced by this telescope is: $m = -\frac{f_o}{f_e} = \boxed{-800}$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

***36.52** (a) The lensmaker's equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

gives $q=\frac{1}{1/f-1/p}=\frac{1}{\left(\frac{p-f}{fp}\right)}=\frac{fp}{p-f}$

Then, $M=\frac{h'}{h}=-\frac{q}{p}=-\frac{f}{p-f}$

gives $\boxed{h'=\frac{hf}{f-p}}$

(b) For $p>>f$, $f-p\approx -p$. Then, $h'=\boxed{-\frac{hf}{p}}$.

(c) Suppose the telescope observes the space station at the zenith:

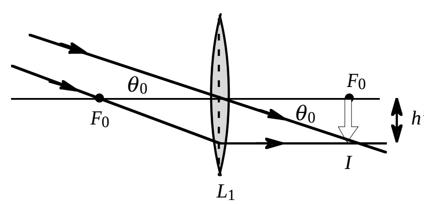
$$h'=-\frac{hf}{p}=-\frac{(108.6\text{ m})(4.00\text{ m})}{407\times 10^{3}\text{ m}}=\boxed{-1.07\text{ mm}}$$

***36.53** (b) Call the focal length of the objective f_o and that of the eyepiece $-|f_e|$. The distance between the lenses is $f_o-|f_e|$. The objective forms a real diminished inverted image of a very distant object at $q_1=f_o$. This image is a virtual object for the eyepiece at $p_2=-|f_e|$.

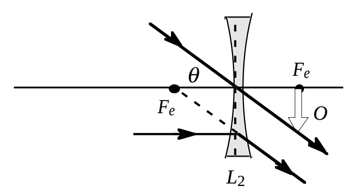
For it $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{-|f_e|}+\frac{1}{q}=\frac{1}{-|f_e|}$, $\frac{1}{q_2}=0$

and $\boxed{q_2=\infty}$

(a) The user views the image as $\boxed{\text{virtual}}$. Letting h' represent the height of the first image, $\theta_o=h'/f_o$ and $\theta=h'/|f_e|$. The angular magnification is



$$m=\frac{\theta}{\theta_o}=\frac{h'/|f_e|}{h'/f_o}=\frac{f_o}{|f_e|}$$



(c) Here, $f_o-|f_e|=10.0$ cm and $\frac{f_o}{|f_e|}=3.00$.

Thus, $|f_e|=\frac{f_o}{3.00}$ and $\frac{2}{3}f_o=10.0$ cm.

$f_o=\boxed{15.0\text{ cm}}$

$|f_e|=5.00$ cm and $f_e=\boxed{-5.00\text{ cm}}$

***36.54** Let I_0 represent the intensity of the light from the nebula and θ_o its angular diameter. With the first telescope, the image diameter h' on the film is given by $\theta_o = -h'/f_o$ as $h' = -\theta_o(2000\text{ mm})$.

The light power captured by the telescope aperture is $P_1 = I_0 A_1 = I_0\left[\pi(200\text{ mm})^2/4\right]$, and the light energy focused on the film during the exposure is $E_1 = P_1 t_1 = I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is $P_2 = I_0 A_2 = I_0\left[\pi(60.0\text{ mm})^2/4\right]$ and the light energy is $E_2 = I_0\left[\pi(60.0\text{ mm})^2/4\right]t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0\text{ mm})^2/4\right]t_2}{\pi\left[\theta_o(900\text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})}{\pi\left[\theta_o(2000\text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$t_2 = \frac{(200\text{ mm})^2(900\text{ mm})^2}{(60.0\text{ mm})^2(2000\text{ mm})^2}(1.50\text{ min}) = \boxed{3.38\text{ min}}$$

36.55 Only a diverging lens gives an upright diminished image. The image is virtual and

$d = p - |q| = p + q$: $\quad M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md}$$

$$f = \frac{-Md}{(1-M)^2} = \frac{-(0.500)(20.0\text{ cm})}{(1-0.500)^2} = \boxed{-40.0\text{ cm}}$$

36.56 If $M < 1$, the lens is diverging and the image is virtual. $\quad d = p - |q| = p + q$

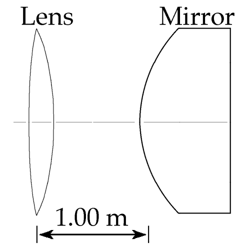
$M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$$p = \frac{d}{1-M}: \qquad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M+1}{-Mp} = \frac{(1-M)^2}{-Md} \qquad \boxed{f = \frac{-Md}{(1-M)^2}}$$

If $M > 1$, the lens is converging and the image is still virtual.

Now $d = -q - p$. We obtain in this case $\quad \boxed{f = \dfrac{Md}{(M-1)^2}}$

***36.57** Start with the first pass through the lens.



$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0 \text{ cm}} - \frac{1}{100 \text{ cm}} \qquad q_1 = 400 \text{ cm to right of lens}$$

For the mirror, $p_2 = -300$ cm

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-50.0 \text{ cm}} - \frac{1}{-300 \text{ cm}} \qquad q_2 = -60.0 \text{ cm}$$

For the second pass through the lens, $p_3 = 160$ cm

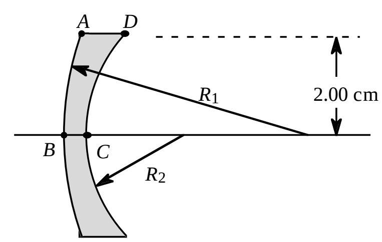
$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0 \text{ cm}} - \frac{1}{160 \text{ cm}} \qquad q_3 = \boxed{160 \text{ cm to left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400 \text{ cm}}{100 \text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{-60.0 \text{ cm}}{-300 \text{ cm}} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160 \text{ cm}}{160 \text{ cm}} = -1 \qquad M = M_1 M_2 M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

***36.58** (a) $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



$$\frac{1}{-65.0 \text{ cm}} = (1.66-1)\left(\frac{1}{50.0 \text{ cm}} - \frac{1}{R_2}\right)$$

$$\frac{1}{R_2} = \frac{1}{50.0 \text{ cm}} + \frac{1}{42.9 \text{ cm}} \qquad \text{so} \qquad R_2 = \boxed{23.1 \text{ cm}}$$

(b) The distance along the axis from B to A is

$$R_1 - \sqrt{R_1^2 - (2.00 \text{ cm})^2} = 50.0 \text{ cm} - \sqrt{(50.0 \text{ cm})^2 - (2.00 \text{ cm})^2} = 0.0400 \text{ cm}$$

Similarly, the axial distance from C to D is

$$23.1 \text{ cm} - \sqrt{(23.1 \text{ cm})^2 - (2.00 \text{ cm})^2} = 0.0868 \text{ cm}$$

Then, $AD = 0.100 \text{ cm} - 0.0400 \text{ cm} + 0.0868 \text{ cm} = \boxed{0.147 \text{ cm}}$.

***36.59** $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0\text{ cm}} - \frac{1}{12.5\text{ cm}}$ so $q_1 = 50.0$ cm (to left of mirror)

This serves as an object for the lens (a virtual object), so

$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-16.7\text{ cm}} - \frac{1}{-25.0\text{ cm}}$ $q_2 = -50.3$ cm (to right of lens)

Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{-50.3\text{ cm}}{-25.0\text{ cm}} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

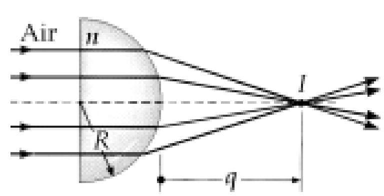
36.60 We first find the focal length of the mirror.

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0\text{ cm}} + \frac{1}{8.00\text{ cm}} = \frac{9}{40.0\text{ cm}}$ and $f = 4.44$ cm

Hence, if $p = 20.0$ cm, $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{4.44\text{ cm}} - \frac{1}{20.0\text{ cm}} = \frac{15.56}{88.8\text{ cm}}$

Thus, $q = \boxed{5.71\text{ cm}}$, real

36.61 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm



The incident rays are parallel, so $p = \infty$.

Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $0 + \frac{1}{q} = \frac{(1.00 - 1.56)}{-6.00\text{ cm}}$ and $\boxed{q = 10.7\text{ cm}}$

***36.62** (a) $I = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{4.50\text{ W}}{4\pi\left(1.60\times10^{-2}\text{ m}\right)^2} = \boxed{1.40\text{ kW/m}^2}$

(b) $I = \dfrac{\mathcal{P}}{4\pi r^2} = \dfrac{4.50\text{ W}}{4\pi(7.20\text{ m})^2} = \boxed{6.91\text{ mW/m}^2}$

(c) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} \Rightarrow \dfrac{1}{7.20\text{ m}} + \dfrac{1}{q} = \dfrac{1}{0.350\text{ m}}$ so $q = 0.368\text{ m}$ and

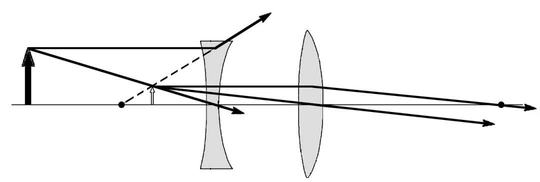
$M = \dfrac{h'}{3.20\text{ cm}} = -\dfrac{q}{p} = -\dfrac{0.368\text{ m}}{7.20\text{ m}}$ $\qquad h' = \boxed{0.164\text{ cm}}$

(d) The lens intercepts power given by $\mathcal{P} = IA = \left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\frac{1}{4}\pi(0.150\text{ m})^2\right]$

and puts it all onto the image where $I = \dfrac{\mathcal{P}}{A} = \dfrac{\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\pi(15.0\text{ cm})^2/4\right]}{\pi(0.164\text{ cm})^2/4} = \boxed{58.1\text{ W/m}^2}$

36.63 From the thin lens equation, $q_1 = \dfrac{f_1 p_1}{p_1 - f_1} = \dfrac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$;



In this case, $p_2 = d - (-4.00\text{ cm})$

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$

and $d = \boxed{8.00\text{ cm}}$

***36.64** (a) For the light the mirror intercepts, $\mathcal{P} = I_0 A = I_0 \pi R_a^2$

$350\text{ W} = \left(1000\text{ W/m}^2\right)\pi R_a^2$ and $R_a = \boxed{0.334\text{ m or larger}}$

(b) In $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{2}{R}$ we have $p \to \infty$ so $q = \dfrac{R}{2}$.

$M = \dfrac{h'}{h} = -\dfrac{q}{p}$, so $h' = -q(h/p) = -\left(\dfrac{R}{2}\right)\left[0.533°\left(\dfrac{\pi\text{ rad}}{180°}\right)\right] = -\left(\dfrac{R}{2}\right)(9.30\text{ m rad})$

where h/p is the angle the Sun subtends. The intensity at the image is then

$$I = \frac{\mathcal{P}}{\pi h'^2/4} = \frac{4I_0\pi R_a^2}{\pi h'^2} = \frac{4I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$$

$120\times10^3\text{ W/m}^2 = \dfrac{16\left(1000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$ so $\boxed{\dfrac{R_a}{R} = 0.0255\text{ or larger}}$

36.65 For the mirror, $f = R/2 = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other figures, we can take $p = \infty$. The mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \text{ then gives } q = f = \boxed{1.50 \text{ m}}.$$

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$, the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi}{180} \text{ rad/deg}\right)(1.50 \text{ m}) = -0.140 \text{ m} = \boxed{-1.40 \text{ cm}}$$

36.66 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, becomes:

$$\frac{1}{5.00 \text{ cm}} = (n-1)\left[\frac{1}{9.00 \text{ cm}} - \frac{1}{(-11.0 \text{ cm})}\right] \quad \text{giving} \quad n = \boxed{1.99}.$$

(b) As the light passes through the lens for the first time, the thin lens equation

$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$ becomes: $\frac{1}{8.00 \text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00 \text{ cm}}$

or $q_1 = 13.3$ cm, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3 \text{ cm}}{8.00 \text{ cm}} = -1.67$

This image becomes the object for the concave mirror with:

$$p_m = 20.0 \text{ cm} - q_1 = 20.0 \text{ cm} - 13.3 \text{ cm} = 6.67 \text{ cm}, \text{ and } f = \frac{R}{2} = +4.00 \text{ cm}.$$

The mirror equation becomes: $\frac{1}{6.67 \text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00 \text{ cm}}$

giving $q_m = 10.0$ cm and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0 \text{ cm}}{6.67 \text{ cm}} = -1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0 \text{ cm} - q_m = +10.0 \text{ cm}$$

The thin lens equation yields: $\frac{1}{10.0 \text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00 \text{ cm}}$

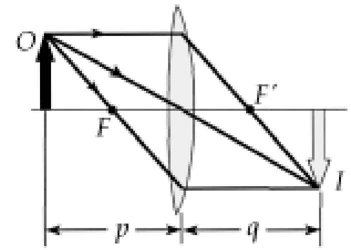
or $q_3 = 10.0$ cm, and $M_3 = -\frac{q_3}{p_3} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00.$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{total} = M_1 M_2 M_3 = \boxed{-2.50}$.

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

36.67 In the original situation, $p_1 + q_1 = 1.50\text{ m}$



In the final situation, $p_2 = p_1 + 0.900\text{ m}$

and $q_2 = q_1 - 0.900\text{ m}.$

Our lens equation is $$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$$

Substituting, we have $$\frac{1}{p_1} + \frac{1}{1.50\text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}$$

Adding the fractions, $$\frac{1.50\text{ m} - p_1 + p_1}{p_1(1.50\text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$$

Simplified, this becomes $$p_1(1.50\text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$$

(a) Thus, $$p_1 = \frac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$$

$$p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$$

(b) $$\frac{1}{f} = \frac{1}{0.300\text{ m}} + \frac{1}{1.50\text{ m} - 0.300\text{ m}} \quad \text{and} \quad f = \boxed{0.240\text{ m}}$$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$, with $M = -\dfrac{q_2}{p_2} = \boxed{-0.250}$

36.68 As the light passes through, the lens attempts to form an image at distance q_1 where

$$\frac{1}{q_1} = \frac{1}{f} - \frac{1}{p_1} \qquad \text{or} \qquad q_1 = \frac{fp_1}{p_1 - f}$$

This image serves as a virtual object for the mirror with $p_2 = -q_1$. The plane mirror then forms an image located at $q_2 = -p_2 = +q_1$ above the mirror and lens.

This second image serves as a virtual object ($p_3 = -q_2 = -q_1$) for the lens as the light makes a return passage through the lens. The final image formed by the lens is located at distance q_3 above the lens where

$$\frac{1}{q_3} = \frac{1}{f} - \frac{1}{p_3} = \frac{1}{f} + \frac{1}{q_1} = \frac{1}{f} + \frac{p_1 - f}{fp_1} = \frac{2p_1 - f}{fp_1} \qquad \text{or} \qquad q_3 = \frac{fp_1}{2p_1 - f}$$

If the final image coincides with the object, it is necessary to require $q_3 = p_1$, or $\dfrac{fp_1}{2p_1 - f} = p_1$.

This yields the solution $\boxed{p_1 = f}$ or $\boxed{\text{the object must be located at the focal point of the lens}}$.

36.69 For the objective: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ becomes $\frac{1}{3.40\text{ mm}}+\frac{1}{q}=\frac{1}{3.00\text{ mm}}$ so $q=25.5\text{ mm}$

The objective produces magnification $M_1=-q/p=-\frac{25.5\text{ mm}}{3.40\text{ mm}}=-7.50$

For the eyepiece as a simple magnifier, $m_e=\frac{25.0\text{ cm}}{f}=\frac{25.0\text{ cm}}{2.50\text{ cm}}=10.0$

and overall $M=M_1 m_e=\boxed{-75.0}$

36.70 (a) Start with the second lens: This lens must form a virtual image located 19.0 cm to the left of it (i.e., $q_2=-19.0\text{ cm}$). The required object distance for this lens is then

$$p_2=\frac{q_2 f_2}{q_2-f_2}=\frac{(-19.0\text{ cm})(20.0\text{ cm})}{-19.0\text{ cm}-20.0\text{ cm}}=\frac{380\text{ cm}}{39.0}$$

The image formed by the first lens serves as the object for the second lens. Therefore, the image distance for the first lens is

$$q_1=50.0\text{ cm}-p_2=50.0\text{ cm}-\frac{380\text{ cm}}{39.0}=\frac{1570\text{ cm}}{39.0}$$

The distance the original object must be located to the left of the first lens is then given by

$$\frac{1}{p_1}=\frac{1}{f_1}-\frac{1}{q_1}=\frac{1}{10.0\text{ cm}}-\frac{39.0}{1570\text{ cm}}=\frac{157-39.0}{1570\text{ cm}}=\frac{118}{1570\text{ cm}} \quad\text{or}\quad p_1=\frac{1570\text{ cm}}{118}=\boxed{13.3\text{ cm}}$$

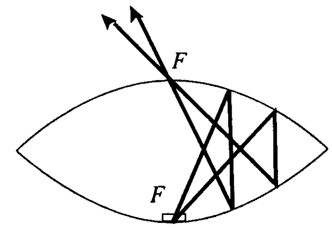
(b) $$M=M_1M_2=\left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)=\left[\left(\frac{1570\text{ cm}}{39.0}\right)\left(\frac{118}{1570\text{ cm}}\right)\right]\left[\frac{(-19.0\text{ cm})(39.0)}{380\text{ cm}}\right]=\boxed{-5.90}$$

(c) Since $M<0$, the final image is $\boxed{\text{inverted}}$.

36.71 (a) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.0224\text{ m})}+\frac{1}{\infty}=\boxed{44.6\text{ diopters}}$$

(b) $$P=\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{(0.330\text{ m})}+\frac{1}{\infty}=\boxed{3.03\text{ diopters}}$$

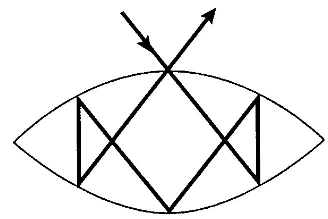
36.72 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).



The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. Thus, the **image is real, inverted, and actual size**.

For the upper mirror:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{7.50\text{ cm}}+\frac{1}{q_1}=\frac{1}{7.50\text{ cm}}: \; q_1=\infty$$



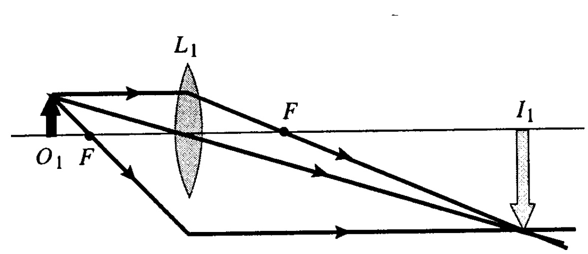
For the lower mirror:

$$\frac{1}{\infty}+\frac{1}{q_2}=\frac{1}{7.50\text{ cm}}: \qquad q_2=7.50\text{ cm}$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

36.73 (a) Lens one:

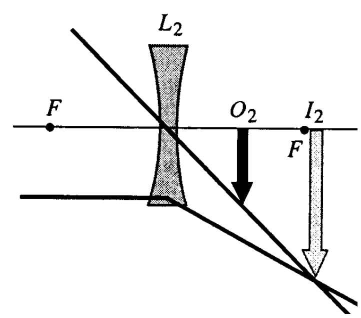
$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}: \qquad q_1=120\text{ cm}$$



$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

This real image is a virtual object for the second lens, at

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$



$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}: \qquad q_2=\boxed{20.0\text{ cm}}$$

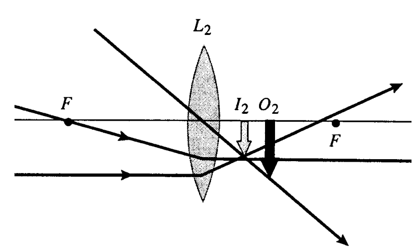
$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is **inverted**.

(c) Lens two converging: $$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2=\boxed{6.67\text{ cm}}$$



$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

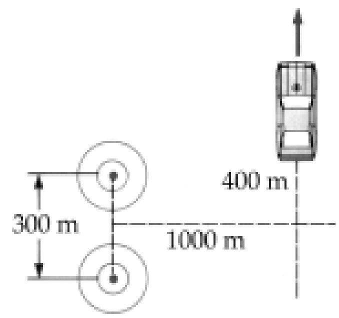
Again, $M_{\text{overall}}<0$ and the final image is **inverted**.

Chapter 37 Solutions

37.1 $\Delta y_{\text{bright}} = \frac{\lambda L}{d} = \frac{(632.8 \times 10^{-9})(5.00)}{2.00 \times 10^{-4}}$ m = $\boxed{1.58 \text{ cm}}$

37.2 $y_{\text{bright}} = \frac{\lambda L}{d} m$ For $m = 1$, $\lambda = \frac{yd}{L} = \frac{(3.40 \times 10^{-3} \text{ m})(5.00 \times 10^{-4} \text{ m})}{3.30 \text{ m}} = \boxed{515 \text{ nm}}$

37.3 Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. The approach to be used is outlined below.



(a) At the $m = 2$ maximum, $\tan\theta = \frac{400 \text{ m}}{1000 \text{ m}} = 0.400$

$\theta = 21.8°$ so $\lambda = \frac{d \sin\theta}{m} = \frac{(300 \text{ m}) \sin 21.8°}{2} = \boxed{55.7 \text{ m}}$

(b) The next minimum encountered is the $m = 2$ minimum; and at that point,

$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ which becomes $d \sin\theta = \frac{5}{2}\lambda$

or $\sin\theta = \frac{5\lambda}{2d} = \frac{5(55.7 \text{ m})}{2(300 \text{ m})} = 0.464$ and $\theta = 27.7°$

so $y = (1000 \text{ m}) \tan 27.7° = 524 \text{ m}$

Therefore, the car must travel an additional $\boxed{124 \text{ m}}$.

37.4 $\lambda = \frac{v}{f} = \frac{354 \text{ m/s}}{2000/\text{s}} = 0.177 \text{ m}$

(a) $d \sin\theta = m\lambda$ so $(0.300 \text{ m}) \sin\theta = 1(0.177 \text{ m})$ and $\theta = \boxed{36.2°}$

(b) $d \sin\theta = m\lambda$ so $d \sin 36.2° = 1(0.0300 \text{ m})$ and $d = \boxed{5.08 \text{ cm}}$

(c) $(1.00 \times 10^{-6} \text{ m}) \sin 36.2° = 1\lambda$ so $\lambda = 590 \text{ nm}$

$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8} \text{ m/s}}{5.90 \times 10^{-7} \text{ m}} = \boxed{508 \text{ THz}}$

37.5 For the tenth minimum, $m = 9$. Using Equation 37.3, $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = \frac{y}{L}$. For small θ, $\sin\theta \approx \tan\theta$. Thus,

$$d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y} = \frac{9.5(589 \times 10^{-9}\text{ m})(2.00\text{ m})}{7.26 \times 10^{-3}\text{ m}} = 1.54 \times 10^{-3}\text{ m} = \boxed{1.54\text{ mm}}$$

Goal Solution

Young's double-slit experiment is performed with 589-nm light and a slit-to-screen distance of 2.00 m. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

G: For the situation described, the observed interference pattern is very narrow, (the minima are less than 1 mm apart when the screen is 2 m away). In fact, the minima and maxima are so close together that it would probably be difficult to resolve adjacent maxima, so the pattern might look like a solid blur to the naked eye. Since the angular spacing of the pattern is inversely proportional to the slit width, we should expect that for this narrow pattern, the space between the slits will be larger than the typical fraction of a millimeter, and certainly much greater than the wavelength of the light ($d >> \lambda = 589$ nm).

O: Since we are given the location of the tenth minimum for this interference pattern, we should use the equation for **destructive interference** from a double slit. The figure for Problem 7 shows the critical variables for this problem.

A: In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$,

The first minimum is described by $m = 0$ and the tenth by $m = 9$: $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = y/L$, but for small θ, $\sin\theta \approx \tan\theta$. Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$

$$d = \frac{9.5(5890 \cdot 10^{-10}\text{ m})(2.00\text{ m})}{7.26 \cdot 10^{-3}\text{ m}} = 1.54 \cdot 10^{-3}\text{ m} = 1.54\text{ mm} = 1.54\text{ mm}$$

L: The spacing between the slits is relatively large, as we expected (about 3 000 times greater than the wavelength of the light). In order to more clearly distinguish between maxima and minima, the pattern could be expanded by increasing the distance to the screen. However, as L is increased, the overall pattern would be less bright as the light expands over a larger area, so that beyond some distance, the light would be too dim to see.

***37.6** $\lambda = \dfrac{340 \text{ m/s}}{2000 \text{ Hz}} = 0.170 \text{ m}$

Maxima are at $d \sin\theta = m\lambda$:

$m = 0$ gives $\theta = 0°$

$m = 1$ gives $\sin\theta = \dfrac{\lambda}{d} = \dfrac{0.170 \text{ m}}{0.350 \text{ m}}$ $\theta = 29.1°$

$m = 2$ gives $\sin\theta = \dfrac{2\lambda}{d} = 0.971$ $\theta = 76.3°$

$m = 3$ gives $\sin\theta = 1.46$ No solution.

Minima at $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$:

$m = 0$ gives $\sin\theta = \dfrac{\lambda}{2d} = 0.243$ $\theta = 14.1°$

$m = 1$ gives $\sin\theta = \dfrac{3\lambda}{2d} = 0.729$ $\theta = 46.8°$

$m = 2$ gives No solution.

$\boxed{\text{So we have maxima at } 0°, 29.1°, \text{ and } 76.3° \text{ and minima at } 14.1° \text{ and } 46.8°.}$

37.7 (a) For the bright fringe,

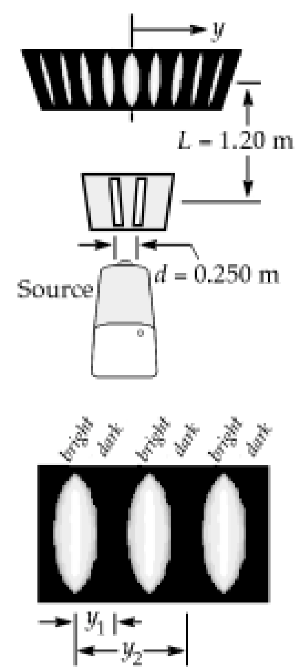
$y_{\text{bright}} = \dfrac{m\lambda L}{d}$ where $m = 1$

$$y = \frac{(546.1\times10^{-9} \text{ m})(1.20 \text{ m})}{0.250\times10^{-3} \text{ m}} = 2.62\times10^{-3} \text{ m} = \boxed{2.62 \text{ mm}}$$

(b) For the dark bands, $y_{\text{dark}} = \dfrac{\lambda L}{d}\left(m + \dfrac{1}{2}\right)$; $m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right) - \left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1) = \frac{(546.1\times10^{-9} \text{ m})(1.20 \text{ m})}{0.250\times10^{-3} \text{ m}}$$

$\Delta y = \boxed{2.62 \text{ mm}}$



Figures for Goal Solution

Goal Solution

A pair of narrow, parallel slits separated by 0.250 mm is illuminated by green light (λ = 546.1 nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands.

G: The spacing between adjacent maxima and minima should be fairly uniform across the pattern as long as the width of the pattern is much less than the distance to the screen (so that the small angle approximation is valid). The separation between fringes should be at least a millimeter if the pattern can be easily observed with a naked eye.

O: The bright regions are areas of constructive interference and the dark bands are destructive interference, so the corresponding double-slit equations will be used to find the y distances.

It can be confusing to keep track of four different symbols for distances. Three are shown in the drawing to the right. Note that:

y is the unknown distance from the bright central maximum (m = 0) to another maximum or minimum on either side of the center of the interference pattern.

λ is the wavelength of the light, determined by the source.

A: (a) For **very small** θ $\quad \sin\theta \approx \tan\theta$ and $\tan\theta = y/L$

and the equation for constructive interference $\quad \sin\theta = m\lambda/d$ (Eq. 37.2)

becomes $\quad y_{\text{bright}} \approx (\lambda L/d)m$ (Eq. 37.5)

Substituting values, $\quad y_{\text{bright}} = \dfrac{(546\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}}(1) = 2.62\text{ mm}$

(b) If you have trouble remembering whether Equation 37.5 or Eq. 37.6 applies to a given situation, you can instead remember that the first bright band is in the center, and dark bands are halfway between bright bands. Thus, Eq. 37.5 describes them all, with $m = 0,\ 1,\ 2\ .\ .\ .$ for bright bands, and with $m = 0.5,\ 1.5,\ 2.5\ .\ .\ .$ for dark bands. The dark band version of Eq. 37.5 is simply Eq. 37.6:

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \tfrac{1}{2}\right)$$

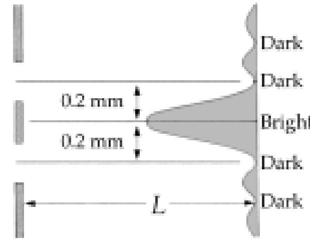
$$\Delta y_{\text{dark}} = \left(1+\tfrac{1}{2}\right)\frac{\lambda L}{d} - \left(0+\tfrac{1}{2}\right)\frac{\lambda L}{d} = \frac{\lambda L}{d} = 2.62\text{ mm}$$

L: This spacing is large enough for easy resolution of adjacent fringes. The distance between minima is the same as the distance between maxima. We expected this equality since the angles are small:

$$\theta = (2.62\text{ mm})/(1.20\text{ m}) = 0.00218\text{ rad} = 0.125°$$

When the angular spacing exceeds about 3°, then $\sin\theta$ differs from $\tan\theta$ when written to three significant figures.

37.8 Taking $m = 0$ and $y = 0.200$ mm in Equation 37.6 gives



$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400\times 10^{-3}\text{ m})(0.200\times 10^{-3}\text{ m})}{442\times 10^{-9}\text{ m}} = 0.362\text{ m}$$

$$L \approx \boxed{36.2\text{ cm}}$$

Geometric optics incorrectly predicts bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

37.9 Location of A = central maximum,

Location of B = first minimum.

So, $$\Delta y = [y_{\min} - y_{\max}] = \frac{\lambda L}{d}\left(0+\frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}$$

Thus, $$d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$$

37.10 At 30.0°, $d\sin\theta = m\lambda$

$(3.20\times 10^{-4}\text{ m})\sin 30.0° = m(500\times 10^{-9}\text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641\text{ maxima}}$.

37.11 $$\phi = \frac{2\pi}{\lambda}d\sin\theta \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$$

(a) $$\phi = \frac{2\pi}{(5.00\times 10^{-7}\text{ m})}(1.20\times 10^{-4}\text{ m})\sin(0.500^\circ) = \boxed{13.2\text{ rad}}$$

(b) $$\phi = \frac{2\pi}{(5.00\times 10^{-7}\text{ m})}(1.20\times 10^{-4}\text{ m})\left(\frac{5.00\times 10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333 \text{ rad} = \dfrac{2\pi d \sin\theta}{\lambda}$, $\theta = \sin^{-1}\left(\dfrac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\dfrac{(5.00\times10^{-7}\text{ m})(0.333\text{ rad})}{2\pi(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$$

(d) If $d\sin\theta = \dfrac{\lambda}{4}$, $\theta = \sin^{-1}\left(\dfrac{\lambda}{4d}\right) = \sin^{-1}\left[\dfrac{5\times10^{-7}\text{ m}}{4(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$$

37.12 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$$\boxed{d(\sin\theta_1 - \sin\theta_2) = m\lambda}$$

37.13 (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta \approx \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $\dfrac{\delta}{\lambda} = \dfrac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\ \lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ since the path difference is an integer multiple of the wavelength.

37.14 (a) $\dfrac{I}{I_{\max}} = \cos^2\left(\dfrac{\phi}{2}\right)$ (Equation 37.11)

Therefore, $\phi = 2\cos^{-1}\left(\dfrac{I}{I_{\max}}\right)^{1/2} = 2\cos^{-1}(0.640)^{1/2} = \boxed{1.29\text{ rad}}$

(b) $\delta = \dfrac{\lambda\phi}{2\pi} = \dfrac{(486\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.8\text{ nm}}$

37.15 $I_{av} = I_{max}\cos^2\left(\dfrac{\pi d \sin\theta}{\lambda}\right)$

For small θ, $\sin\theta = \dfrac{y}{L}$ and $I_{av} = 0.750 I_{max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{I_{av}}{I_{max}}\right)^{1/2}$$

$$y = \frac{(6.00\times10^{-7})(1.20\text{ m})}{\pi(2.50\times10^{-3}\text{ m})}\cos^{-1}\left(\frac{0.750 I_{max}}{I_{max}}\right)^{1/2} = \boxed{48.0\ \mu\text{m}}$$

37.16 $I = I_{max}\cos^2\left(\dfrac{\pi y d}{\lambda L}\right)$

$$\frac{I}{I_{max}} = \cos^2\left[\frac{\pi\left(6.00\times10^{-3}\text{ m}\right)\left(1.80\times10^{-4}\text{ m}\right)}{\left(656.3\times10^{-9}\text{ m}\right)(0.800\text{ m})}\right] = \boxed{0.987}$$

37.17 (a) From Equation 37.8,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2 + D^2}}$$

$$\phi \approx \frac{2\pi y d}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{max}} = \frac{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta_{max}\right)} = \frac{\cos^2\dfrac{\phi}{2}}{\cos^2 m\pi}$$

$$\frac{I}{I_{max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

Goal Solution

Two narrow parallel slits separated by 0.850 mm are illuminated by 600-nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?

G: It is difficult to accurately predict the relative intensity at the point of interest without actually doing the calculation. The waves from each slit could meet in phase ($\phi = 0$) to produce a bright spot of **constructive interference**, out of phase ($\phi = 180°$) to produce a dark region of **destructive interference**, or most likely the phase difference will be somewhere between these extremes, $0 < \phi < 180°$, so that the relative intensity will be $0 < I/I_{max} < 1$.

O: The phase angle depends on the path difference of the waves according to Equation 37.8. This phase difference is used to find the average intensity at the point of interest. Then the relative intensity is simply this intensity divided by the maximum intensity.

A: (a) Using the variables shown in the diagram for problem 7 we have,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\left(\frac{y}{\sqrt{y^2+L^2}}\right) \cong \frac{2\pi yd}{\lambda L} = \frac{2\pi\left(0.850\cdot 10^{-3}\text{ m}\right)\left(0.00250\text{ m}\right)}{\left(600\cdot 10^{-9}\text{ m}\right)\left(2.80\text{ m}\right)} = 7.95\text{ rad} = 2\pi + 1.66\text{ rad} = 95.5$$

(b) $$\frac{I}{I_{max}} = \frac{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta_{max}\right)} = \frac{\cos^2\left(\frac{\phi}{2}\right)}{\cos^2(m\pi)} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{95.5}{2}\right) = 0.452$$

L: It appears that at this point, the waves show **partial interference** so that the combination is about half the brightness found at the central maximum. We should remember that the equations used in this solution do not account for the diffraction caused by the finite width of each slit. This diffraction effect creates an "envelope" that diminishes in intensity away from the central maximum as shown by the dotted line in Figures 37.13 and P37.60. Therefore, the relative intensity at y = 2.50 mm will actually be slightly less than 0.452.

37.18 (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t+\phi) + E_0\sin(\omega t+2\phi), \quad \text{where} \quad \phi = \frac{2\pi}{\lambda}d\sin\theta.$$

$$E_r = E_0\left(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos 2\phi + \cos\omega t\sin 2\phi\right)$$

$$E_r = E_0(\sin\omega t)\left(1+\cos\phi + 2\cos^2\phi - 1\right) + E_0(\cos\omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1+2\cos\phi)(\sin\omega t\cos\phi + \cos\omega t\sin\phi) = E_0(1+2\cos\phi)\sin(\omega t+\phi)$$

Then the intensity is $I \propto E_r^2 = E_0^2 (1 + 2\cos\phi)^2 \left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t + \phi)$ is 1/2.

From one slit alone we would get intensity $I_{max} \propto E_0^2 \left(\frac{1}{2}\right)$ so $\boxed{I = I_{max}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2}$

(b) Look at the $N = 3$ graph in Figure 37.13. Minimum intensity is zero, attained where $\cos\phi = -1/2$. One relative maximum occurs at $\cos\phi = -1.00$, where $I = I_{max}$.

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = 9.00\, I_0$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

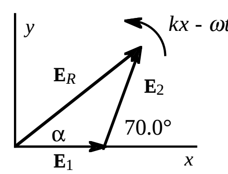
***37.19** (a) We can use $\sin A + \sin B = 2\sin(A/2 + B/2)\cos(A/2 - B/2)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0 \text{ kN/C})\sin(15x - 4.5t + 35.0°)\cos 35.0°$$

$$E_1 + E_2 = (19.7 \text{ kN/C})\sin(15x - 4.5t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7 \text{ kN/C}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) In units of kN/C, the resultant phasor is



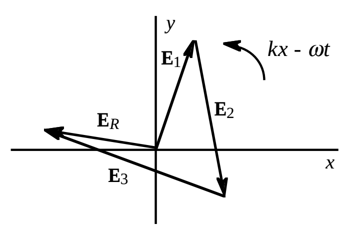
$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 = (12.0\,\mathbf{i}) + (12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}) = 16.1\,\mathbf{i} + 11.3\,\mathbf{j}$$

$$\mathbf{E}_R = \sqrt{(16.1)^2 + (11.3)^2} \text{ at } \tan^{-1}(11.3/16.1) = \boxed{19.7 \text{ kN/C at } 35.0°}$$

(c) $\mathbf{E}_R = 12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}$

$+15.5\cos 80.0°\,\mathbf{i} - 15.5\sin 80.0°\,\mathbf{j}$

$+17.0\cos 160°\,\mathbf{i} + 17.0\sin 160°\,\mathbf{j}$



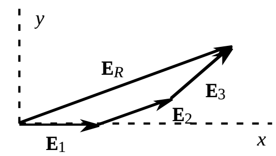
$$\mathbf{E}_R = -9.18\,\mathbf{i} + 1.83\,\mathbf{j} = \boxed{9.36 \text{ kN/C at } 169°}$$

The wave function of the total wave is $E_P = (9.36 \text{ kN/C})\sin(15x - 4.5t + 169°)$

37.20 (a) $\mathbf{E}_R = E_0\left[\mathbf{i} + (\mathbf{i}\cos 20.0° + \mathbf{j}\sin 20.0°) + (\mathbf{i}\cos 40.0° + \mathbf{j}\sin 40.0°)\right]$

$\mathbf{E}_R = E_0[2.71\mathbf{i} + 0.985\mathbf{j}] = 2.88E_0$ at $20.0° =$ $\boxed{2.88E_0 \text{ at } 0.349 \text{ rad}}$

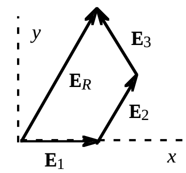
$E_P = 2.88E_0 \sin(\omega t + 0.349)$



(b) $\mathbf{E}_R = E_0\left[\mathbf{i} + (\mathbf{i}\cos 60.0° + \mathbf{j}\sin 60.0°) + (\mathbf{i}\cos 120° + \mathbf{j}\sin 120°)\right]$

$\mathbf{E}_R = E_0[1.00\mathbf{i} + 1.73\mathbf{j}] = 2.00E_0$ at $60.0° =$ $\boxed{2.00E_0 \text{ at } \pi/3 \text{ rad}}$

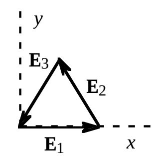
$E_P = 2.00E_0 \sin(\omega t + \pi/3)$



(c) $\mathbf{E}_R = E_0\left[\mathbf{i} + (\mathbf{i}\cos 120° + \mathbf{j}\sin 120°) + (\mathbf{i}\cos 240° + \mathbf{j}\sin 240°)\right]$

$\mathbf{E}_R = E_0[0\mathbf{i} + 0\mathbf{j}] = \boxed{0}$

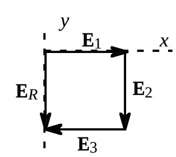
$E_P = 0$



(d) $\mathbf{E}_R = E_0\left[\mathbf{i} + (\mathbf{i}\cos 3\pi/2 + \mathbf{j}\sin 3\pi/2) + (\mathbf{i}\cos 3\pi + \mathbf{j}\sin 3\pi)\right]$

$\mathbf{E}_R = E_0[0\mathbf{i} - 1.00\mathbf{j}] = E_0$ at $270° =$ $\boxed{E_0 \text{ at } 3\pi/2 \text{ rad}}$

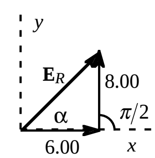
$E_P = E_0 \sin(\omega t + 3\pi/2)$



37.21 $\mathbf{E}_R = 6.00\mathbf{i} + 8.00\mathbf{j} = \sqrt{(6.00)^2 + (8.00)^2}$ at $\tan^{-1}(8.00/6.00)$

$\mathbf{E}_R = 10.0$ at $53.1° = 10.0$ at 0.927 rad

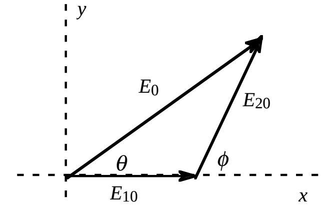
$E_P =$ $\boxed{10.0 \sin(100\pi t + 0.927)}$



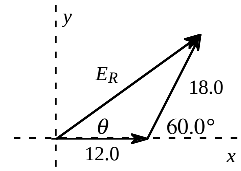
37.22 If $E_1 = E_{10}\sin\omega t$ and $E_2 = E_{20}\sin(\omega t + \phi)$, then by phasor addition, the amplitude of $\mathbf{E}$ is

$$E_0 = \sqrt{(E_{10} + E_{20}\cos\phi)^2 + (E_{20}\sin\phi)^2} = \boxed{\sqrt{E_{10}^2 + 2E_{10}E_{20}\cos\phi + E_{20}^2}}$$

and the phase angle is found from $\boxed{\sin\theta = \dfrac{E_{20}\sin\phi}{E_0}}$



37.23 $\mathbf{E}_R = 12.0\,\mathbf{i} + \left(18.0\cos 60.0°\,\mathbf{i} + 18.0\sin 60.0°\,\mathbf{j}\right)$



$\mathbf{E}_R = 21.0\,\mathbf{i} + 15.6\,\mathbf{j} = 26.2$ at $36.6°$

$E_P = \boxed{26.2\sin\left(\omega t + 36.6°\right)}$

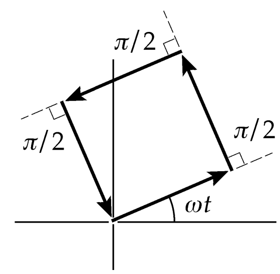
37.24 Constructive interference occurs where $m = 0, 1, 2, 3, \ldots$, for

$$\left(\frac{2\pi x_1}{\lambda} - 2\pi f t + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi f t + \frac{\pi}{8}\right) = 2\pi m \quad \frac{2\pi(x_1 - x_2)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

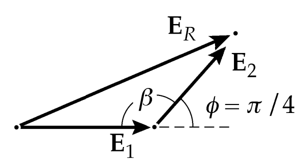
$$\frac{(x_1 - x_2)}{\lambda} + \frac{1}{12} - \frac{1}{16} = m \qquad \boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)\lambda \quad m = 0, 1, 2, 3, \ldots}$$

37.25 See the figure to the right:

$\boxed{\phi = \pi/2}$



37.26 $E_R^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\beta$, where $\beta = 180 - \phi$.



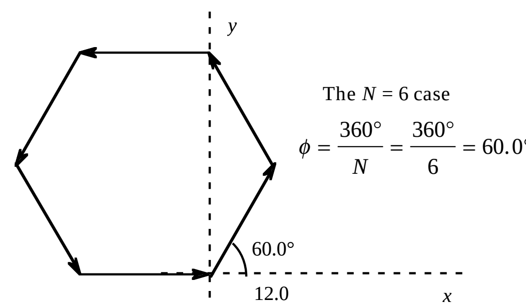
Since $I \propto E^2$,

$$I_R = \boxed{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

37.27 Take $\boxed{\phi = 360°/N}$ where N defines the number of coherent sources. Then,

$$E_R = \sum_{m=1}^{N} E_0 \sin\left(\omega t + m\phi\right) = 0$$

In essence, the set of N electric field components complete a full circle and return to zero.



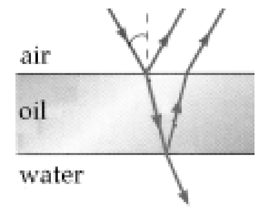
***37.28** Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \frac{\lambda}{n}$ is the wavelength in the material. Then $$2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$$

$$\lambda = 4nt = 4 \times 1.33 \times 115 \text{ nm} = \boxed{612 \text{ nm}}$$

37.29 (a) The light reflected from the top of the oil film undergoes phase reversal. Since 1.45 > 1.33, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have



$$2nt = \left(m + \frac{1}{2}\right)\lambda \qquad \text{or} \qquad \lambda_m = \frac{2nt}{\left(m + \frac{1}{2}\right)} = \frac{2(1.45)(280 \text{ nm})}{\left(m + \frac{1}{2}\right)}$$

Substituting for m, we have $m = 0$: $\lambda_0 = 1620$ nm (infrared)

$m = 1$: $\lambda_1 = 541$ nm (green)

$m = 2$: $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in the reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda \qquad \text{or} \qquad \lambda_m = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these. the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

37.30 Since 1 < 1.25 < 1.33, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $2t = \dfrac{m\lambda_{\text{cons}}}{n}$

and for destructive interference, $2t = \dfrac{\left(m + \frac{1}{2}\right)\lambda_{\text{des}}}{n}$

Then $\dfrac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}} = 1 + \dfrac{1}{2m} = \dfrac{640 \text{ nm}}{512 \text{ nm}} = 1.25$ and $m = 2$

Therefore, $t = \dfrac{2(640 \text{ nm})}{2(1.25)} = \boxed{512 \text{ nm}}$

37.31 Treating the anti-reflectance coating like a camera-lens coating, $2t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{n}$

Let $m = 0$: $t = \dfrac{\lambda}{4n} = \dfrac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

37.32 $2nt = \left(m + \dfrac{1}{2}\right)\lambda$ so $t = \left(m + \dfrac{1}{2}\right)\dfrac{\lambda}{2n}$

Minimum $t = \left(\dfrac{1}{2}\right)\dfrac{(500 \text{ nm})}{2(1.30)} = \boxed{96.2 \text{ nm}}$

37.33 Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt = m\lambda$, or $\lambda = 2nt/m$. The film thickness is $t = 1.00 \times 10^{-5} \text{ cm} = 1.00 \times 10^{-7} \text{ m} = 100 \text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2(1.38)(100 \text{ nm})}{m} = \frac{276 \text{ nm}}{m} \quad \text{where} \quad m = 1, 2, 3, \ldots$$

or $\lambda_1 = 276 \text{ nm}$, $\lambda_2 = 138 \text{ nm}$, ... All reflection maxima are in the ultraviolet and beyond. **[No visible wavelengths are intensified.]**

***37.34** (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \lambda/n$.

$$t = \frac{\lambda}{2n} = \frac{656.3 \text{ nm}}{2(1.378)} = \boxed{238 \text{ nm}}$$

(b) The filter will expand. As t increases in $2nt = \lambda$, so does $\boxed{\lambda \text{ increase}}$.

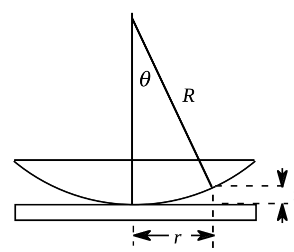
(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238 \text{ nm}) = \boxed{328 \text{ nm}}$ (near ultraviolet) .

37.35 If the path length $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580 \text{ nm}}{2} = \boxed{290 \text{ nm}}$$

37.36 The condition for bright fringes is



$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1,\ 2,\ 3,\ \ldots$$

From the sketch, observe that

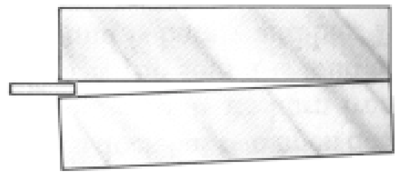
$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

The condition for a bright fringe becomes $\dfrac{r^2}{R} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{n}$.

Thus, for fixed m and λ, $nr^2 = $ constant.

Therefore, $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$ and $n_{\text{liquid}} = (1.00)\dfrac{(1.50 \text{ cm})^2}{(1.31 \text{ cm})^2} = \boxed{1.31}$

37.37 For destructive interference in the air, $2t = m\lambda$.

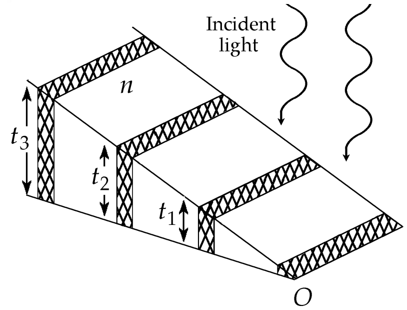


For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600\text{ nm})}{2} = 8.70 \times 10^{-6}\text{ m}$$

Therefore, the <u>radius</u> of the wire is

$$r = \frac{d}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$



<u>Goal Solution</u>

An air wedge is formed between two glass plates separated at one edge by a very fine wire as shown in Figure P37.37. When the wedge is illuminated from above by 600-nm light, 30 dark fringes are observed. Calculate the radius of the wire.

G: The radius of the wire is probably less than 0.1 mm since it is described as a "very fine wire."

O: Light reflecting from the bottom surface of the top plate undergoes no phase shift, while light reflecting from the top surface of the bottom plate is shifted by π, and also has to travel an extra distance $2t$, where t is the thickness of the air wedge.

For destructive interference, $2t = m\lambda \quad (m = 0, 1, 2, 3, \ldots)$

The first dark fringe appears where $m = 0$ at the line of contact between the plates. The 30th dark fringe gives for the diameter of the wire $2t = 29\lambda$, and $t = 14.5\lambda$.

A: $r = \frac{t}{2} = 7.25\lambda = 7.25\left(600 \times 10^{-9}\text{ m}\right) = 4.35\ \mu\text{m}$

L: This wire is not only less than 0.1 mm; it is even thinner than a typical human hair (~ 50 μm).

37.38 For destructive interference, $2t = \frac{\lambda m}{n}$.

At the position of the maximum thickness of the air film,

$$m = \frac{2tn}{\lambda} = \frac{2(4.00 \times 10^{-5}\text{ m})(1.00)}{5.461 \times 10^{-7}\text{ m}} = 146.5$$

The greatest integer value is $m = 146$.

Therefore, including the dark band at zero thickness, there are $\boxed{147\text{ dark fringes}}$.

***37.39** For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}}t = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

The least common multiple of these two wavelengths is 1200 nm, so we get no reflected light at $2(1.00)t = 3(400\text{ nm}) = 2(600\text{ nm}) = 1200\text{ nm}$, so t = 600 nm at this second dark fringe.

By similar triangles, $\dfrac{600\text{ nm}}{x} = \dfrac{0.0500\text{ mm}}{10.0\text{ cm}}$,

or the distance from the contact point is $x = \left(600\times10^{-9}\text{ m}\right)\left(\dfrac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right) = \boxed{1.20\text{ mm}}$

37.40 $2t = m\lambda \quad \Rightarrow \quad m = \dfrac{2t}{\lambda} = \dfrac{2(1.80\times10^{-4}\text{ m})}{550.5\times10^{-9}\text{ m}} = \boxed{654\text{ dark fringes}}$

37.41 When the mirror on one arm is displaced by Δl, the path difference increases by $2\Delta l$. A shift resulting in the formation of successive dark (or bright) fringes requires a path length change of one-half wavelength. Therefore, $2\Delta l = m\lambda/2$, where in this case, m = 250.

$$\Delta l = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

37.42 Distance $= 2(3.82\times10^{-4}\text{ m}) = 1700\lambda \qquad \lambda = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$

37.43 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \dfrac{2L}{\lambda}$. It changes to $m_2 = \dfrac{2L}{\lambda/n} = \dfrac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \dfrac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{35\left(633\times10^{-9}\text{ m}\right)}{2(0.0300\text{ m})} = \boxed{1.000\ 369}$$

37.44 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1 + \frac{N\lambda}{2L}}$$

37.45 The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60.0 \times 10^6 \text{ s}^{-1}} = 5.00 \text{ m}$.

Along the line *AB* the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from *B*, when it arrives at *A*, will always be in phase with transmitter *B*. Since *B* is 180° out of phase with *A*, the two signals always interfere destructively at the position of *A*.

The first antinode (point of constructive interference) is located at distance

$$\frac{\lambda}{4} = \frac{5.00 \text{ m}}{4} = \boxed{1.25 \text{ m}}$$ from the node at *A*.

***37.46** My middle finger has width $d = 2$ cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$d \sin\theta = m\lambda$ $\theta_0 = 0$ $(2 \times 10^{-2} \text{ m}) \sin\theta_1 = 1\,(6 \times 10^{-7} \text{ m})$

Thus, $\theta_1 = 2 \times 10^{-3}$ degree

and $\theta_1 - \theta_0 \boxed{\sim 10^{-3} \text{ degree}}$

(b) Choose $\theta_1 = 20°$ $2 \times 10^{-2} \text{ m} \sin 20° = 1\lambda$ $\lambda = 7$ mm

Millimeter waves are $\boxed{\text{microwaves}}$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{7 \times 10^{-3} \text{ m}} \boxed{\sim 10^{11} \text{ Hz}}$$

37.47 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic *t*.

$\frac{t}{\lambda}+\frac{1}{2}=\frac{t}{(\lambda/n)}=\frac{nt}{\lambda}$ or $t=\boxed{\frac{\lambda}{2(n-1)}}$ where n is the index of refraction for the plastic.

***37.48** No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\lambda/2$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is then $\delta = 2nt + \lambda/2$. For constructive interference, $\delta = m\lambda$, or $2(1.00)t + \lambda/2 = m\lambda$. Thus, the film thickness for the mth order bright fringe is:

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4},$$

and the thickness for the $m - 1$ bright fringe is: $$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}.$$

Therefore, the change in thickness required to go from one bright fringe to the next is $\Delta t = t_m - t_{m-1} = \lambda/2$. To go through 200 bright fringes, the change in thickness of the air film must be: $200(\lambda/2) = 100\lambda$. Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00 \times 10^{-7}\text{ m}\right) = 5.00 \times 10^{-5}\text{ m},$$

From $\Delta L = L_i \alpha(\Delta T)$, we have: $$\alpha = \frac{\Delta L}{L_i(\Delta T)} = \frac{5.00 \times 10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})} = \boxed{20.0 \times 10^{-6}\,°\text{C}^{-1}}$$

***37.49** Since 1 < 1.25 < 1.34, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, which must be equal to

$$m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\text{ nm})}{2(1.25)} = 200\text{ nm}$$

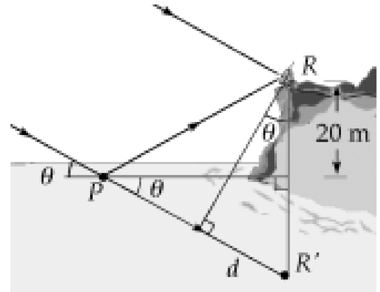
The volume we assume to be constant: $1.00\text{ m}^3 = (200\text{ nm})A$

$$A = \frac{1.00\text{ m}^3}{200(10^{-9}\text{ m})} = 5.00 \times 10^6\text{ m}^2 = \boxed{5.00\text{ km}^2}$$

37.50 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\pi/2$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Using Equation 37.5,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00 \times 10^{-7}\text{ m})(100\text{ m})}{(2.00 \times 10^{-2}\text{ m})} = \boxed{2.50\text{ mm}}$$

37.51 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P.



Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad (1)$$

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

It is equally far from P to R as from P to R', the mirror image of the telescope.

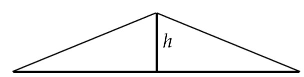
So the path difference is $d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$

The wavelength is $\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{60.0\times10^6\text{ Hz}} = 5.00\text{ m}$

Substituting for d and λ in Equation (1), $(40.0\text{ m})\sin\theta = \frac{5.00\text{ m}}{2}$

Solving for the angle θ, $\sin\theta = \frac{5.00\text{ m}}{80.0\text{ m}}$ and $\boxed{\theta = 3.58°}$

37.52 $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$



$(15.0\text{ km})^2 + h^2 = 227.63$ $h = \boxed{1.62\text{ km}}$

37.53 From Equation 37.13, $\frac{I}{I_{max}} = \cos^2\left(\frac{\pi y d}{\lambda L}\right)$

Let λ_2 equal the wavelength for which $\frac{I}{I_{max}} \rightarrow \frac{I_2}{I_{max}} = 0.640$

Then $\lambda_2 = \frac{\pi y d/L}{\cos^{-1}(I_2/I_{max})^{1/2}}$

But $\frac{\pi y d}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{max}}\right)^{1/2} = (600\text{ nm})\cos^{-1}(0.900) = 271\text{ nm}$

Substituting this value into the expression for λ_2, $\lambda_2 = \frac{271\text{ nm}}{\cos^{-1}(0.640^{1/2})} = \boxed{421\text{ nm}}$

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{max}}\right)^{1/2}$ must be expressed in radians.

37.54 For Young's experiment, use $\delta = d\sin\theta = m\lambda$. Then, at the point where the two bright lines coincide,

$d\sin\theta = m_1\lambda_1 = m_2\lambda_2$ so $\dfrac{\lambda_1}{\lambda_2} = \dfrac{540}{450} = \dfrac{m_2}{m_1} = \dfrac{6}{5}$

$$\sin\theta = \frac{6\lambda_2}{d} = \frac{6(450\text{ nm})}{0.150\text{ mm}} = 0.0180$$

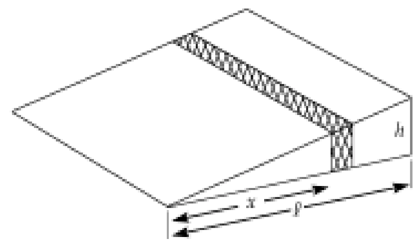
Since $\sin\theta \approx \theta$ and $L = 1.40$ m, $x = \theta L = (0.0180)(1.40\text{ m}) = \boxed{2.52\text{ cm}}$

37.55 For dark fringes, $2nt = m\lambda$

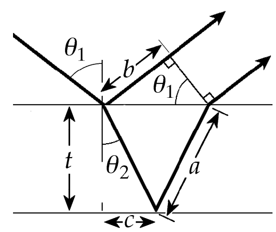
and at the edge of the wedge, $t = \dfrac{84(500\text{ nm})}{2}$.

When submerged in water, $2nt = m\lambda$

$m = \dfrac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$ so $m + 1 = \boxed{113\text{ dark fringes}}$



***37.56** At entrance, $1.00\sin 30.0° = 1.38\sin\theta_2$ $\theta_2 = 21.2°$



Call t the unknown thickness. Then

$\cos 21.2° = \dfrac{t}{a}$ $a = \dfrac{t}{\cos 21.2°}$

$\tan 21.2° = \dfrac{c}{t}$ $c = t\tan 21.2°$

$\sin\theta_1 = \dfrac{b}{2c}$ $b = 2t\tan 21.2°\sin 30.0°$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor n accounts for the shorter wavelength in the film. For constructive interference, we require

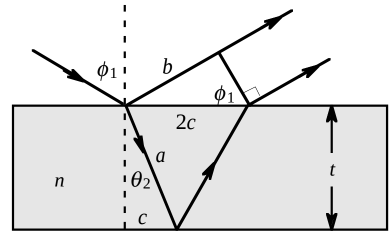
$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will be given by $2an - b - \dfrac{\lambda}{2} = 0.$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t(\tan 21.2°)\sin 30.0°$$

$\dfrac{590\text{ nm}}{2} = \left(\dfrac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2°\sin 30.0°\right)t = 2.57t$ $t = \boxed{115\text{ nm}}$

37.57 The shift between the two reflected waves is $\delta = 2na - b - \lambda/2$ where a and b are as shown in the ray diagram, n is the index of refraction, and the factor of $\lambda/2$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes



$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

From the figure's geometry, $a = \dfrac{t}{\cos\theta_2}$, $c = a\sin\theta_2 = \dfrac{t\sin\theta_2}{\cos\theta_2}$, $b = 2c\sin\phi_1 = \dfrac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$. Thus, $b = \dfrac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda \quad \text{or} \quad \boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$$

37.58 (a) Minimum: $2nt = m\lambda_2$ $\qquad m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ $\qquad m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$ so $m' = m - 1$

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$ so $\boxed{m = \dfrac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$

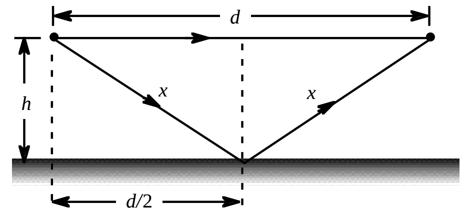
(b) $m = \dfrac{500}{2(500 - 370)} = 1.92 \to 2$ (wavelengths measured to ± 5 nm)

[Minimum]: $2nt = m\lambda_2$ $\qquad 2(1.40)t = 2(370\text{ nm})$ $\qquad t = 264$ nm

[Maximum]: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$ $\quad 2(1.40)t = (1.5)500$ nm $\qquad t = 268$ nm

Film thickness = $\boxed{266\text{ nm}}$

37.59 From the sketch, observe that



$$x = \sqrt{h^2 + (d/2)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \lambda/2$.

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where m = 0, 1, 2, 3, ...

Thus, $$2x - d = \left(m + \frac{1}{2}\right)\lambda \quad \text{or} \quad \lambda = \frac{4x - 2d}{2m + 1}$$

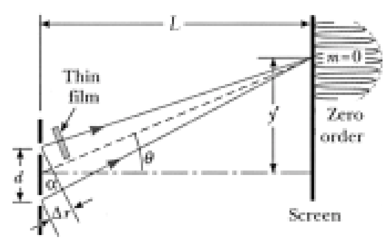
For the longest wavelength, m = 0, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$

(b) For destructive interference, $$\delta = \left(m - \frac{1}{2}\right)\lambda \quad \text{where } m = 1, 2, 3, \ldots.$$

Thus, $$2x - d = m\lambda \quad \text{or} \quad \lambda = \frac{2x - d}{m}.$$

For the longest wavelength, m = 1 giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$

37.60 Call t the thickness of the sheet. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. As light advances through distance t in air, the number of cycles it goes through is t / λ_a.



The number of cycles in the sheet is $$\frac{t}{(\lambda_a / n)} = \frac{nt}{\lambda_a}$$

Thus, the sheet introduces phase difference $$\phi = 2\pi\left(\frac{nt}{\lambda_a} - \frac{t}{\lambda_a}\right)$$

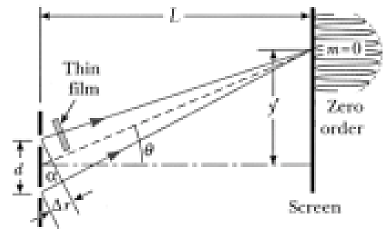
The corresponding difference in **path length** is $$\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = \frac{2\pi}{\lambda_a}(nt - t)\frac{\lambda_a}{2\pi} = (n - 1)t$$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel, so the angle θ may be expressed as $\tan\theta = \Delta r/d = y'/L$.

Substituting for Δr and solving for y' gives

$$y' = \Delta r\left(\frac{L}{d}\right) = \frac{t(n - 1)L}{d} = \frac{(5.00 \times 10^{-5}\text{ m})(1.50 - 1)(1.00\text{ m})}{(3.00 \times 10^{-4}\text{ m})} = 0.0833\text{ m} = \boxed{8.33\text{ cm}}$$

37.61 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is



$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)$$

The corresponding difference in **path length** Δr is $\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$

Eliminating Δr by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$

Goal Solution

Consider the double-slit arrangement shown in Figure P37.60, where the slit separation is d and the slit to screen distance is L. A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y'. Find y'.

G: Since the film shifts the pattern upward, we should expect y' to be proportional to n, t, and L.

O: The film increases the optical path length of the light passing through the upper slit, so the physical distance of this path must be shorter for the waves to meet in phase ($\phi = 0$) to produce the central maximum. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film.

A: First calculate the additional phase difference due to the plastic. Recall that the relation between phase difference and path difference is $\phi = 2\pi\delta/\lambda$. The presence of plastic affects this by changing the wavelength of the light, so that the phase change of the light in air and plastic, as it travels over the thickness t is

$$\phi_{air} = \frac{2\pi t}{\lambda_{air}} \quad \text{and} \quad \phi_{plastic} = \frac{2\pi t}{\lambda_{air}/n}$$

Thus, plastic causes an additional phase change of $\Delta\phi = \frac{2\pi t}{\lambda_{air}}(n-1)$

Next, in order to interfere constructively, we must calculate the additional distance that the light from the bottom slit must travel.

$$\Delta r = \frac{\Delta\phi\,\lambda_{air}}{2\pi} = t(n-1)$$

In the small angle approximation we can write $\Delta r = y'd/L$, so $y' = \frac{t(n-1)L}{dL}$

L: As expected, y' is proportional to t and L. It increases with increasing n, being proportional to $(n-1)$. It is also inversely proportional to the slit separation d, which makes sense since slits that are closer together make a wider interference pattern.

37.62 $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50 \times 10^6 \text{ Hz}} = 200 \text{ m}$

For destructive interference, the path difference is one-half wavelength.

Thus, $$\frac{\lambda}{2} = 100 \text{ m} = x + \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2} - 2.00 \times 10^4 \text{ m},$$

or $$2.01 \times 10^4 \text{ m} - x = \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2}$$

Squaring and solving, $x = \boxed{99.8 \text{ m}}$

37.63 (a) Constructive interference in the reflected light requires $2t = \left(m + \frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5(650 \times 10^{-9} \text{ m})}{2} = 17.7 \ \mu\text{m}$$

Now from the geometry in Figure 37.18, the distance from the center of curvature down to the flat side of the lens is

$$\sqrt{R^2 - r^2} = R - t \qquad \text{or} \qquad R^2 - r^2 = R^2 - 2Rt + t^2$$

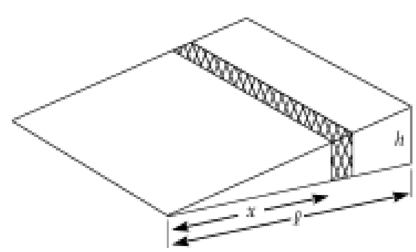
$$R = \frac{r^2 + t^2}{2t} = \frac{(5.00 \times 10^{-2} \text{ m})^2 + (1.77 \times 10^{-5} \text{ m})^2}{2(1.77 \times 10^{-5} \text{ m})} = \boxed{70.6 \text{ m}}$$

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_2}\right) = 0.520\left(\frac{1}{\infty} - \frac{1}{-70.6 \text{ m}}\right)$ so $f = \boxed{136 \text{ m}}$

37.64 Bright fringes occur when $2t = \frac{\lambda}{n}\left(m + \frac{1}{2}\right)$

and dark fringes occur when $2t = \left(\frac{\lambda}{n}\right)m$

The thickness of the film at x is $t = \left(\frac{h}{\ell}\right)x$.

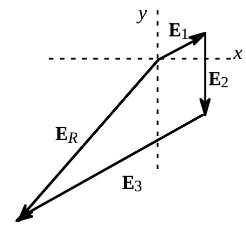


Therefore, $\boxed{x_{\text{bright}} = \frac{\lambda \ell}{2hn}\left(m + \frac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \frac{\lambda \ell m}{2hn}}$

37.65 $\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \left[\cos\frac{\pi}{6} + 3.00\cos\frac{7\pi}{2} + 6.00\cos\frac{4\pi}{3}\right]\mathbf{i} + \left[\sin\frac{\pi}{6} + 3.00\sin\frac{7\pi}{2} + 6.00\sin\frac{4\pi}{3}\right]\mathbf{j}$

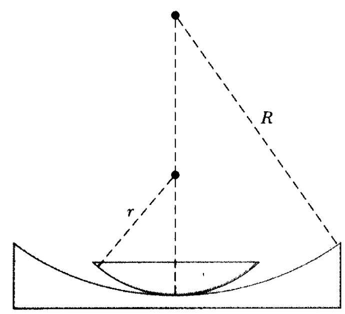
$\mathbf{E}_R = -2.13\,\mathbf{i} - 7.70\,\mathbf{j}$

$\mathbf{E}_R = \sqrt{(-2.13)^2 + (-7.70)^2}$ at $\tan^{-1}\left(\frac{-7.70}{-2.13}\right) = 7.99$ at 4.44 rad



Thus, $E_P = \boxed{7.99\sin(\omega t + 4.44\text{ rad})}$

37.66 For bright rings the gap t between surfaces is given by $2t = \left(m + \frac{1}{2}\right)\lambda$ The first bright ring has $m = 0$ and the hundredth has $m = 99$.



So, $t = \frac{1}{2}(99.5)\left(500\times10^{-9}\text{ m}\right) = 24.9\ \mu\text{m}$.

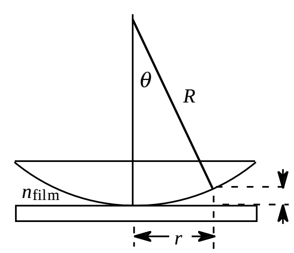
Call r_b the ring radius. From the geometry of the figure at the right,

$$t = r - \sqrt{r^2 - r_b^2} - \left(R - \sqrt{R^2 - r_b^2}\right)$$

Since $r_b << r$, we can expand in series: $t = r - r\left(1 - \frac{1}{2}\frac{r_b^2}{r^2}\right) - R + R\left(1 - \frac{1}{2}\frac{r_b^2}{R^2}\right) = \frac{1}{2}\frac{r_b^2}{r} - \frac{1}{2}\frac{r_b^2}{R}$

$$r_b = \left[\frac{2t}{1/r - 1/R}\right]^{1/2} = \left[\frac{2\left(24.9\times10^{-6}\text{ m}\right)}{1/4.00\text{ m} - 1/12.0\text{ m}}\right]^{1/2} = \boxed{1.73\text{ cm}}$$

***37.67** The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2t\,n_{\text{film}} + (\lambda/2)$, with the factor of $\lambda/2$ being due to a phase reversal at *one* of the surfaces.



For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that $t = m\lambda/2n_{\text{film}}$.

To find t in terms of r and R, $R^2 = r^2 + (R-t)^2$ so $r^2 = 2Rt + t^2$

Since t is much smaller than R, $t^2 << 2Rt$ and $r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$.

Thus, where m is an integer, $\boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$

37.68 (a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$

Hence, the first bright band ($m = 0$) corresponds to $nt = \lambda/4$.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$, we have $x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$ $t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63\times 10^{-6}\text{ rad}}$

37.69 $2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right) = \frac{1}{2}\lambda$ so $h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})(606\times 10^{-9}\text{ m})}{2(1.2\times 10^{-3}\text{ m})} = \boxed{0.505\text{ mm}}$

37.70 Superposing the two vectors, $E_R = |\mathbf{E}_1 + \mathbf{E}_2|$

$$E_R = |\mathbf{E}_1 + \mathbf{E}_2| = \sqrt{\left(E_0 + \frac{E_0}{3}\cos\phi\right)^2 + \left(\frac{E_0}{3}\sin\phi\right)^2} = \sqrt{E_0^2 + \frac{2}{3}E_0^2\cos\phi + \frac{E_0^2}{9}\cos^2\phi + \frac{E_0^2}{9}\sin^2\phi}$$

$$E_R = \sqrt{\frac{10}{9}E_0^2 + \frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\cos\phi$$

Using the trigonometric identity $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$, this becomes

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2} - 1\right) = \frac{4}{9}I_{\max} + \frac{4}{3}I_{\max}\cos^2\frac{\phi}{2},$$

or $\boxed{I = \frac{4}{9}I_{\max}\left(1 + 3\cos^2\frac{\phi}{2}\right)}$

CHAPTER 38

38.1 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{6.328\times10^{-7}}{3.00\times10^{-4}} = 2.11\times10^{-3}$

$\dfrac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta = \theta$ (for small θ)

$2y = \boxed{4.22\text{ mm}}$

38.2 The positions of the first-order minima are $y/L \approx \sin\theta = \pm\lambda/a$. Thus, the spacing between these two minima is $\Delta y = 2(\lambda/a)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}$$

38.3 $\dfrac{y}{L} = \sin\theta = \dfrac{m\lambda}{a}$ $\quad\Delta y = 3.00\times10^{-3}$ m $\quad\Delta m = 3-1 = 2$ $\quad$ and $\quad a = \dfrac{\Delta m\lambda L}{\Delta y}$

$$a = \frac{(2)(690\times10^{-9}\text{ m})(0.500\text{ m})}{3.00\times10^{-3}\text{ m}} = \boxed{2.30\times10^{-4}\text{ m}}$$

***38.4** For destructive interference,

$\sin\theta = m\dfrac{\lambda}{a} = \dfrac{\lambda}{a} = \dfrac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$ $\quad$ and $\quad\theta = 7.98°$

$\dfrac{d}{L} = \tan\theta$ gives $d = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912$ m

$d = \boxed{91.2\text{ cm}}$

***38.5** If the speed of sound is 340 m/s,

$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{650\text{ /s}} = 0.523\text{ m}$$

Diffraction minima occur at angles described by $a\sin\theta = m\lambda$

$1.10\text{ m}\sin\theta_1 = 1(0.523\text{ m})$ $\theta_1 = 28.4°$

$1.10\text{ m}\sin\theta_2 = 2(0.523\text{ m})$ $\theta_2 = 72.0°$

$1.10\text{ m}\sin\theta_3 = 3(0.523\text{ m})$ θ_3 nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10\text{ m})\sin\theta_x = 1.5(0.523\text{ m})$ $\theta_x \boxed{\approx 46°}$

There is no solution to $a\sin\theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

38.6 (a) $\sin\theta = \dfrac{y}{L} = \dfrac{m\lambda}{a}$

Therefore, for first minimum, $m = 1$ and

$$L = \frac{ay}{m\lambda} = \frac{\left(7.50\times10^{-4}\text{ m}\right)\left(8.50\times10^{-4}\text{ m}\right)}{(1)\left(587.5\times10^{-9}\text{ m}\right)} = \boxed{1.09\text{ m}}$$

(b) $w = 2y_1$ yields $y_1 = 0.850$ mm

$$w = 2\left(0.850\times10^{-3}\text{ m}\right) = \boxed{1.70\text{ mm}}$$

38.7 $\sin\theta \approx \dfrac{y}{L} = \dfrac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}$

$$\frac{\beta}{2} = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi\left(4.00\times10^{-4}\text{ m}\right)}{546.1\times10^{-9}\text{ m}}\left(\frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = 7.86\text{ rad}$$

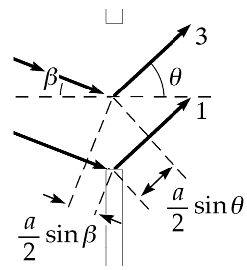
$$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62\times10^{-2}}$$

38.8 Bright fringes will be located approximately midway between adjacent dark fringes. Therefore, for the second bright fringe, let m = 2.5 and use

$$\sin\theta = m\lambda/a \approx y/L\ .$$

The wavelength will be $\lambda \approx \frac{ay}{mL} = \frac{(0.800\times10^{-3}\text{ m})(1.40\times10^{-3}\text{ m})}{2.5(0.800\text{ m})} = 5.60\times10^{-7}\text{ m} = \boxed{560\text{ nm}}$

38.9 Equation 38.1 states that $\sin\theta = m\lambda/a$, where $m = \pm 1, \pm 2, \pm 3, \ldots$. The requirement for m = 1 is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in Figure 38.5. This extra distance must be equal to $\lambda/2$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to ray 3) of $(a/2)\sin\beta$ Then, for destructive interference,



$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \qquad \text{so} \qquad \sin\theta = \frac{\lambda}{a} - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \frac{2\lambda}{a} - \sin\beta$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \frac{3\lambda}{a} - \sin\beta$

Generalizing, we obtain the condition for the mth order minimum: $\sin\theta = \frac{m\lambda}{a} - \sin\beta$

***38.10** (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$,
$\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) = \text{nonexistent}$.

Thus, there are $5+5+1 = \boxed{\text{11 directions for interference maxima}}$.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For m = 1, $(0.700\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$ and $\theta_1 = 45.8°$.

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°, \pm 10.3°, \pm 21.0°, \pm 32.5°, \text{ and } \pm 63.6°}$.

(c) $I = I_{max}\left[\dfrac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $\dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.5015\ \mu\text{m}} = 0.785\text{ rad} = 45.0°$

$$\frac{I}{I_{max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a\sin\theta}{\lambda} = 1.57\text{ rad} = 90.0°$ and $\dfrac{I}{I_{max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\dfrac{\pi a\sin\theta}{\lambda} = 2.36\text{ rad} = 135°$ and $\dfrac{I}{I_{max}} = \boxed{0.0901}$

At $\theta = 63.6°$, $\dfrac{\pi a\sin\theta}{\lambda} = 3.93\text{ rad} = 225°$ and $\dfrac{I}{I_{max}} = \boxed{0.0324}$

38.11 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{5.00\times 10^{-7}\text{ m}}{5.00\times 10^{-4}} = \boxed{1.00\times 10^{-3}\text{ rad}}$

38.12 $\theta_{min} = \dfrac{y}{L} = 1.22\dfrac{\lambda}{D}$

$y = \dfrac{(1.22)(5.00\times 10^{-7})(0.0300)}{7.00\times 10^{-3}} = \boxed{2.61\ \mu\text{m}}$

y = radius of star-image
L = length of eye
λ = 500 nm
D = pupil diameter
θ = half angle

38.13 Following Equation 38.9 for diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{(632.8\times 10^{-9}\text{ m})}{(0.00500\text{ m})} = 1.54\times 10^{-4}\text{ rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$r_{beam} = \theta_{min}\,(1.00\times 10^4\text{ m}) = 1.544\text{ m}$

and its diameter is $d_{beam} = 2r_{beam} = \boxed{3.09\text{ m}}$

Goal Solution

A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

G: A typical laser pointer makes a spot about 5 cm in diameter at 100 m, so the spot size at 10 km would be about 100 times bigger, or about 5 m across. Assuming that this HeNe laser is similar, we could expect a comparable beam diameter.

O: We assume that the light is parallel and not diverging as it passes through and fills the circular aperture. However, as the light passes through the circular aperture, it will spread from diffraction according to Equation 38.9.

A: The beam spreads into a cone of half-angle $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{(632.8\times10^{-9}\text{ m})}{(0.00500\text{ m})} = 1.54\times10^{-4}\text{ rad}$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\text{min}}(1.00\times10^{4}\text{ m}) = 1.54\text{ m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = 3.09\text{ m}$

L: The beam is several meters across as expected, and is about 600 times larger than the laser aperture. Since most HeNe lasers are low power units in the mW range, the beam at this range would be so spread out that it would be too dim to see on a screen.

38.14 $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D} = \dfrac{d}{L}$ $\qquad 1.22\left(\dfrac{5.80\times10^{-7}\text{ m}}{4.00\times10^{-3}\text{ m}}\right) = \dfrac{d}{1.80\text{ mi}}\left(\dfrac{1\text{ mi}}{1609\text{ m}}\right)$ $\qquad d = \boxed{0.512\text{ m}}$

The shortening of the wavelength inside the patriot's eye does not change the answer.

38.15 By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap when

$$\theta_{\text{min}} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $L = \dfrac{dD}{1.22\lambda} = \dfrac{(2.00\times10^{-3}\text{ m})(4.00\times10^{-3}\text{ m})}{1.22(500\times10^{-9}\text{ nm})} = \boxed{13.1\text{ m}}$

38.16 $D = 1.22\dfrac{\lambda}{\theta_{\text{min}}} = \dfrac{1.22(5.00\times10^{-7})}{1.00\times10^{-5}}\text{ m} = \boxed{6.10\text{ cm}}$

38.17 $\theta_{min} = 1.22\left(\frac{\text{wavelength}}{\text{pupil diameter}}\right) = \frac{(\text{distance between sources})}{L}$

Thus, $\frac{1.22\lambda}{d} = \frac{w}{vt}$, or $w = \frac{1.22\lambda(vt)}{d}$

Taillights are red. Take $\lambda \approx 650$ nm: $w \approx \frac{1.22(650\times10^{-9}\text{ m})(20.0\text{ m/s})(600\text{ s})}{5.00\times10^{-3}\text{ m}} = \boxed{1.90\text{ m}}$

38.18 $\theta_{min} = 1.22\left(\frac{\text{wavelength}}{\text{pupil diameter}}\right) = \frac{(\text{distance between sources})}{L}$ so $\frac{1.22\lambda}{d} = \frac{w}{vt}$

$w = \boxed{\frac{1.22\lambda(vt)}{d}}$ where $\lambda \approx 650$ nm is the average wavelength radiated by the red taillights.

38.19 $\frac{1.22\lambda}{D} = \frac{d}{L}$ $\lambda = \frac{c}{f} = 0.0200$ m $D = 2.10$ m $L = 9000$ m

$d = 1.22\frac{(0.0200\text{ m})(9000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$

38.20 Apply Rayleigh's criterion, $\theta_{min} = \frac{x}{D} = 1.22\frac{\lambda}{d}$

where $\theta_{min} =$ half-angle of light cone, $x =$ radius of spot, $\lambda =$ wavelength of light, $d =$ diameter of telescope, $D =$ distance to Moon.

Then, the diameter of the spot on the Moon is

$$2x = 2\left(1.22\frac{\lambda D}{d}\right) = \frac{2(1.22)(694.3\times10^{-9}\text{ m})(3.84\times10^{8}\text{ m})}{2.70\text{ m}} = \boxed{241\text{ m}}$$

38.21 For 0.100° angular resolution, $1.22\frac{(3.00\times10^{-3}\text{ m})}{D} = (0.100°)\left(\frac{\pi}{180°}\right)$ $D = \boxed{2.10\text{ m}}$

38.22 $L = 88.6\times10^{9}$ m, $D = 0.300$ m, $\lambda = 590\times10^{-9}$ m

(a) $1.22\frac{\lambda}{D} = \theta_{min} = \boxed{2.40\times10^{-6}\text{ rad}}$

(b) $d = \theta_{min}L = \boxed{213\text{ km}}$

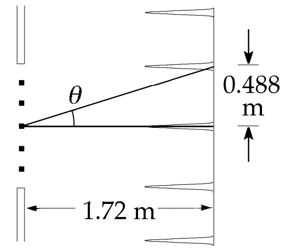
38.23 $d = \frac{1.00 \text{ cm}}{2000} = \frac{1.00 \times 10^{-2} \text{ m}}{2000} = 5.00 \ \mu\text{m}$

$\sin \theta = \frac{m\lambda}{d} = \frac{1\left(640 \times 10^{-9} \text{ m}\right)}{5.00 \times 10^{-6} \text{ m}} = 0.128 \qquad \theta = \boxed{7.35°}$

38.24 The principal maxima are defined by

$$d \sin \theta = m\lambda \quad m = 0, 1, 2, \ldots$$

For $m = 1$, $\lambda = d \sin \theta$



where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$\tan \theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} = 0.284$ so $\theta = 15.8°$ and $\sin \theta = 0.273$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$d = \frac{1}{5310 \text{ cm}^{-1}} = 1.88 \times 10^{-4} \text{ cm} = 1.88 \times 10^{3} \text{ nm}$

The wavelength is $\lambda = d \sin \theta = (1.88 \times 10^{3} \text{ nm})(0.273) = \boxed{514 \text{ nm}}$

38.25 The grating spacing is $d = \frac{(1.00 \times 10^{-2} \text{ m})}{4500} = 2.22 \times 10^{-6} \text{ m}$

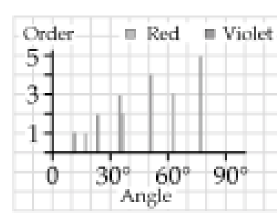


Figure for Goal Solution

In the 1st-order spectrum, diffraction angles are given by

$\sin \theta = \frac{\lambda}{d}$: $\sin \theta_1 = \frac{656 \times 10^{-9} \text{ m}}{2.22 \times 10^{-6} \text{ m}} = 0.295$

so that for red $\theta_1 = 17.17°$

and for violet $\sin \theta_2 = \frac{434 \times 10^{-9} \text{ m}}{2.22 \times 10^{-6} \text{ m}} = 0.195$

so that $\theta_2 = 11.26°$

The angular separation is in first-order, $\Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$

In the second-order spectrum, $\Delta\theta = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) - \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \boxed{13.2°}$

Again, in the third order, $\Delta\theta = \sin^{-1}\left(\frac{3\lambda_1}{d}\right) - \sin^{-1}\left(\frac{3\lambda_2}{d}\right) = \boxed{26.5°}$

Since the red line does not appear in the fourth-order spectrum, the answer is complete.

Goal Solution

The hydrogen spectrum has a red line at 656 nm and a violet line at 434 nm. What is the angular separation between two spectral lines obtained with a diffraction grating that has 4500 lines/cm?

G: Most diffraction gratings yield several spectral orders within the 180° viewing range, which means that the angle between red and violet lines is probably 10° to 30°.

O: The angular separation is the difference between the angles corresponding to the red and violet wavelengths for each visible spectral order according to the diffraction grating equation, $d\sin\theta = m\lambda$.

A: The grating spacing is $d = \left(1.00\times10^{-2}\text{ m}\right)/4500\text{ lines} = 2.22\times10^{-6}\text{ m}$

In the first-order spectrum (m = 1), the angles of diffraction are given by $\sin\ \theta = \lambda/d$:

$\sin\theta_{1r} = \frac{656\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.295$ so $\theta_{1r} = 17.17°$

$\sin\theta_{1v} = \frac{434\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.195$ so $\theta_{1v} = 11.26°$

The angular separation is $\Delta\theta_1 = \theta_{1r} - \theta_{1v} = 17.17 - 11.26 = 5.91$

In the 2nd-order ($m = 2$) $\Delta\theta_2 = \sin^{-1}\left(\frac{2\lambda_r}{d}\right) - \sin^{-1}\left(\frac{2\lambda_v}{d}\right) = 13.2°$

In the third order ($m = 3$), $\Delta\theta_3 = \sin^{-1}\left(\frac{3\lambda_r}{d}\right) - \sin^{-1}\left(\frac{3\lambda_v}{d}\right) = 26.5°$

In the fourth order, the red line is not visible: $\theta_{4r} = \sin^{-1}(4\lambda_r / d) = \sin^{-1}(1.18)$ does not exist

L: The full spectrum is visible in the first 3 orders with this diffraction grating, and the fourth is partially visible. We can also see that the pattern is dispersed more for higher spectral orders so that the angular separation between the red and blue lines increases as m increases. It is also worth noting that the spectral orders can overlap (as is the case for the second and third order spectra above), which makes the pattern look confusing if you do not know what you are looking for.

38.26 $\sin\theta = 0.350$: $d = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8 \text{ nm}}{0.350} = 1.81 \times 10^3 \text{ nm}$

Line spacing = $\boxed{1.81\ \mu\text{m}}$

***38.27** (a) $d = \dfrac{1}{3660 \text{ lines/cm}} = 2.732 \times 10^{-4} \text{ cm} = 2.732 \times 10^{-6} \text{ m} = 2732 \text{ nm}$

$\lambda = \dfrac{d\sin\theta}{m}$: At $\theta = 10.09°$ $\lambda = \boxed{478.7 \text{ nm}}$

At $\theta = 13.71°$, $\lambda = \boxed{647.6 \text{ nm}}$

At $\theta = 14.77°$, $\lambda = \boxed{696.6 \text{ nm}}$

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ and $\lambda = d\sin\theta_2$ so $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\left(\dfrac{\lambda}{\sin\theta_1}\right)} = 2\sin\theta_1$

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$

38.28 $d = \dfrac{1}{800/\text{mm}} = 1.25 \times 10^{-6} \text{ m}$

The blue light goes off at angles $\sin\theta_m = \dfrac{m\lambda}{d}$: $\theta_1 = \sin^{-1}\left(\dfrac{1 \times 5.00 \times 10^{-7} \text{ m}}{1.25 \times 10^{-6} \text{ m}}\right) = 23.6°$

$\theta_2 = \sin^{-1}(2 \times 0.400) = 53.1°$

$\theta_3 = \sin^{-1}(3 \times 0.400) =$ nonexistent

The red end of the spectrum is at $\theta_1 = \sin^{-1}\left(\dfrac{1 \times 7.00 \times 10^{-7} \text{ m}}{1.25 \times 10^{-6} \text{ m}}\right) = 34.1°$

$\theta_2 = \sin^{-1}(2 \times 0.560) =$ nonexistent

So only the first-order spectrum is complete, and $\boxed{\text{it does not overlap}}$ the second-order spectrum.

38.29 (a) From Equation 38.12, $R = Nm$ where $N = (3000\ \text{lines/cm})(4.00\ \text{cm}) = 1.20 \times 10^4$ lines.

In the 1st order, $R = (1)(1.20 \times 10^4\ \text{lines}) = \boxed{1.20 \times 10^4}$

In the 2nd order, $R = (2)(1.20 \times 10^4\ \text{lines}) = \boxed{2.40 \times 10^4}$

In the 3rd order, $R = (3)(1.20 \times 10^4\ \text{lines}) = \boxed{3.60 \times 10^4}$

(b) From Equation 38.11, $R = \dfrac{\lambda}{\Delta\lambda}$:

In the 3rd order, $\Delta\lambda = \dfrac{\lambda}{R} = \dfrac{400\ \text{nm}}{3.60 \times 10^4} = 0.0111\ \text{nm} = \boxed{11.1\ \text{pm}}$

38.30 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends the different order spectra are:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1500\ \text{nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{2\lambda_v}{d} = \dfrac{1200\ \text{nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing d.

38.31 (a) $Nm = \dfrac{\lambda}{\Delta\lambda}$ $\quad N(1) = \dfrac{531.7\ \text{nm}}{0.19\ \text{nm}} = \boxed{2800}$

(b) $\dfrac{1.32 \times 10^{-2}\ \text{m}}{2800} = \boxed{4.72\ \mu\text{m}}$

38.32 $d\sin\theta = m\lambda$ and, differentiating, $d(\cos\theta)d\theta = md\lambda$ or $d\sqrt{1-\sin^2\theta}\,\Delta\theta \approx m\Delta\lambda$

$d\sqrt{1 - m^2\lambda^2/d^2}\,\Delta\theta \approx m\Delta\lambda$ so $\boxed{\Delta\theta \approx \dfrac{\Delta\lambda}{\sqrt{d^2/m^2 - \lambda^2}}}$

38.33 $$d = \frac{1.00\times10^{-3}\ \text{m/mm}}{250\ \text{lines/mm}} = 4.00\times10^{-6}\ \text{m} = 4000\ \text{nm}$$

$$d\sin\theta = m\lambda \quad\Rightarrow\quad m = \frac{d\sin\theta}{\lambda}$$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\text{max}} = \frac{d\sin\theta_{\text{max}}}{\lambda} = \frac{(4000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71 \quad \text{or} \quad \boxed{\text{5 orders is the maximum}}\ .$$

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\text{max}} = \frac{d\sin\theta_{\text{max}}}{\lambda} = \frac{(4000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0 \quad \text{or} \quad \boxed{\text{10 orders in the short - wavelength region}}$$

38.34 $$d = \frac{1}{4200/\text{cm}} = 2.38\times10^{-6}\ \text{m} = 2380\ \text{nm}$$

$$d\sin\theta = m\lambda \qquad \text{or} \qquad \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \qquad \text{and} \qquad y = L\tan\theta = L\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$$

Thus, $$\Delta y = L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}$$

For $m = 1$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{589}{2380}\right)\right]\right\} = 0.554\ \text{mm}$$

For $m = 2$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{2(589)}{2380}\right)\right]\right\} = 1.54\ \text{mm}$$

For $m = 3$, $$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3(589)}{2380}\right)\right]\right\} = 5.04\ \text{mm}$$

Thus, the observed order must be $\boxed{m = 2}$.

38.35 $2d\sin\theta = m\lambda$: $$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.353\times10^{-9}\ \text{m})\sin(7.60°)}{(1)} = 9.34\times10^{-11}\ \text{m} = \boxed{0.0934\ \text{nm}}$$

38.36 $$2d\sin\theta = m\lambda \Rightarrow d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.129\ \text{nm})}{2\sin(8.15°)} = \boxed{0.455\ \text{nm}}$$

38.37 $2d\sin\theta = m\lambda$ so $\sin\theta = \dfrac{m\lambda}{2d} = \dfrac{1(0.140\times10^{-9}\text{ m})}{2(0.281\times10^{-9}\text{ m})} = 0.249$ and $\boxed{\theta = 14.4°}$

38.38 $\sin\theta_m = \dfrac{m\lambda}{2d}$: $\sin 12.6° = \dfrac{1\lambda}{2d} = 0.218$

$\sin\theta_2 = \dfrac{2\lambda}{2d} = 2(0.218)$ so $\theta_2 = 25.9°$

$\boxed{\text{Three}}$ other orders appear: $\theta_3 = \sin^{-1}(3\times0.218) = 40.9°$

$\theta_4 = \sin^{-1}(4\times0.218) = 60.8°$

$\theta_5 = \sin^{-1}(5\times0.218) =$ nonexistent

38.39 $2d\sin\theta = m\lambda$ $\theta = \sin^{-1}\left[\dfrac{m\lambda}{2d}\right] = \sin^{-1}\left[\dfrac{2\times0.166}{2\times0.314}\right] = \boxed{31.9°}$

***38.40** Figure 38.25 of the text shows the situation. $2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$m=1 \Rightarrow \lambda_1 = \dfrac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$

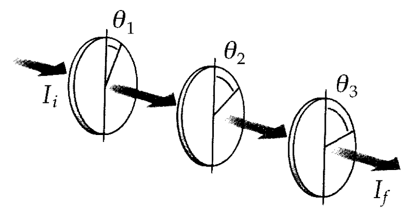
$m=2 \Rightarrow \lambda_2 = \dfrac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$

$m=3 \Rightarrow \lambda_3 = \dfrac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$

***38.41** The average value of the cosine-squared function is one-half, so the first polarizer transmits $\dfrac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \dfrac{3}{4}$.

$I_f = \dfrac{1}{2}\times\dfrac{3}{4}\,I_i = \boxed{\dfrac{3}{8}I_i}$

38.42 (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, $\theta_3 = 60.0°$



$I_f = I_i \cos^2(\theta_1 - 0°)\cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$

$I_f = (10.0 \text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°) = \boxed{6.89 \text{ units}}$

(b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, $\theta_3 = 60.0°$

$I_f = (10.0 \text{ units})\cos^2(0°)\cos^2(30.0°)\cos^2(30.0°) = \boxed{5.63 \text{ units}}$

38.43 $I = I_{max}\cos^2\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{I}{I_{max}}\right)^{1/2}$

(a) $\frac{I}{I_{max}} = \frac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3.00}\right)^{1/2} = \boxed{54.7°}$

(b) $\frac{I}{I_{max}} = \frac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{5.00}\right)^{1/2} = \boxed{63.4°}$

(c) $\frac{I}{I_{max}} = \frac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{10.0}\right)^{1/2} = \boxed{71.6°}$

38.44 By Brewster's law, $n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$

38.45 $\sin\theta_c = \frac{1}{n}$ or $n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 34.4°} = 1.77$

Also, $\tan\theta_p = n$. Thus, $\theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}$

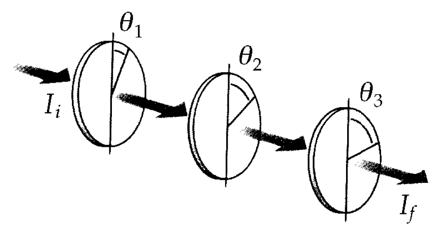
38.46 $\sin\theta_c = \frac{1}{n}$ and $\tan\theta_p = n$

Thus, $\sin\theta_c = \frac{1}{\tan\theta_p}$ or $\boxed{\cot\theta_p = \sin\theta_c}$

38.47 Complete polarization occurs at Brewster's angle $\tan\theta_p = 1.33$ $\theta_p = 53.1°$

Thus, the Moon is $\boxed{36.9°}$ above the horizon.

38.48 For incident unpolarized light of intensity $I_{\max}$:



After transmitting 1st disk: $I = \left(\frac{1}{2}\right) I_{\max}$

After transmitting 2nd disk: $I = \left(\frac{1}{2}\right) I_{\max} \cos^2\theta$

After transmitting 3rd disk: $I = \left(\frac{1}{2}\right) I_{\max}\left(\cos^2\theta\right)\cos^2(90° - \theta)$

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and $\cos^2(90 - \theta) = \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

we have $I = \frac{1}{2} I_{\max}\left[\frac{(1+\cos 2\theta)}{2}\right]\left[\frac{(1-\cos 2\theta)}{2}\right] = \frac{1}{8} I_{\max}(1 - \cos^2 2\theta) = \frac{1}{8} I_{\max}\left(\frac{1}{2}\right)(1 - \cos 4\theta)$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \frac{1}{16} I_{\max}(1 - \cos 4\omega t)}$

38.49 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $I_{\max}\cos^2\theta$.

The second sheet passes $I_{\max}\cos^4\theta$,

nd the nth sheet lets through $I_{\max}\cos^{2n}\theta \geq 0.90 I_{\max}$ where $\theta = 45°/n$

Try different integers to find $\cos^{2\times 5}\left(\frac{45°}{5}\right) = 0.885,$ $\cos^{2\times 6}\left(\frac{45°}{6}\right) = 0.902,$

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

***38.50** Consider vocal sound moving at 340 m/s and of frequency 3000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{3000\ \text{Hz}} = 0.113\ \text{m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113\ \text{m}}{0.600\ \text{m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

38.51 The first minimum is at $a\sin\theta = 1\lambda$.

This has no solution if $\dfrac{\lambda}{a} > 1$

or if $a < \lambda = \boxed{632.8\ \text{nm}}$

38.52 $$x = 1.22\frac{\lambda}{d}D = 1.22\left(\frac{5.00\times10^{-7}\ \text{m}}{5.00\times10^{-3}\ \text{m}}\right)\left(250\times10^{3}\ \text{m}\right) = \boxed{30.5\ \text{m}}$$

$D = 250\times10^{3}$ m
$\lambda = 5.00\times10^{-7}$ m
$d = 5.00\times10^{-3}$ m

38.53 $$d = \frac{1}{400/\text{mm}} = 2.50\times10^{-6}\ \text{m}$$

(a) $d\sin\theta = m\lambda$ $\theta_a = \sin^{-1}\left(\dfrac{2\times541\times10^{-9}\ \text{m}}{2.50\times10^{-6}\ \text{m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541\times10^{-9}\ \text{m}}{1.33} = 4.07\times10^{-7}\ \text{m}$

$$\theta_b = \sin^{-1}\left(\frac{2\times4.07\times10^{-7}\ \text{m}}{2.50\times10^{-6}\ \text{m}}\right) = \boxed{19.0°}$$

(c) $d\sin\theta_a = 2\lambda$ $\quad d\sin\theta_b = \dfrac{2\lambda}{n}$ $\quad n\sin\theta_b = 1\sin\theta_a$

***38.54** (a) $\lambda = \dfrac{v}{f} = \dfrac{3.00\times 10^{8}\ \text{m/s}}{1.40\times 10^{9}\ \text{s}^{-1}} = 0.214\ \text{m}$

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{0.214\ \text{m}}{3.60\times 10^{4}\ \text{m}}\right) = \boxed{7.26\ \mu\text{rad}} = 7.26\ \mu\text{rad}\left(\frac{180\times 60\times 60\ \text{s}}{\pi}\right) = \boxed{1.50\ \text{arc seconds}}$$

(b) $\theta_{\min} = \dfrac{d}{L}$: $\quad d = \theta_{\min}L = \left(7.26\times 10^{-6}\ \text{rad}\right)\left(26\ 000\ \text{ly}\right) = \boxed{0.189\ \text{ly}}$

(c) $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\left(\dfrac{500\times 10^{-9}\ \text{m}}{12.0\times 10^{-3}\ \text{m}}\right) = \boxed{50.8\ \mu\text{rad}}$ $\quad$ (10.5 seconds of arc)

(d) $d = \theta_{\min}L = \left(50.8\times 10^{-6}\ \text{rad}\right)\left(30.0\ \text{m}\right) = 1.52\times 10^{-3}\ \text{m} = \boxed{1.52\ \text{mm}}$

38.55 $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{(2.00\ \text{m})}{(10.0\ \text{m})} = \boxed{0.244\ \text{rad} = 14.0°}$

38.56 With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan 54.0° = 1.38$

In the liquid, $\lambda_n = \lambda / n = 750\ \text{nm} / 1.38 = \boxed{545\ \text{nm}}$

38.57 (a) $d\sin\theta = m\lambda$, or $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{3\left(500\times 10^{-9}\ \text{m}\right)}{\sin 32.0°} = 2.83\ \mu\text{m}$

Therefore, lines per unit length $= \dfrac{1}{d} = \dfrac{1}{2.83\times 10^{-6}\ \text{m}}$

or lines per unit length $= 3.53\times 10^{5}/\text{m} = \boxed{3.53\times 10^{3}/\text{cm}}$.

(b) $\sin\theta = \dfrac{m\lambda}{d} = \dfrac{m\left(500\times 10^{-9}\ \text{m}\right)}{2.83\times 10^{-6}\ \text{m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $\quad m(0.177) \le 1.00$ or $m \le 5.65$

Therefore, the highest order observed is $\quad m = 5$

Total number primary maxima observed is $\quad 2m+1 = \boxed{11}$

Goal Solution

Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

G: The diffraction pattern described in this problem seems to be similar to previous problems that have diffraction gratings with 2 000 to 5 000 lines/mm. With the third-order maximum at 32°, there are probably 5 or 6 maxima on each side of the central bright fringe, for a total of 11 or 13 primary maxima.

O: The diffraction grating equation can be used to find the grating spacing and the angles of the other maxima that should be visible within the 180° viewing range.

A: (a) Use Equation 38.10, $d\sin\theta = m\lambda$

$$d = \frac{m\lambda}{\sin\theta} = \frac{(3)(5.00\times10^{-7}\text{ m})}{\sin(32.0°)} = 2.83\times10^{-6}\text{ m}$$

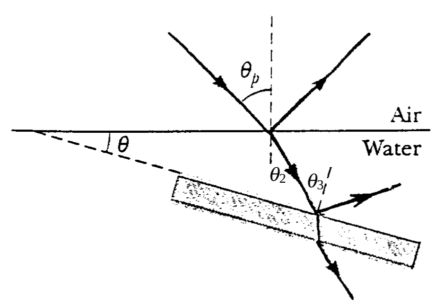
Thus, the grating gauge is $\frac{1}{d} = 3.534\times10^{5}\text{ lines / m} = 3530\text{ lines / cm}$ ◊

(b) $$\sin\theta = m\left(\frac{\lambda}{d}\right) = \frac{m(5.00\times10^{-7}\text{ m})}{2.83\times10^{-6}\text{ m}} = m(0.177)$$

For $\sin\theta \le 1$, we require that $m(1.77) \le 1$ or $m \le 5.65$. Since m must be an integer, its maximum value is really 5. Therefore, the total number of maxima is $2m + 1 = 11$

L: The results agree with our predictions, and apparently there are 5 maxima on either side of the central maximum. If more maxima were desired, a grating with **fewer** lines/cm would be required; however, this would reduce the ability to resolve the difference between lines that appear close together.

38.58 For the air-to-water interface,



$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

For the water-to-glass interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{1.50}{1.33}$ so $\theta_3 = 48.4°$

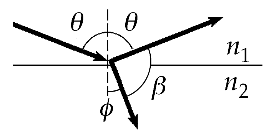
The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{11.5°}$

38.59 The limiting resolution between lines $\theta_{\min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}$ rad

Assuming a picture screen with vertical dimension l, the minimum viewing distance for no visible lines is found from $\theta_{\min} = \left(l/485\right)/L$. The desired ratio is then

$$\frac{L}{l} = \frac{1}{485\,\theta_{\min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}$$

38.60 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch, we also see that:



$$\theta + \phi + \beta = \pi, \quad \text{or} \quad \phi = \pi - (\theta + \beta)$$

Using the given identity: $\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$,

which reduces to: $\sin\phi = \sin(\theta+\beta)$.

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$

Snell's law then becomes: $n_2\left(\sin\theta\cos\beta + \cos\theta\sin\beta\right) = n_1\sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta + \sin\beta) = n_1\tan\theta$.

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2\sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$$\tan\theta = \frac{n(1.00)}{1.00 - 0}, \quad \text{or} \quad n = \tan\theta, \quad \text{which is Brewster's law.}$$

38.61 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$

In this case $m = 1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{7.50\times10^{9}\text{ Hz}} = 4.00\times10^{-2}$ m

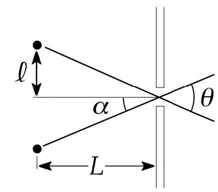
Thus, $\theta = \sin^{-1}\left(\dfrac{4.00\times10^{-2}\text{ m}}{6.00\times10^{-2}\text{ m}}\right) = \boxed{41.8°}$

(b) From Equation 38.4, $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$

When $\theta = 15.0°$, $\beta = \dfrac{2\pi(0.0600\text{ m})\sin 15.0°}{0.0400\text{ m}} = 2.44\text{ rad}$

and $\dfrac{I}{I_{max}} = \left[\dfrac{\sin(1.22\text{ rad})}{1.22\text{ rad}}\right]^2 = \boxed{0.593}$

(c) $\sin\theta = \dfrac{\lambda}{a}$ so $\theta = 41.8°$:



This is the minimum angle subtended by the two sources at the slit. Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

$L = \ell\cot\alpha = (0.100\text{ m})\cot(41.8/2) = \boxed{0.262\text{ m}}$

38.62 $\dfrac{I}{I_{max}} = \dfrac{1}{2}(\cos^2 45.0°)(\cos^2 45.0°) = \boxed{\dfrac{1}{8}}$

38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$\theta = (2\pi/\lambda)\delta$

after traveling distance d through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length d and the index of refraction. Therefore,

$\delta = |dn_O - dn_E|$

The absolute value is used since n_O/n_E may be more or less than unity. Therefore,

$\theta = \left(\dfrac{2\pi}{\lambda}\right)|dn_O - dn_E| = \boxed{\left(\dfrac{2\pi}{\lambda}\right)d|n_O - n_E|}$

(b) $d = \dfrac{\lambda\theta}{2\pi|n_O - n_E|} = \dfrac{(550\times10^{-9}\text{ m})(\pi/2)}{2\pi|1.544 - 1.553|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$

***38.64** For a diffraction grating, the locations of the principal maxima for wavelength λ are given by $\sin\theta = m\lambda/d \approx y/L$. The grating spacing may be expressed as $d = a/N$ where a is the width of the grating and N is the number of slits. Thus, the screen locations of the maxima become

$y = NLm\lambda / a$. If two nearly equal wavelengths are present, the difference in the screen locations of corresponding maxima is

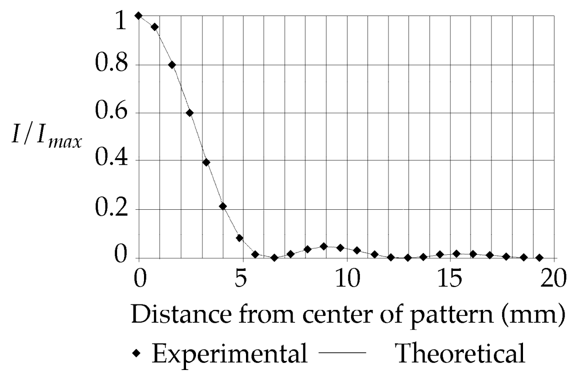
$$\Delta y = \frac{NLm(\Delta\lambda)}{a}$$

For a single slit of width a, the location of the first diffraction minimum is $\sin\theta = \lambda/a \approx y/L$, or $y = (L/a)\lambda$. If the two wavelengths are to be just resolved by Rayleigh's criterion, $y = \Delta y$ from above. Therefore,

$$\left(\frac{L}{a}\right)\lambda = \frac{NLm(\Delta\lambda)}{a}$$ or the resolving power of the grating is $\boxed{R'' \frac{\lambda}{\Delta\lambda} = Nm}$.

38.65 The first minimum in the single-slit diffraction pattern occurs at

$\sin\theta = \frac{\lambda}{a}$ » $\frac{y_{\min}}{L}$



Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36 \text{ mm} \pm 0.08 \text{ mm}$,

the width is $a = \frac{(632.8 \cdot 10^{-9} \text{ m})(1.00 \text{ m})}{6.36 \cdot 10^{-3} \text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$

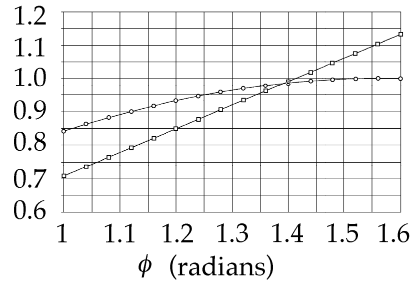
38.66 (a) From Equation 38.4, $$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$$

If we define $\phi \equiv \beta/2$ this becomes $\frac{I}{I_{\max}} = \left[\frac{\sin\phi}{\phi}\right]^2$

Therefore, when $\frac{I}{I_{\max}} = \frac{1}{2}$ we must have $\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$

(b) Let $y_1 = \sin\phi$ and $y_2 = \dfrac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \pi/2$ is shown to the right.



The solution to the transcendental equation is found to be $\boxed{\phi = 1.39 \text{ rad}}$.

(c) $\beta = \dfrac{2\pi a \sin\theta}{\lambda} = 2\phi$

gives $\quad \sin\theta = \left(\dfrac{\phi}{\pi}\right)\dfrac{\lambda}{a} = 0.443\dfrac{\lambda}{a}$.

If $\dfrac{\lambda}{a}$ is small, then $\theta \approx 0.443\dfrac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}$$

38.67

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.3917	smaller
1.3915	1.39154	bigger
1.39152	1.39155	bigger
1.3916	1.391568	smaller
1.39158	1.391563	
1.39157	1.391560	
1.39156	1.391558	
1.391559	1.3915578	
1.391558	1.3915575	
1.391557	1.3915573	
1.3915574	1.3915574	

We get the answer to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

***38.68** In $I = I_{\max}\left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$ find $\frac{dI}{d\beta} = I_{\max}\left(\frac{2\sin(\beta/2)}{(\beta/2)}\right)\left[\frac{(\beta/2)\cos(\beta/2)(1/2) - \sin(\beta/2)(1/2)}{(\beta/2)^2}\right]$

and require that it be zero. The possibility $\sin(\beta/2) = 0$ locates all of the minima and the central maximum, according to

$\beta/2 = 0,\ \pi,\ 2\pi,\ \ldots;$ $\beta = \frac{2\pi a\sin\theta}{\lambda} = 0,\ 2\pi,\ 4\pi,\ \ldots;$ $a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots.$

The side maxima are found from $\frac{\beta}{2}\cos\left(\frac{\beta}{2}\right) - \sin\left(\frac{\beta}{2}\right) = 0$, or $\tan\left(\frac{\beta}{2}\right) = \frac{\beta}{2}$.

This has solutions $\boxed{\frac{\beta}{2} = 4.4934}$, $\boxed{\frac{\beta}{2} = 7.7253}$, and others, giving

(a) $\pi a\sin\theta = 4.4934\lambda$ $\boxed{a\sin\theta = 1.4303\lambda}$

(b) $\pi a\sin\theta = 7.7253\lambda$ $\boxed{a\sin\theta = 2.4590\lambda}$

***38.69** (a) We require $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D}{2L}$.

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44(500\times10^{-9}\text{ m})(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

Chapter 39 Solutions

39.1 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2000 \text{ kg})(20.0 \text{ m/s}) + (1500 \text{ kg})(0 \text{ m/s}) = 4.00 \times 10^4 \text{ kg} \cdot \text{m/s}$$

$$p_f = (m_1 + m_2) v_f = (2000 \text{ kg} + 1500 \text{ kg}) v_f$$

Since $p_i = p_f$, $\quad v_f = \dfrac{4.00 \times 10^4 \text{ kg} \cdot \text{m/s}}{2000 \text{ kg} + 1500 \text{ kg}} = 11.429 \text{ m/s}$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0 \text{ m/s} - (+10.0 \text{ m/s}) = 10.0 \text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0 \text{ m/s} - (+10.0 \text{ m/s}) = -10.0 \text{ m/s}$$

$$v'_f = 11.429 \text{ m/s} - (+10.0 \text{ m/s}) = 1.429 \text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2000 \text{ kg})(10.0 \text{ m/s}) + (1500 \text{ kg})(-10.0 \text{ m/s}) = 5000 \text{ kg} \cdot \text{m/s}$$

and our final momentum is

$$p'_f = (2000 \text{ kg} + 1500 \text{ kg}) v'_f = (3500 \text{ kg})(1.429 \text{ m/s}) = 5000 \text{ kg} \cdot \text{m/s}$$

39.2 (a) $v = v_T + v_B = \boxed{60.0 \text{ m/s}}$

(b) $v = v_T - v_B = \boxed{20.0 \text{ m/s}}$

(c) $v = \sqrt{v_T^2 + v_B^2} = \sqrt{20^2 + 40^2} = \boxed{44.7 \text{ m/s}}$

39.3 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object v_1. The second observer has constant velocity v_{21} relative to the first, and measures the object to have velocity $v_2 = v_1 - v_{21}$.

The second observer measures an acceleration of $\quad a_2 = \dfrac{dv_2}{dt} = \dfrac{dv_1}{dt}$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma F = ma$.

39.4 The laboratory observer notes Newton's second law to hold: $F_1 = ma_1$

(where the subscript 1 implies the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $a_2 = a_1 - a'$

(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation

$F_2 = ma_2$ or $F_1 = ma_2$

(since $F_1 = F_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $F_2 = ma_2 - ma'$ which is *not* Newton's second law.

***39.5** $L = L_p\sqrt{1 - v^2/c^2} \Rightarrow v = c\sqrt{1 - \left(L/L_p\right)^2}$

Taking $L = L_p/2$ where $L_p = 1.00$ m gives $v = c\sqrt{1 - \left(\frac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \frac{1}{4}} = \boxed{0.866c}$

39.6 $\Delta t = \frac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ so $v = c\left[1 - \left(\frac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\Delta t = 2\Delta t_p \Rightarrow v = c\left[1 - \left(\frac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \frac{1}{4}\right]^{1/2} = \boxed{0.866c}$

***39.7** (a) $\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = \frac{1}{\sqrt{1 - (0.500)^2}} = \frac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma\Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}$$

Thus, the Earth observer records a pulse rate of $\frac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

39.8 The observed length of an object moving at speed v is $L = L_p\sqrt{1 - v^2/c^2}$ with L_p as the proper length. For the two ships, we know $L_2 = L_1$, $L_{2p} = 3L_{1p}$, and $v_1 = 0.350c$

Thus, $L_2^2 = L_1^2$ and $9L_{1p}^2\left(1 - \frac{v_2^2}{c^2}\right) = L_{1p}^2\left[1 - (0.350)^2\right]$

giving $9 - 9\frac{v_2^2}{c^2} = 0.878$, or $v_2 = \boxed{0.950c}$

***39.9** $\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1 - v^2/c^2}\right)\Delta t \approx \left(1 - \frac{v^2}{2c^2}\right)\Delta t$ and $\Delta t - \Delta t_p = \left(\frac{v^2}{2c^2}\right)\Delta t$

If $v = 1000\ \text{km/h} = \frac{1.00 \times 10^6\ \text{m}}{3600\ \text{s}} = 277.8\ \text{m/s}$, then $\frac{v}{c} = 9.26 \times 10^{-7}$

and $\left(\Delta t - \Delta t_p\right) = (4.28 \times 10^{-13})(3600\ \text{s}) = 1.54 \times 10^{-9}\ \text{s} = \boxed{1.54\ \text{ns}}$

39.10 $\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - (0.950)^2} = 0.312$

(a) astronauts' time: $\Delta t_p = \gamma^{-1}\Delta t = (0.312)(4.42\ \text{yr}) = \boxed{1.38\ \text{yr}}$

(b) astronauts' distance: $L = \gamma^{-1}\Delta L_p = (0.312)(4.20\ \text{ly}) = \boxed{1.31\ \text{ly}}$

39.11 The spaceship appears length-contracted to the Earth observer as given by

$L = L_p\sqrt{1 - v^2/c^2}$ or $L^2 = L_p^2\left(1 - v^2/c^2\right)$

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \frac{v^2}{c^2}(ct)^2$

Equating these two expressions gives $L_p^2 - L_p^2\frac{v^2}{c^2} = (ct)^2\frac{v^2}{c^2}$ or $\left[L_p^2 + (ct)^2\right]\frac{v^2}{c^2} = L_p^2$

Using the given values: $L_p = 300\ \text{m}$ and $t = 7.50 \times 10^{-7}\ \text{s}$

this becomes $(1.41 \times 10^5\ \text{m}^2)\frac{v^2}{c^2} = 9.00 \times 10^4\ \text{m}^2$

giving $v = \boxed{0.800c}$

Goal Solution

A spaceship with a proper length of 300 m takes 0.750 μs seconds to pass an Earth observer. Determine its speed as measured by the Earth observer.

G: We should first determine if the spaceship is traveling at a relativistic speed: classically, $v = (300\text{m})/(0.750\ \mu\text{s}) = 4.00 \times 10^8$ m/s, which is faster than the speed of light (impossible)! Quite clearly, the relativistic correction must be used to find the correct speed of the spaceship, which we can guess will be close to the speed of light.

O: We can use the contracted length equation to find the speed of the spaceship in terms of the proper length and the time. The time of 0.750 μs is the **proper time** measured by the Earth observer, because it is the time interval between two events that she sees as happening at the same point in space. The two events are the passage of the front end of the spaceship over her stopwatch, and the passage of the back end of the ship.

A: $L = L_p / \gamma$, with $L = v\Delta t$: $v\Delta t = L_p\left(1 - v^2 / c^2\right)^{1/2}$

Squaring both sides, $v^2\Delta t^2 = L_p{}^2\left(1 - v^2 / c^2\right)$

$$v^2c^2 = L_p{}^2c^2 / \Delta t^2 - v^2L_p{}^2 / \Delta t^2$$

Solving for the velocity, $v = \dfrac{cL_p / \Delta t}{\sqrt{c^2 + L_p{}^2 / \Delta t^2}}$

So $v = \dfrac{\left(3.00 \times 10^8\right)(300\ \text{m})\Big/\left(0.750 \times 10^{-6}\ \text{s}\right)}{\sqrt{\left(3.00 \times 10^8\right)^2 + (300\ \text{m})^2\Big/\left(0.750 \times 10^{-6}\ \text{s}\right)^2}} = 2.40 \times 10^8\ \text{m / s}$

L: The spaceship is traveling at 0.8c. We can also verify that the general equation for the speed reduces to the classical relation $v = L_p / \Delta t$ when the time is relatively large.

39.12 The spaceship appears to be of length L to Earth observers,

where $L = L_p\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}$ and $L = vt$

$vt = L_p\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}$ so $v^2t^2 = L_p{}^2\left(1 - \dfrac{v^2}{c^2}\right)$

Solving for v, $v^2\left(t^2 + \dfrac{L_p{}^2}{c^2}\right) = L_p{}^2$ $\boxed{\dfrac{v}{c} = L_p\left(c^2t^2 + L_p{}^2\right)^{-1/2}}$

***39.13** For $\frac{v}{c} = 0.990, \quad \gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is $\Delta t = \frac{4.60 \text{ km}}{0.990c}$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60 \times 10^3 \text{ m}}{0.990\left(3.00 \times 10^8 \text{ m/s}\right)}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1 - (v/c)^2} = \frac{L_p}{\gamma} = \frac{4.60 \times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$

39.14 We find Carpenter's speed: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.37 \times 10^6 + 0.160 \times 10^6)}\right]^{1/2} = 7.82 \text{ km / s}$$

Then the time period of one orbit, $T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53 \times 10^6)}{7.82 \times 10^3} = 5.25 \times 10^3 \text{ s}$

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right](22T)$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82 \times 10^3 \text{ m / s}}{3 \times 10^8 \text{ m / s}}\right)^2 22\left(5.25 \times 10^3 \text{ s}\right) = \boxed{39.2\ \mu\text{s}}$$

(b) For one orbit, $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

39.15 For pion to travel 10.0 m in Δt in our frame, $10.0 \text{ m} = v\Delta t = v(\gamma \Delta t_p) = \frac{v(26.0 \times 10^{-9} \text{ s})}{\sqrt{1 - v^2 / c^2}}$

Solving for the velocity, $(3.85 \times 10^8 \text{ m / s})^2(1 - v^2 / c^2) = v^2$

$$1.48 \times 10^{17} \text{ m}^2 / \text{s}^2 = v^2(1 + 1.64)$$

$$v = 2.37 \times 10^8 \text{ m / s} = \boxed{0.789c}$$

***39.16** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.01$ so $\boxed{v = 0.140c}$

***39.17** (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you see its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have $L = L_p\sqrt{1 - v^2/c^2}$

from which $\dfrac{L}{L_p} = \dfrac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1 - \dfrac{v^2}{c^2}}$ and $\boxed{v = 0.312c}$

***39.18** (a) $\Delta t = \gamma\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$

(d) Mission control gets signals for 21.0 yr while the battery is operating, and then for 14.7 years after the battery stops powering the transmitter, 14.7 ly away: $21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$

***39.19** The orbital speed of the Earth is as described by $\Sigma F = ma$: $\dfrac{Gm_Sm_E}{r^2} = \dfrac{m_Ev^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{1.496\times10^{11}\text{ m}}} = 2.98\times10^4\text{ m s}$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 57.005\,66\times10^6\text{ Hz}$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}} = 56.994\,34\times10^6\text{ Hz}$$

The difference, which lets them figure out the speed of our planet, is

$$(57.005\,66 - 56.994\,34)\times10^6\text{ Hz} = \boxed{1.13\times10^4\text{ Hz}}$$

39.20 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\dfrac{c+v}{c-v}}$

and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\dfrac{c+v}{c-v}}$

Combining gives $\boxed{f = f_{\text{source}}\dfrac{(c+v)}{(c-v)}}$

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$

The beat frequency is then $f_b = f - f_{\text{source}} = \dfrac{2f_{\text{source}}v}{c} = \boxed{\dfrac{2v}{\lambda}}$

(c) $$f_b = \frac{(2)(30.0\ \text{m/s})(10.0\times10^9\ \text{Hz})}{3.00\times10^8\ \text{m/s}} = \frac{(2)(30.0\ \text{m/s})}{(0.0300\ \text{m})} = 2000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\ \text{m/s}}{10.0\times10^9\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $v = \dfrac{f_b\lambda}{2}$ so $\Delta v = \dfrac{\Delta f_b\lambda}{2} = \dfrac{(5\ \text{Hz})(0.0300\ \text{m})}{2} = \boxed{0.0750\ \text{m/s} \approx 0.2\ \text{mi / h}}$

39.21 (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c-v_s}{c+v_s}\right)^{1/2} \qquad \text{where} \qquad v_s = v_{\text{source}}$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1-\left(\frac{v_s}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1-\frac{v_s}{2c}\right)\left(1-\frac{v_s}{2c}\right) \approx \left(1-\frac{v_s}{c}\right)$$

So, $$f_{\text{obs}} \approx f_{\text{source}}\left(1-\frac{v_s}{c}\right)$$

The observed wavelength is found from $c = \lambda_{\text{obs}} f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1-v_s/c)} = \frac{\lambda}{1-v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{1}{1-v_s/c}-1\right) = \lambda\left(\frac{v_s/c}{1-v_s/c}\right)$$

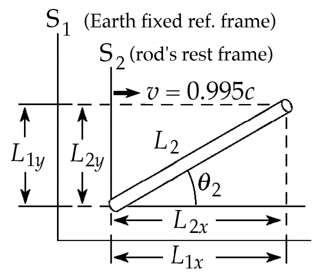
Since $1 - v_s/c \approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda} \approx \dfrac{v_{\text{source}}}{c}}$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\ \text{nm}}{397\ \text{nm}}\right) = \boxed{0.050\,4c}$$

39.22 $u_x' = \dfrac{u_x - v}{1 - u_x v / c^2} = \dfrac{0.950c - 0.750c}{1 - 0.950 \times 0.750} = \boxed{0.696c}$

39.23 $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2} = \dfrac{-0.750c - 0.750c}{1 - (-0.750)(0.750)} = \boxed{-0.960c}$

***39.24** $\gamma = 10.0$ We are also given: $L_1 = 2.00$ m, and $\theta_1 = 30.0°$ (both measured in a reference frame moving relative to the rod).



Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00 \text{ m})(0.867) = 1.73$ m

and $L_{1y} = L_1 \sin\theta_1 = (2.00 \text{ m})(0.500) = 1.00$ m

L_{2x} = a "proper length" is related to L_{1x}

by $L_{1x} = L_{2x}/\gamma$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3$ m and $L_{2y} = L_{1y} = 1.00$ m

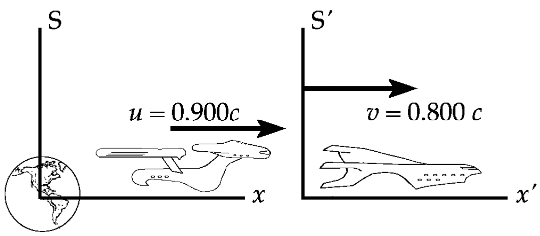
(Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4 \text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

39.25 u_x = Enterprise velocity

v = Klingon velocity



From Equation 39.16,

$u_x' = \dfrac{u_x - v}{1 - \dfrac{u_x v}{c^2}} = \dfrac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$

***39.26** (a) From Equation 39.13, $\Delta x' = \gamma(\Delta x - v\Delta t)$,

$$0 = \gamma\left[2.00 \text{ m} - v\left(8.00\times10^{-9} \text{ s}\right)\right]$$

$$v = \frac{2.00 \text{ m}}{8.00\times10^{-9} \text{ s}} = \boxed{2.50\times10^{8} \text{ m/s}}$$

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8} \text{ m/s}\right)^2 / \left(3.00\times10^{8} \text{ m/s}\right)^2}} = 1.81$$

(b) From Equation 39.11, $x' = \gamma(x - vt) = 1.81\left[3.00 \text{ m} - \left(2.50\times10^{8} \text{ m/s}\right)\left(1.00\times10^{-9} \text{ s}\right)\right] = \boxed{4.97 \text{ m}}$

(c)
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = 1.81\left[1.00\times10^{-9} \text{ s} - \frac{\left(2.50\times10^{8} \text{ m/s}\right)}{\left(3.00\times10^{8} \text{ m/s}\right)^2}(3.00 \text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8} \text{ s}}$$

39.27 $p = \gamma mu$

(a) For an electron moving at $0.0100c$, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = \dfrac{1}{\sqrt{1-(0.0100)^2}} = 1.00005 \approx 1.00$

Thus, $p = 1.00\left(9.11\times10^{-31} \text{ kg}\right)(0.0100)\left(3.00\times10^{8} \text{ m / s}\right) = \boxed{2.73\times10^{-24} \text{ kg}\cdot\text{m/s}}$

(b) Following the same steps as used in part (a), we find at $0.500c$

$\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22} \text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22} \text{ kg}\cdot\text{m/s}}$

***39.28** Using the relativistic form, $p = \dfrac{mu}{\sqrt{1-(u/c)^2}} = \gamma mu$,

we find the difference Δp from the classical momentum, mu: $\Delta p = \gamma mu - mu = (\gamma - 1)mu$

(a) The difference is 1.00% when $(\gamma - 1)mu = 0.0100\gamma mu$:

$$\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(u/c)^2}} \Rightarrow 1-(u/c)^2 = (0.990)^2 \quad \text{or} \quad u = \boxed{0.141c}$$

(b) The difference is 10.0% when $(\gamma - 1)mu = 0.100\gamma mu$:

$$\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(u/c)^2}} \Rightarrow 1-(u/c)^2 = (0.900)^2 \quad \text{or} \quad u = \boxed{0.436c}$$

***39.29** $\frac{p-mu}{mu}=\frac{\gamma mu-mu}{mu}=\gamma-1$

$$\gamma-1=\frac{1}{\sqrt{1-(u/c)^2}}-1\approx 1+\frac{1}{2}\left(\frac{u}{c}\right)^2-1=\frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p-mu}{mu}=\frac{1}{2}\left(\frac{90.0\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2=\boxed{4.50\times10^{-14}}$$

39.30 $p=\frac{mu}{\sqrt{1-(u/c)^2}}$ becomes $1-\frac{u^2}{c^2}=\frac{m^2u^2}{p^2}$

which gives: $1=u^2\left(\frac{m^2}{p^2}+\frac{1}{c^2}\right)$

or $c^2=u^2\left(\frac{m^2c^2}{p^2}+1\right)$ and $\boxed{u=\frac{c}{\sqrt{\frac{m^2c^2}{p^2}+1}}}$

***39.31** Relativistic momentum must be conserved:

For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2=p_1$

or $\gamma_2m_2u_2=\gamma_1m_1u_1=\frac{2.50\times10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$

or $\frac{(1.67\times10^{-27}\ \text{kg})u_2}{\sqrt{1-(u_2/c)^2}}=(4.960\times10^{-28}\ \text{kg})c$

and $u_2=\boxed{0.285c}$

Goal Solution

An unstable particle at rest breaks into two fragments of **unequal** mass. The rest mass of the lighter fragment is 2.50×10^{-28} kg, and that of the heavier fragment is 1.67×10^{-27} kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

G: The heavier fragment should have a speed less than that of the lighter piece since the momentum of the system must be conserved. However, due to the relativistic factor, the ratio of the speeds will not equal the simple ratio of the particle masses, which would give a speed of $0.134c$ for the heavier particle.

O: Relativistic momentum of the system must be conserved. For the total momentum to be zero after the fission, as it was before, $\mathbf{p}_1 + \mathbf{p}_2 = 0$, where we will refer to the lighter particle with the subscript '1', and to the heavier particle with the subscript '2.'

A: $\gamma_2 m_2 v_2 + \gamma_1 m_1 v_1 = 0$ so $\gamma_2 m_2 v_2 + \left(\dfrac{2.50 \times 10^{-28}\text{ kg}}{\sqrt{1-0.893^2}}\right)(0.893c) = 0$

Rearranging, $\left(\dfrac{1.67 \times 10^{-27}\text{ kg}}{\sqrt{1 - v_2^2/c^2}}\right)\dfrac{v_2}{c} = -4.96 \times 10^{-28}\text{ kg}$

Squaring both sides, $\left(2.79 \times 10^{-54}\right)\left(\dfrac{v_2}{c}\right)^2 = \left(2.46 \times 10^{-55}\right)\left(1 - \dfrac{v_2^2}{c^2}\right)$ and $v_2 = -0.285c$

We choose the negative sign only to mean that the two particles must move in opposite directions. The speed, then, is $|v_2| = 0.285c$

L: The speed of the heavier particle is less than the lighter particle, as expected. We can also see that for this situation, the relativistic speed of the heavier particle is about twice as great as was predicted by a simple non-relativistic calculation.

39.32 $\Delta E = (\gamma_1 - \gamma_2)mc^2$. For an electron, $mc^2 = 0.511$ MeV.

(a) $\Delta E = \left(\sqrt{\dfrac{1}{(1-0.810)}} - \sqrt{\dfrac{1}{(1-0.250)}}\right)mc^2 = \boxed{0.582\text{ MeV}}$

(b) $\Delta E = \left(\sqrt{\dfrac{1}{1-(0.990)^2}} - \sqrt{\dfrac{1}{1-0.810}}\right)mc^2 = \boxed{2.45\text{ MeV}}$

39.33 $E = \gamma mc^2 = 2mc^2$, or $\gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1-(1/\gamma)^2} = \dfrac{\sqrt{3}}{2}$, or $u = \dfrac{c\sqrt{3}}{2}$.

The momentum is then $p = \gamma mu = 2m\left(\dfrac{c\sqrt{3}}{2}\right) = \left(\dfrac{mc^2}{c}\right)\sqrt{3} = \left(\dfrac{938.3\text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63 \times 10^3\ \dfrac{\text{MeV}}{c}}$

***39.34** The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$

For $u = 0.100c$, $K_r = \left(\dfrac{1}{\sqrt{1-0.0100}} - 1\right)mc^2 = 0.005038\,mc^2$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $K_c = \frac{1}{2}m(0.100\,c)^2 = 0.005000\,mc^2$

different by $\dfrac{0.005038 - 0.005000}{0.005038} = 0.751\%$

For still smaller speeds the agreement will be still better.

39.35 (a) $E_R = mc^2 = (1.67\times10^{-27}\text{ kg})(2.998\times10^8\text{ m / s})^2 = 1.50\times10^{-10}\text{ J} = \boxed{938\text{ MeV}}$

(b) $E = \gamma mc^2 = \dfrac{1.50\times10^{-10}\text{ J}}{[1-(0.95c\,/\,c)^2]^{1/2}} = 4.81\times10^{-10}\text{ J} = \boxed{3.00\times10^3\text{ MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\text{ J} - 1.50\times10^{-10}\text{ J} = 3.31\times10^{-10}\text{ J} = \boxed{2.07\times10^3\text{ MeV}}$

***39.36** (a) $KE = E - E_R = 5E_R$

$E = 6E_R = 6(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m / s})^2 = 4.92\times10^{-13}\text{ J} = \boxed{3.07\text{ MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R$

Thus, $\gamma = \dfrac{E}{E_r} = 6 = \dfrac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

39.37 The relativistic density is

$$\frac{E_R}{c^2 V} = \frac{mc^2}{c^2 V} = \frac{m}{V} = \frac{m}{(L_p)(L_p)\left[L_p\sqrt{1-(u/c)^2}\right]} = \frac{8.00\text{ g}}{(1.00\text{ cm})^3\sqrt{1-(0.900)^2}} = \boxed{18.4\text{ g/cm}^3}$$

***39.38** We must conserve both mass-energy and relativistic momentum. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \qquad \text{and} \qquad \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22$$

Conservation of mass-energy gives $E_1 + E_2 = E_{\text{total}}$ which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34 \times 10^{-27}$ kg

This reduces to: $m_1 + 3.09 m_2 = 1.66 \times 10^{-27}$ kg [1]

Since the momentum after must equal zero, $p_1 = p_2$ gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c)m_1 = (6.22)(0.987c)m_2$

which becomes $m_1 = 3.52 m_2$ [2]

Solving [1] and [2] simultaneously, $m_1 = \boxed{8.84 \times 10^{-28}\ \text{kg}}$ and $m_2 = \boxed{2.51 \times 10^{-28}\ \text{kg}}$

39.39 $E = \gamma mc^2$, $p = \gamma mu$; $E^2 = (\gamma mc^2)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2 c^2 = \gamma^2\left((mc^2)^2 - (mc)^2 u^2\right) = (mc^2)^2\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1} = (mc^2)^2 \quad \text{Q.E.D.}$$

39.40 (a) $K = 50.0$ GeV

$$mc^2 = \left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.998 \times 10^8\ \text{m/s}\right)^2\left(\frac{1}{1.60 \times 10^{-10}\ \text{J/GeV}}\right) = 0.938\ \text{GeV}$$

$$E = K + mc^2 = 50.0\ \text{GeV} + 0.938\ \text{GeV} = 50.938\ \text{GeV}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \quad \Rightarrow \quad p = \sqrt{\frac{E^2 - \left(mc^2\right)^2}{c^2}} = \sqrt{\frac{(50.938\ \text{GeV})^2 - (0.938\ \text{GeV})^2}{c^2}}$$

$$p = 50.9\ \frac{\text{GeV}}{c} = \left(\frac{50.9\ \text{GeV}}{3.00 \times 10^8\ \text{m/s}}\right)\left(\frac{1.60 \times 10^{-10}\ \text{J}}{1\ \text{GeV}}\right) = \boxed{2.72 \times 10^{-17}\ \text{kg} \cdot \text{m/s}}$$

(b) $$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - (u/c)^2}} \quad \Rightarrow \quad u = c\sqrt{1 - \left(mc^2/E\right)^2}$$

$$v = \left(3.00 \times 10^8\ \text{m/s}\right)\sqrt{1 - \left(\frac{0.938\ \text{GeV}}{50.938\ \text{GeV}}\right)^2} = \boxed{2.9995 \times 10^8\ \text{m/s}}$$

39.41 (a) $q(\Delta V) = K = (\gamma - 1)m_e c^2$

Thus, $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 1 + \dfrac{q(\Delta V)}{m_e c^2}$ from which $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1)m_e c^2 = q(\Delta V) = \left(1.60\times10^{-19}\text{ C}\right)\left(2.50\times10^{4}\text{ J/C}\right) = \boxed{4.00\times10^{-15}\text{ J}}$

39.42 (a) $E = \gamma mc^2 = 20.0\text{ GeV}$ with $mc^2 = 0.511\text{ MeV}$ for electrons. Thus, $\gamma = \dfrac{20.0\times10^{9}\text{ eV}}{0.511\times10^{6}\text{ eV}} = \boxed{3.91\times10^{4}}$

(b) $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = 3.91\times10^{4}$ from which $\boxed{u = 0.999\,999\,999\,7c}$

(c) $L = L_p\sqrt{1-(u/c)^2} = \dfrac{L_p}{\gamma} = \dfrac{3.00\times10^{3}\text{ m}}{3.91\times10^{4}} = 7.67\times10^{-2}\text{ m} = \boxed{7.67\text{ cm}}$

39.43 Conserving total momentum, $p_{\text{Before decay}} = p_{\text{after decay}} = 0$: $\quad p_\nu = p_\mu = \gamma m_\mu u = \gamma(206\,m_e)u$

Conservation of mass-energy gives: $\quad E_\mu + E_\nu = E_\pi$

$$\gamma m_\mu c^2 + p_\nu c = m_\pi c^2$$

$$\gamma(206\,m_e) + \frac{p_\nu}{c} = 270\,m_e$$

Substituting from the momentum equation above, $\quad \gamma(206\,m_e) + \gamma(206\,m_e)\dfrac{u}{c} = 270\,m_e$

or $\quad \gamma\left(1+\dfrac{u}{c}\right) = \dfrac{270}{206} = 1.31 \Rightarrow \dfrac{u}{c} = 0.264$

Then, $K_\mu = (\gamma-1)m_\mu c^2 = (\gamma-1)206\left(m_e c^2\right) = \left(\dfrac{1}{\sqrt{1-(0.264)^2}} - 1\right)206(0.511\text{ MeV}) = \boxed{3.88\text{ MeV}}$

Also, $E_\nu = E_\pi - E_\mu = m_\pi c^2 - \gamma m_\mu c^2 = (270 - 206\gamma)m_e c^2$

$$E_\nu = \left(270 - \frac{206}{\sqrt{1-(0.264)^2}}\right)(0.511\text{ MeV}) = \boxed{28.8\text{ MeV}}$$

***39.44** Let a 0.3-kg flag be run up a flagpole 7 m high.

We put into it energy $\quad mgh = 0.3\text{ kg}(9.8\text{ m/s}^2)\,7\text{ m} \approx 20\text{ J}$

So we put into it extra mass $\Delta m = \dfrac{E}{c^2} = \dfrac{20\text{ J}}{(3\times10^8\text{ m/s})^2} = 2\times10^{-16}\text{ kg}$

for a fractional increase of $\dfrac{2\times10^{16}\text{ kg}}{0.3\text{ kg}}\ \boxed{\sim10^{-15}}$

***39.45** $E = 2.86\times10^5$ J. Also, the mass-energy relation says that $E = mc^2$.

Therefore, $m = \dfrac{E}{c^2} = \dfrac{2.86\times10^5\text{ J}}{(3.00\times10^8\text{ m/s})^2} = \boxed{3.18\times10^{-12}\text{ kg}}$

No, a mass loss of this magnitude (out of a total of 9.00 g) $\boxed{\text{could not be detected}}$.

39.46 (a) $K = (\gamma-1)mc^2 = \left(\dfrac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2 = 0.25\,mc^2 = \boxed{2.25\times10^{22}\text{ J}}$

(b) $E = m_{\text{fuel}}c^2$ so $m_{\text{fuel}} = \dfrac{2.25\times10^{22}}{9.00\times10^{16}} = \boxed{2.50\times10^5\text{ kg}}$

39.47 $\Delta m = \dfrac{E}{c^2} = \dfrac{\mathcal{P}t}{c^2} = \dfrac{0.800\left(1.00\times10^9\text{ J/s}\right)\left(3.00\text{ yr}\right)\left(3.16\times10^7\text{ s/yr}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2} = \boxed{0.842\text{ kg}}$

39.48 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with $mc^2 = 8.60\times10^{-9}\text{ J} = 53.8\text{ GeV}$

Thus, $\left(mc^2+K\right)^2 = (14.0\text{ keV})^2 + \left(mc^2\right)^2$ or $\left(1+\dfrac{K}{mc^2}\right)^2 = \left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2 + 1$

So $1+\dfrac{K}{mc^2} = \sqrt{1+\left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1+\dfrac{1}{2}\left(\dfrac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx \dfrac{(14.0\text{ keV})^2}{2\,mc^2} = \dfrac{\left(14.0\times10^3\text{ eV}\right)^2}{2\left(53.8\times10^9\text{ eV}\right)} = \boxed{1.82\times10^{-3}\text{ eV}}$

39.49 $\mathcal{P} = \dfrac{dE}{dt} = \dfrac{d\left(mc^2\right)}{dt} = c^2\dfrac{dm}{dt} = 3.77\times10^{26}\text{ W}$

Thus, $\dfrac{dm}{dt} = \dfrac{3.77\times10^{26}\ \text{J/s}}{\left(3.00\times10^{8}\ \text{m/s}\right)^2} = \boxed{4.19\times10^{9}\ \text{kg/s}}$

39.50 $2m_ec^2 = 1.02\ \text{MeV}$: $E_\gamma \geq \boxed{1.02\ \text{MeV}}$

39.51 The moving observer sees the charge as stationary, so she says it feels no magnetic force.

$q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = q(\mathbf{E}'+0)$ and $\boxed{\mathbf{E}' = \mathbf{E}+\mathbf{v}\times\mathbf{B}}$

***39.52** (a) When $K_e = K_p$, $m_ec^2(\gamma_e-1) = m_pc^2(\gamma_p-1)$

In this case, $m_ec^2 = 0.511\ \text{MeV}$, $m_pc^2 = 938\ \text{MeV}$ and $\gamma_e = \left[1-(0.750)^2\right]^{-1/2} = 1.5119$

Substituting, $\gamma_p = 1+\dfrac{m_ec^2(\gamma_e-1)}{m_pc^2} = 1+\dfrac{(0.511\ \text{MeV})(1.5119-1)}{938\ \text{MeV}} = 1.000279$

but $\gamma_p = \dfrac{1}{\left[1-\left(u_p/c\right)^2\right]^{1/2}}$. Therefore, $u_p = c\sqrt{1-\gamma_p^{-2}} = \boxed{0.0236c}$

(b) When $p_e = p_p$, $\gamma_pm_pu_p = \gamma_em_eu_e$ or $\gamma_pu_p = \dfrac{\gamma_em_eu_e}{m_p}$.

Thus, $\gamma_pu_p = \dfrac{(1.5119)\left(0.511\ \text{MeV}/c^2\right)(0.750c)}{938\ \text{MeV}/c^2} = 6.1772\times10^{-4}\,c$

and $\dfrac{u_p}{c} = 6.1772\times10^{-4}\sqrt{1-\left(\dfrac{u_p}{c}\right)^2}$ which yields $u_p = \boxed{6.18\times10^{-4}\,c} = 185\ \text{km/s}$

39.53 (a) $10^{13}\ \text{MeV} = (\gamma-1)m_pc^2$ so $\gamma \approx 10^{10}$ $v_p \approx c$

$t' = \dfrac{t}{\gamma} = \dfrac{10^5\ \text{yr}}{10^{10}} = 10^{-5}\ \text{yr} \sim \boxed{10^2\ \text{s}}$

(b) $d' = ct'$ $\boxed{\sim10^{11}\ \text{m}}$

Goal Solution

The cosmic rays of highest energy are protons, which have kinetic energy on the order of 10^{13} MeV. (a) How long would it take a proton of this energy to travel across the Milky Way galaxy, having a diameter on the order of ~10^5 light-years, as measured in the proton's frame? (b) From the point of view of the proton, how many kilometers across is the galaxy?

G: We can guess that the energetic cosmic rays will be traveling close to the speed of light, so the time it takes a proton to traverse the Milky Way will be much less in the proton's frame than 10^5 years. The galaxy will also appear smaller to the high-speed protons than the galaxy's proper diameter of 10^5 light-years.

O: The kinetic energy of the protons can be used to determine the relativistic γ-factor, which can then be applied to the time dilation and length contraction equations to find the time and distance in the proton's frame of reference.

A: The relativistic kinetic energy of a proton is $K = (\gamma - 1)mc^2 = 10^{13}$ MeV

Its rest energy is $$mc^2 = \left(1.67 \times 10^{-27}\text{ kg}\right)\left(2.998 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 \left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ kg}\cdot\text{m}^2/\text{s}^2}\right) = 938\text{ MeV}$$

So $10^{13}\text{ MeV} = (\gamma - 1)(938\text{ MeV})$, and therefore $\gamma = 1.07 \times 10^{10}$

The proton's speed in the galaxy's reference frame can be found from $\gamma = 1\big/\sqrt{1 - v^2/c^2}$:

$$1 - v^2/c^2 = 8.80 \times 10^{-21} \quad \text{and} \quad v = c\sqrt{1 - 8.80 \times 10^{-21}} = \left(1 - 4.40 \times 10^{-21}\right)c \approx 3.00 \times 10^8\text{ m/s}$$

The proton's speed is nearly as large as the speed of light. In the galaxy frame, the traversal time is $\Delta t = x/v = 10^5\text{ light-years}/c = 10^5\text{ years}$

(a) This is dilated from the proper time measured in the proton's frame. The proper time is found from $\Delta t = \gamma \Delta t_p$:

$$\Delta t_p = \Delta t/\gamma = 10^5\text{ yr}\big/1.07 \times 10^{10} = 9.38 \times 10^{-6}\text{ years} = 296\text{ s} \sim \text{a few hundred seconds}$$

(b) The proton sees the galaxy moving by at a speed nearly equal to c, passing in 296 s:

$$\Delta L_p = v\Delta t_p = \left(3.00 \times 10^8\right)(296\text{ s}) = 8.88 \times 10^7\text{ km} \sim 10^8\text{ km}$$

$$\Delta L_p = \left(8.88 \times 10^{10}\text{ m}\right)\left(9.46 \times 10^{15}\text{ m/ly}\right) = 9.39 \times 10^{-6}\text{ ly} \sim 10^{-5}\text{ ly}$$

L: The results agree with our predictions, although we may not have guessed that the protons would be traveling so close to the speed of light! The calculated results should be rounded to zero significant figures since we were given order of magnitude data. We should also note that the relative speed of motion v and the value of γ are the same in both the proton and galaxy reference frames.

39.54 Take the primed frame as:

(a) The mother ship: $u_x = \dfrac{u'_{x'} + v}{1 + u'_{x'}v/c^2} = \dfrac{v+v}{1+v^2/c^2} = \dfrac{2v}{1+v^2/c^2} = \dfrac{2(0.500c)}{1+(0.500)^2} = \boxed{0.800c}$

(b) The shuttle: $u_x = \dfrac{v + \dfrac{2v}{1+v^2/c^2}}{1 + \dfrac{v}{c^2}\left(\dfrac{2v}{1+v^2/c^2}\right)} = \dfrac{3v + v^3/c^2}{1+3v^2/c^2} = \dfrac{3(0.500c) + (0.500c)^3/c^2}{1+3(0.500)^2} = \boxed{0.929c}$

39.55 $\dfrac{\Delta mc^2}{mc^2} = \dfrac{4(938.78\text{ MeV}) - 3728.4\text{ MeV}}{4(938.78\text{ MeV})} \times 100\% = \boxed{0.712\%}$

39.56 $d_{\text{earth}} = vt_{\text{earth}} = v\gamma t_{\text{astro}}$ so $2.00\times10^6\text{ yr}\cdot c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\text{ yr}$

$\sqrt{1-v^2/c^2} = (v/c)(1.50\times10^{-5})$ $\quad 1 - \dfrac{v^2}{c^2} = \dfrac{v^2}{c^2}(2.25\times10^{-10})$

$1 = \dfrac{v^2}{c^2}(1+2.25\times10^{-10})$ so $\dfrac{v}{c} = \left(1+2.25\times10^{-10}\right)^{-1/2} = 1 - \tfrac{1}{2}(2.25\times10^{-10})$

$\boxed{\dfrac{v}{c} = 1 - 1.12\times10^{-10}}$

***39.57** (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$. Then $u'_x = +0.800c$, and

$$u_x = \frac{u'_x + v}{1+(u'_x v)/c^2} = \frac{0.800c + 0.600c}{1+(0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \dfrac{L_p}{\gamma} = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$. Thus,

$$\text{time} = \frac{0.160\text{ ly}}{0.800c + 0.600c} = \boxed{0.114\text{ yr}}$$

(d) $K = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2 = \left(\dfrac{1}{\sqrt{1-(0.946)^2}} - 1\right)(4.00\times10^5\text{ kg})(3.00\times10^8\text{ m/s})^2 = \boxed{7.50\times10^{22}\text{ J}}$

39.58 In this case, the proper time is T_0 (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$

where $\Delta t = T + t$. Here T is the time she waits before sending a signal and t is the time required for the signal to reach the students.

Thus, we have: $$T + t = \gamma T_0 \qquad (1)$$

To determine the travel time t, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$

The time required for the signal to travel this distance is: $t = \frac{d}{c} = \left(\frac{v}{c}\right)(T + t)$

Solving for t gives: $$t = \frac{(v/c)T}{1 - (v/c)}$$

Substituting this into equation (1) yields: $$T + \frac{(v/c)T}{1 - (v/c)} = \gamma T_0$$

or $$T = (1 - v/c)^{-1} = \gamma T_0$$

Then $$T = T_0 \frac{1 - (v/c)}{\sqrt{1 - (v^2/c^2)}} = T_0 \frac{1 - (v/c)}{\sqrt{[1 + (v/c)][1 - (v/c)]}} = \boxed{T_0 \sqrt{\frac{1 - (v/c)}{1 + (v/c)}}}$$

39.59 Look at the situation from the instructor's viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v + u'}{1 + vu'/c^2} = \frac{(-0.280c) - (0.600c)}{1 + (-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock)}$$

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$$\Delta t = \gamma \Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - (0.753c)^2 / c^2}} = 1.52$$

Thus, the students measure the exam to last $T = 1.52(50.0 \text{ min}) = \boxed{76.0 \text{ minutes}}$

(b) The duration of the exam as measured by observers on Earth is:

$$\Delta t = \gamma \Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - (0.280c)^2/c^2}} \quad \text{so} \quad T = 1.04(50.0 \text{ min}) = \boxed{52.1 \text{ minutes}}$$

***39.60** The energy which arrives in one year is $E = \mathcal{P}t = \left(1.79\times10^{17}\text{ J / s}\right)\left(3.16\times10^{7}\text{ s}\right) = 5.66\times10^{24}\text{ J}$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66\times10^{24}\text{ J}}{\left(3.00\times10^{8}\text{ m / s}\right)^2} = \boxed{6.28\times10^{7}\text{ kg}}$

***39.61** The observer sees the proper length of the tunnel, 50.0 m, but sees the train contracted to length

$L = L_p\sqrt{1-v^2/c^2} = 100\text{ m}\sqrt{1-(0.950)^2} = 31.2\text{ m}$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8\text{ m}}$ so it is completely within the tunnel.

***39.62** If the energy required to remove a mass m from the surface is equal to its mass energy mc^2, then

$$\frac{GM_s m}{R_g} = mc^2$$

and
$$R_g = \frac{GM_s}{c^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2\text{ / kg}^2)(1.99\times10^{30}\text{ kg})}{(3.00\times10^{8}\text{ m / s})^2} = 1.47\times10^{3}\text{ m} = \boxed{1.47\text{ km}}$$

39.63 (a) At any speed, the momentum of the particle is given by $p = \gamma mu = \dfrac{mu}{\sqrt{1-(u/c)^2}}$

Since $F = qE = \dfrac{dp}{dt}$
$$qE = \frac{d}{dt}\left[mu\left(1-u^2/c^2\right)^{-1/2}\right]$$

$$qE = m\left(1-u^2/c^2\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-u^2/c^2\right)^{-3/2}\left(2u/c^2\right)\frac{du}{dt}$$

So $\dfrac{qE}{m} = \dfrac{du}{dt}\left[\dfrac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$ and $\boxed{a = \dfrac{du}{dt} = \dfrac{qE}{m}\left(1-\dfrac{u^2}{c^2}\right)^{3/2}}$

(b) As $u \to c$, $\boxed{a \to 0}$

(c) $\displaystyle\int_0^v \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_{t=0}^{t}\frac{qE}{m}dt$ so $\boxed{u = \dfrac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}} = \boxed{\frac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2} - mc\right)}$$

***39.64** (a) $f_{\text{observed}} = f_{\text{source}}\sqrt{\dfrac{1+v/c}{1-v/c}}$ implies $\dfrac{c}{\lambda+\Delta\lambda} = \dfrac{c}{\lambda}\sqrt{\dfrac{1+v/c}{1-v/c}}$,

or $$\sqrt{\frac{1-v/c}{1+v/c}} = \frac{\lambda+\Delta\lambda}{\lambda}$$

and $$\boxed{1+\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1-v/c}{1+v/c}}}$$

(b) $$1+\frac{550\text{ nm}-650\text{ nm}}{650\text{ nm}} = \sqrt{\frac{1-v/c}{1+v/c}} = 0.846$$

$$1-\frac{v}{c} = (0.846)^2\left(1+\frac{v}{c}\right) = 0.716+0.716\left(\frac{v}{c}\right)$$

$$v = 0.166c = \boxed{4.97\times10^7\text{ m/s}}$$

39.65 (a) An observer at rest relative to the mirror sees the light travel a distance

$$D = 2d - x = 2\left(1.80\times10^{12}\text{ m}\right) - (0.800c)t$$

where $x = (0.800c)t$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2(1.80\times10^{12}\text{ m})}{3.00\times10^8\text{ m / s}} - 0.800t = \boxed{6.67\times10^3\text{ s}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of:

$$L = d\sqrt{1-\frac{v^2}{c^2}} = \left(1.80\times10^{12}\text{ m}\right)\sqrt{1-\frac{(0.800c)^2}{c^2}} = 1.08\times10^{12}\text{ m},$$

and the mirror moving toward the ship at speed $v = 0.800c$. Thus, he measures the distance the light travels as:

$$D = 2\left(1.08\times10^{12}\text{ m} - y\right)$$

where $y = (0.800c)(t/2)$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left[1.08\times10^{12}\text{ m} - (0.800c)\frac{t}{2}\right],$$ which gives $t = \boxed{4.00\times10^3\text{ s}}$

39.66 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t = \frac{D}{c} = \frac{2d - vt}{c} = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of

$$L = d\sqrt{1 - \frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as

$$D = 2(L - y)$$

where $y = vt/2$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left(d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right) \quad \text{so} \quad (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \quad \text{or} \quad \boxed{t = \frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

39.67 (a) Since Mary is in the same reference frame, S′, as Ted, she observes the ball to have the same speed Ted observes, namely $|u_x'| = \boxed{0.800c}$.

(b) $$\Delta t' = \frac{L_p}{|u_x'|} = \frac{1.80 \times 10^{12}\ \text{m}}{0.800(3.00 \times 10^8\ \text{m/s})} = \boxed{7.50 \times 10^3\ \text{s}}$$

(c) $$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = (1.80 \times 10^{12}\ \text{m})\sqrt{1 - \frac{(0.600c)^2}{c^2}} = \boxed{1.44 \times 10^{12}\ \text{m}}$$

Since $v = 0.600c$ and $u_x' = -0.800c$, the velocity Jim measures for the ball is

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2} = \frac{(-0.800c) + (0.600c)}{1 + (-0.800)(0.600)} = \boxed{-0.385c}$$

(d) Jim observes the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into thi distance from both ends. The gap closes at the rate $0.600c + 0.385c = 0.985c$, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44 \times 10^{12}\ \text{m}}{0.985(3.00 \times 10^{18}\ \text{m / s})} = \boxed{4.88 \times 10^3\ \text{s}}$$

39.68 (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$

The motion is in the x direction: $L_y = L_{0y} = L_0 \sin\theta_0$

$$L_x = L_{0x}\sqrt{1-(v/c)^2} = (L_0 \cos\theta_0)\sqrt{1-(v/c)^2}$$

Thus, $$L^2 = L_0{}^2 \cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0{}^2 \sin^2\theta_0 = L_0{}^2\left[1-\left(\frac{v}{c}\right)^2 \cos^2\theta_0\right]$$

or $$\boxed{L = L_0\left[1-(v/c)^2 \cos^2\theta_0\right]^{1/2}}$$

(b) $$\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma \tan\theta_0}$$

39.69 (a) First, we find the velocity of the stick relative to S′ using $L = L_p\sqrt{1-(u_x')^2/c^2}$

Thus $$u_x' = \pm\, c\sqrt{1-(L/L_p)^2}$$

Selecting the negative sign because the stick moves in the negative x direction in S′ gives:

$$u_x' = -c\sqrt{1-\left(\frac{0.500\text{ m}}{1.00\text{ m}}\right)^2} = -0.866c$$ so the speed is $|u_x'| = \boxed{0.866c}$

Now determine the velocity of the stick relative to S, using the measured velocity of the stick relative to S′ and the velocity of S′ relative to S. From the velocity addition equation, we have:

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{(-0.866c)+(0.600c)}{1+(0.600c)(-0.866c)} = -0.554c$$ and the speed is $|u_x| = \boxed{0.554c}$

(b) Therefore, the contracted length of the stick as measured in S is:

$$L = L_p\sqrt{1-(u_x/c)^2} = (1.00\text{ m})\sqrt{1-(0.554)^2} = \boxed{0.833\text{ m}}$$

39.70 (b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We see the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-(v/c)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$3.60\text{ ly}/1.80c = 2.00\text{ yr}.$$

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We see the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$$3.60\text{ ly}/0.200c = 18.0\text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

***39.71** The unshifted frequency is $$f_{\text{source}} = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{394\times10^{-9}\text{ m}} = 7.61\times10^{14}\text{ Hz}$$

We observe frequency $$f = \frac{3.00\times10^{8}\text{ m/s}}{475\times10^{-9}\text{ m}} = 6.32\times10^{14}\text{ Hz}$$

Then $$f = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}}$$

gives: $$6.32 = 7.61\sqrt{\frac{1+v/c}{1-v/c}}$$

or $$\frac{1+v/c}{1-v/c} = (0.829)^2$$

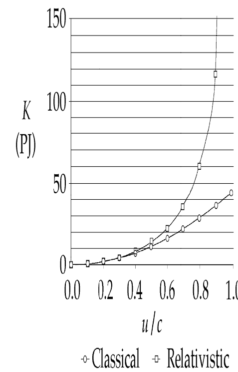
Solving for v yields: $$v = -0.185c = \boxed{0.185c\ (\text{away})}$$

39.72 Take $m = 1.00$ kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\text{ J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)mc^2 = \left(9.00\times10^{16}\text{ J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}}-1\right)$

u/c	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



$K_c = 0.990K_r$ when $\frac{1}{2}(u/c)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}}-1\right]$, yielding $u = \boxed{0.115c}$

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

39.73 $\Delta m = \frac{E}{c^2} = \frac{mc(\Delta T)}{c^2} = \frac{\rho Vc(\Delta T)}{c^2} = \frac{\left(1030\text{ kg / m}^3\right)\left(1.40\times10^9\right)\left(10^3\text{ m}\right)^3\left(4186\text{ J / kg}\cdot{}^\circ\text{C}\right)\left(10.0\ {}^\circ\text{C}\right)}{\left(3.00\times10^8\text{ m / s}\right)^2}$

$\Delta m = \boxed{6.71\times10^8\text{ kg}}$

Chapter 40 Solutions

40.1 $T = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} = \boxed{5.18\times10^{3}\ \text{K}}$

***40.2** (a) $\lambda_{\text{max}} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} \sim \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{4}\ \text{K}}\ \boxed{\sim10^{-7}\ \text{m}}$ $\boxed{\text{ultraviolet}}$.

(b) $\lambda_{\text{max}} \sim \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{10^{7}\ \text{K}}\ \boxed{\sim10^{-10}\ \text{m}}$. $\boxed{\gamma-\text{ray}}$

40.3 (a) Using $\lambda_{\text{max}}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

we get $\lambda_{\text{max}} = \dfrac{2.898\times10^{-3}\ \text{m}}{2900\ \text{K}} = 9.99\times10^{-7}\ \text{m} = \boxed{999\ \text{nm}}$

(b) The $\boxed{\text{peak wavelength is in the infrared}}$ region of the electromagnetic spectrum, which is much wider than the visible region of the spectrum.

40.4 Planck's radiation law gives intensity-per-wavelength. Taking E to be the photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathcal{P} = \frac{2\pi hc^2(\lambda_2-\lambda_1)\pi(d/2)^2}{\left(\frac{\lambda_1+\lambda_2}{2}\right)^5\left(e^{\frac{2hc}{(\lambda_1+\lambda_2)k_BT}}-1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1+\lambda_2}$$

$$n = \frac{\mathcal{P}}{E} = \frac{8\pi^2 c\, d^2(\lambda_2-\lambda_1)}{(\lambda_1+\lambda_2)^4\left(e^{2hc/(\lambda_1+\lambda_2)k_BT}-1\right)} = \frac{8\pi^2\left(3.00\times10^{8}\ \text{m/s}\right)\left(5.00\times10^{-5}\ \text{m}\right)^2\left(1.00\times10^{-9}\ \text{m}\right)}{\left(1001\times10^{-9}\ \text{m}\right)^4\left(e^{\frac{2\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1001\times10^{-9}\ \text{m}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)\left(7.50\times10^{3}\ \text{K}\right)}}-1\right)}$$

$$n = \frac{5.90\times10^{16}\ /\ \text{s}}{\left(e^{3.84}-1\right)} = \boxed{1.30\times10^{15}\ /\ \text{s}}$$

***40.5** (a) $P = eA\sigma T^4 = 1\left(20.0\times10^{-4}\ \text{m}^2\right)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(5000\ \text{K})^4 = \boxed{7.09\times10^4\ \text{W}}$

(b) $\lambda_{\max}T = \lambda_{\max}(5000\ \text{K}) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \ \Rightarrow\ \lambda_{\max} = \boxed{580\ \text{nm}}$

(c) We compute: $\dfrac{hc}{k_B T} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)(5000\ \text{K})} = 2.88\times10^{-6}\ \text{m}$

The power per wavelength interval is $P(\lambda) = A\,I(\lambda) = \dfrac{2\pi hc^2 A}{\lambda^5\left[\exp\left(hc/\lambda k_B T\right)-1\right]}$, and

$$2\pi hc^2 A = 2\pi\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)^2\left(20.0\times10^{-4}\right) = 7.50\times10^{-19}\ \frac{\text{J}\cdot\text{m}^4}{\text{s}}$$

$$P(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\ \text{m}\right)^5\left[\exp\left(2.88\ \mu\text{m}/0.580\ \mu\text{m}\right)-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1} = \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d) - (i) The other values are computed similarly:

	λ	$hc/k_B T$	$e^{hc/\lambda k_B T}-1$	$2\pi hc^2 A/\lambda^5$	$P(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(c)	580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $P(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$P \approx \frac{\left[(5.44+7.99)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \frac{\text{W}}{\text{m}}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$P = 2.13\times10^{4}\ \text{W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

40.6 (a) $\mathsf{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathsf{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.77\times10^{26}\ \text{W}}{1\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right]\left(5.67\times10^{-8}\ \dfrac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)}\right]^{1/4} = \boxed{5.75\times10^{3}\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.75\times10^{3}\ \text{K}} = 5.04\times10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$$

40.7 (a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$$

(d) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

40.8 $$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{589.3\times10^{-9}\ \text{m}} = 3.37\times10^{-19}\ \text{J/photon}$$

$$n = \frac{\mathsf{P}}{E} = \frac{10.0\ \text{J/s}}{3.37\times10^{-19}\ \text{J/photon}} = \boxed{2.96\times10^{19}\ \text{photons/s}}$$

40.9 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $$\frac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photons}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$$

***40.10** Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{500 \times 10^{-9}\ \text{m}} = 3.98 \times 10^{-19}\ \text{J}$$

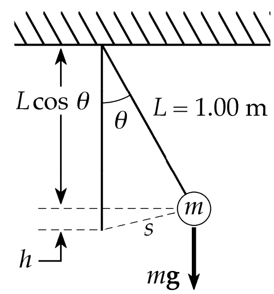
The energy entering the eye each second

$$E = \mathcal{P}t = (IA)t = (4.00 \times 10^{-11}\ \text{W/m}^2)\frac{\pi}{4}(8.50 \times 10^{-3}\ \text{m})^2(1.00\ \text{s}) = 2.27 \times 10^{-15}\ \text{J}$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27 \times 10^{-15}\ \text{J}}{3.98 \times 10^{-19}\ \text{J/photon}} = \boxed{5.71 \times 10^{3}\ \text{photons}}$$

40.11 We take $\theta = 0.0300$ radians. Then the pendulum's total energy is



$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\ \text{kg})(9.80\ \text{m/s}^2)(1.00 - 0.9995) = 4.41 \times 10^{-3}\ \text{J}$$

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{g/L} = 0.498\ \text{Hz}$

The energy is quantized, $E = nhf$

Therefore, $n = \frac{E}{hf} = \frac{4.41 \times 10^{-3}\ \text{J}}{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(0.498\ \text{s}^{-1})} = \boxed{1.34 \times 10^{31}}$

40.12 The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}$$

$$\lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\ \text{nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\ \text{nm}$$

The temperature of the star is given by $\lambda_{\max}T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$:

$$T = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{375 \times 10^{-9}} = \boxed{7.73 \times 10^{3}\ \text{K}}$$

40.13 This follows from the fact that at low T or long λ, the exponential factor in the denominator of Planck's radiation law is large compared to 1, so the factor of 1 in the denominator can be neglected. In this approximation, one arrives at *Wien's radiation law*.

***40.14** Planck's radiation law is $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$

Using the series expansion $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots$

Planck's law reduces to $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[\left(1 + hc/\lambda k_B T + \ldots\right) - 1\right]} \approx \dfrac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)} = \dfrac{2\pi c k_B T}{\lambda^4}$

which is the Rayleigh-Jeans law, for very long wavelengths.

40.15 (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^8\text{ m/s})}{(4.20\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})} = \boxed{296\text{ nm}}$

$f_c = \dfrac{c}{\lambda_c} = \dfrac{3.00 \times 10^8\text{ m/s}}{296 \times 10^{-9}\text{ m}} = \boxed{1.01 \times 10^{15}\text{ Hz}}$

(b) $\dfrac{hc}{\lambda} = \phi + e(\Delta V_S)$: $\dfrac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{180 \times 10^{-9}} = (4.20\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}) + (1.60 \times 10^{-19})(\Delta V_S)$

Therefore, $\boxed{\Delta V_S = 2.71\text{ V}}$

40.16 $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(9.11 \times 10^{-31})(4.60 \times 10^5)^2 = 9.64 \times 10^{-20}\text{ J} = 0.602\text{ eV}$

(a) $\phi = E - K_{max} = \dfrac{1240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ eV} = \boxed{1.38\text{ eV}}$

(b) $f_c = \dfrac{\phi}{h} = \dfrac{1.38\text{ eV}}{6.626 \times 10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60 \times 10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34 \times 10^{14}\text{ Hz}}$

40.17 (a) $\lambda_c = \dfrac{hc}{\phi}$ Li: $\lambda_c = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{(2.30\ \text{eV})(1.60\times10^{-19}\ \text{J / eV})} = 540\ \text{nm}$

Be: $\lambda_c = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{(3.90\ \text{eV})(1.60\times10^{-19}\ \text{J / eV})} = 318\ \text{nm}$

Hg: $\lambda_c = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{(4.50\ \text{eV})(1.60\times10^{-19}\ \text{J / eV})} = 276\ \text{nm}$

$\lambda < \lambda_c$ for photo current. Thus, only lithium will exhibit the photoelectric effect.

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{\max}$

$$\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{400\times10^{-9}\ \text{m}} = (2.30\ \text{eV})(1.60\times10^{-19}) + K_{\max}$$

$$K_{\max} = 1.29\times10^{-19}\ \text{J} = \boxed{0.808\ \text{eV}}$$

40.18 From condition (i), $hf = e(\Delta V_{S1}) + \phi_1$ and $hf = e(\Delta V_{S2}) + \phi_2$

$$(\Delta V_{S1}) = (\Delta V_{S2}) + 1.48\ \text{V}$$

Then $\phi_2 - \phi_1 = 1.48\ \text{eV}$

From condition (ii), $h f_{c1} = \phi_1 = 0.600 h f_{c2} = 0.600\phi_2$

$$\phi_2 - 0.600\phi_2 = 1.48\ \text{eV}$$

$$\boxed{\phi_2 = 3.70\ \text{eV}} \quad \boxed{\phi_1 = 2.22\ \text{eV}}$$

40.19 (a) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi \quad\rightarrow\quad \phi = \dfrac{1240\ \text{nm}\cdot\text{eV}}{546.1\ \text{nm}} - 0.376\ \text{eV} = \boxed{1.90\ \text{eV}}$

(b) $e(\Delta V_S) = \dfrac{hc}{\lambda} - \phi = \dfrac{1240\ \text{nm}\cdot\text{eV}}{587.5\ \text{nm}} - 1.90\ \text{eV} \quad\rightarrow\quad \Delta V_S = \boxed{0.216\ \text{V}}$

Goal Solution

Two light sources are used in a photoelectric experiment to determine the work function for a particular metal surface. When green light from a mercury lamp (λ = 546.1 nm) is used, a retarding potential of 0.376 V reduces the photocurrent to zero. (a) Based on this measurement, what is the work function for this metal? (b) What stopping potential would be observed when using the yellow light from a helium discharge tube (λ = 587.5 nm)?

G: According to Table 40.1, the work function for most metals is on the order of a few eV, so this metal is probably similar. We can expect the stopping potential for the yellow light to be slightly lower than 0.376 V since the yellow light has a longer wavelength (lower frequency) and therefore less energy than the green light.

O: In this photoelectric experiment, the green light has sufficient energy hf to overcome the work function of the metal ϕ so that the ejected electrons have a maximum kinetic energy of 0.376 eV. With this information, we can use the photoelectric effect equation to find the work function, which can then be used to find the stopping potential for the less energetic yellow light.

A: (a) Einstein's photoelectric effect equation is $K_{\max} = hf - \phi$, and the energy required to raise an electron through a 1 V potential is 1 eV, so that $K_{\max} = eV_s = 0.376$ eV.

A photon from the mercury lamp has energy:

$$hf = \frac{hc}{\lambda} = \frac{\left(4.14\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{546.1\times10^{-9}\ \text{m}}$$

$$E = hf = 2.27\ \text{eV}$$

Therefore, the work function for this metal is:

$$\phi = hf - K_{\max} = 2.27\ \text{eV} - (0.376\ \text{eV}) = 1.90\ \text{eV}$$

(b) For the yellow light, $\lambda = 587.5$ nm, and

$$hf = \frac{hc}{\lambda} = \frac{\left(4.14\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m / s}\right)}{587.5\times10^{-9}\ \text{m}}$$

$$E = 2.11\ \text{eV}$$

Therefore, $K_{\max} = hf - \phi = 2.11\ \text{eV} - 1.90\ \text{eV} = 0.216\ \text{eV}$, so $V_s = 0.216$ V

L: The work function for this metal is lower than we expected, and does not correspond with any of the values in Table 40.1. Further examination in the **CRC Handbook of Chemistry and Physics** reveals that all of the metal elements have work functions between 2 and 6 eV. However, a single metal's work function may vary by about 1 eV depending on impurities in the metal, so it is just barely possible that a metal might have a work function of 1.90 eV.

The stopping potential for the yellow light is indeed lower than for the green light as we expected. An interesting calculation is to find the wavelength for the lowest energy light that will eject electrons from this metal. That threshold wavelength for $K_{\max}$ = 0 is 658 nm, which is red light in the visible portion of the electromagnetic spectrum.)

40.20 From the photoelectric equation, we have: $e(\Delta V_{S1}) = E_{\gamma 1} - \phi$ and $e(\Delta V_{S2}) = E_{\gamma 2} - \phi$

Since $\Delta V_{S2} = 0.700(\Delta V_{S1})$, then $e(\Delta V_{S2}) = 0.700(E_{\gamma 1} - \phi) = E_{\gamma 2} - \phi$

or $(1 - 0.700)\phi = E_{\gamma 2} - 0.700 E_{\gamma 1}$

and the work function is: $\phi = \dfrac{E_{\gamma 2} - 0.700 E_{\gamma 1}}{0.300}$

The photon energies are: $E_{\gamma 1} = \dfrac{hc}{\lambda_1} = \dfrac{1240 \text{ nm} \cdot \text{eV}}{410 \text{ eV}} = 3.03 \text{ eV}$

and $E_{\gamma 2} = \dfrac{hc}{\lambda_2} = \dfrac{1240 \text{ nm} \cdot \text{eV}}{445 \text{ eV}} = 2.79 \text{ eV}$

Thus, the work function is $\phi = \dfrac{2.79 \text{ eV} - 0.700(3.03 \text{ eV})}{0.300} = 2.23 \text{ eV}$

and we recognize this as characteristic of $\boxed{\text{potassium}}$.

***40.21** The energy needed is $E = 1.00 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

The energy absorbed in time t is $E = Pt = (IA)t$

so $$t = \frac{E}{IA} = \frac{1.60 \times 10^{-19} \text{ J}}{(500 \text{ J/s} \cdot \text{m}^2)[\pi(2.82 \times 10^{-15} \text{ m})^2]} = 1.28 \times 10^7 \text{ s} = \boxed{148 \text{ days}}$$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

***40.22** Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$, or

$$K_{\max} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{200 \times 10^{-9} \text{ m}} \left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) - 4.70 \text{ eV} = 1.51 \text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{(5.00 \times 10^{-2} \text{ m})(1.51 \text{ N} \cdot \text{m/C})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = \boxed{8.41 \times 10^{-12} \text{ C}}$$

40.23 (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $f' = f\sqrt{\dfrac{1+v/c}{1-v/c}} = \left(7.00\times10^{14}\right)\sqrt{\dfrac{1.28}{0.720}} = 9.33\times10^{14}\ \text{Hz}$

Therefore, $\phi = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(9.33\times10^{14}\ \text{Hz}\right) = 6.18\times10^{-19}\ \text{J} = \boxed{3.87\ \text{eV}}$

(c) At $v = 0.900c$, $f = 3.05\times10^{15}\ \text{Hz}$

and $K_{\max} = hf - \phi = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.05\times10^{15}\ \text{Hz}\right)\left(\dfrac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 3.87\ \text{eV} = \boxed{8.78\ \text{eV}}$

***40.24** $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{700\times10^{-9}\ \text{m}} = 2.84\times10^{-19}\ \text{J} = \boxed{1.78\ \text{eV}}$

$p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{700\times10^{-9}\ \text{m}} = \boxed{9.47\times10^{-28}\ \text{kg}\cdot\text{m/s}}$

40.25 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = \dfrac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\ \text{m}}$

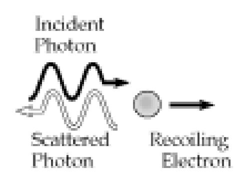
(b) $E_0 = hc/\lambda_0$: $\left(300\times10^{3}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right) = \left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\ \text{m/s}\right)/\lambda_0$

$\lambda_0 = 4.14\times10^{-12}\ \text{m}$ and $\lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\ \text{m}$

$E' = \dfrac{hc}{\lambda'} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{4.63\times10^{-12}\ \text{m}} = 4.30\times10^{14}\ \text{J} = \boxed{268\ \text{keV}}$

(c) $K_e = E_0 - E' = 300\ \text{keV} - 268.5\ \text{keV} = \boxed{31.5\ \text{keV}}$

40.26 This is Compton scattering through 180°:



$E_0 = \dfrac{hc}{\lambda_0} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.110\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J/eV})} = 11.3\ \text{keV}$

$\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = (2.43\times10^{-12}\ \text{m})(1-\cos 180°) = 4.86\times10^{-12}\ \text{m}$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\ \text{nm}$ so $E' = \dfrac{hc}{\lambda'} = 10.8\ \text{keV}$

Momentum conservation: $\dfrac{h}{\lambda_0}\ \mathbf{i} = \dfrac{h}{\lambda'}(-\mathbf{i}) + p_e(\mathbf{i})$ and $p_e = h\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda'}\right)$

$p_e = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(\dfrac{\left(3.00\times10^{8}\ \text{m/s}\right)/c}{1.60\times10^{-19}\ \text{J/eV}}\right)\left(\dfrac{1}{0.110\times10^{-9}\ \text{m}} + \dfrac{1}{0.115\times10^{-9}\ \text{m}}\right) = \boxed{22.1\ \text{keV}/c}$

Energy conservation: $11.3\text{ keV} = 10.8\text{ keV} + K_e$ so that $\boxed{K_e = 478\text{ eV}}$

Check: $E^2 = p^2c^2 + m_e^2c^4$ or $(m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$

$(511\text{ keV} + 0.478\text{ keV})^2 = (22.1\text{ keV})^2 + (511\text{ keV})^2$

$2.62 \times 10^{11} = 2.62 \times 10^{11}$

40.27 $K_e = E_0 - E'$

With $K_e = E'$, $E' = E_0 - E'$: $E' = \dfrac{E_0}{2}$

$\lambda' = \dfrac{hc}{E'} = \dfrac{hc}{\frac{1}{2}E_0} = 2\dfrac{hc}{E_0} = 2\lambda_0$ $\lambda' = \lambda_0 + \lambda_C(1 - \cos\theta)$

$2\lambda_0 = \lambda_0 + \lambda_C(1 - \cos\theta)$ $1 - \cos\theta = \dfrac{\lambda_0}{\lambda_C} = \dfrac{0.00160}{0.00243} \to \theta = \boxed{70.0°}$

40.28 We may write down four equations, not independent, in the three unknowns λ_0, λ', and v using the conservation laws:

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_ec^2 - m_ec^2 \qquad \text{(Energy conservation)}$$

$$\frac{h}{\lambda_0} = \gamma m_e v\cos 20.0° \quad \text{(momentum in } x\text{-direction)}$$

$$0 = \frac{h}{\lambda'} - \gamma m_e v\sin 20.0° \qquad \text{(momentum in } y\text{-direction)}$$

and Compton's equation $\lambda' - \lambda_0 = \dfrac{h}{m_ec}(1 - \cos 90.0°)$.

It is easiest to ignore the energy equation and, using the two momentum equations, write

$$\frac{h/\lambda_0}{h/\lambda'} = \frac{\gamma m_e v\cos 20.0°}{\gamma m_e v\sin 20.0°} \quad \text{or} \quad \lambda_0 = \lambda'\tan 20.0°$$

Then, the Compton equation becomes $\lambda' - \lambda'\tan 20.0° = 0.00243\text{ nm}$,

or $\lambda' = \dfrac{0.00243\text{ nm}}{1 - \tan 20.0°} = 0.00382\text{ nm} = \boxed{3.82\text{ pm}}$

40.29 (a) Conservation of momentum in the x direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $$\frac{h}{\lambda_0} = \left(p_e + \frac{h}{\lambda'}\right)\cos\theta \qquad [1]$$

Conservation of momentum in the y direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$,

which (neglecting the trivial solution $\theta = 0$) gives: $$p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting [2] into [1] gives: $\dfrac{h}{\lambda_0} = \dfrac{2h}{\lambda'}\cos\theta$, or $$\lambda' = 2\lambda_0 \cos\theta \qquad [3]$$

Then the Compton equation is $$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

giving $$2\lambda_0 \cos\theta - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

or $$2\cos\theta - 1 = \frac{hc}{\lambda_0}\frac{1}{m_e c^2}(1 - \cos\theta)$$

Since $E_\gamma = \dfrac{hc}{\lambda_0}$, this may be written as: $$2\cos\theta - 1 = \left(\frac{E_\gamma}{m_e c^2}\right)(1 - \cos\theta)$$

which reduces to: $$\left(2 + \frac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \frac{E_\gamma}{m_e c^2}$$

or $\cos\theta = \dfrac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \dfrac{0.511 \text{ MeV} + 0.880 \text{ MeV}}{1.02 \text{ MeV} + 0.880 \text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0°}$

(b) Using Equation (3): $E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda_0(2\cos\theta)} = \dfrac{E_\gamma}{2\cos\theta} = \dfrac{0.880 \text{ MeV}}{2\cos 43.0°} = 0.602 \text{ MeV} = \boxed{602 \text{ keV}}$

Then, $$p'_\gamma = \frac{E'_\gamma}{c} = 0.602 \text{ MeV}/c = \boxed{3.21\times10^{-22} \text{ kg}\cdot\text{m/s}}$$

(c) From Equation (2), $$p_e = p'_\gamma = 0.602 \text{ MeV}/c = \boxed{3.21\times10^{-22} \text{ kg}\cdot\text{m/s}}$$

From energy conservation: $$K_e = E_\gamma - E'_\gamma = 0.880 \text{ MeV} - 0.602 \text{ MeV} = 0.278 \text{ MeV} = \boxed{278 \text{ keV}}$$

40.30 The energy of the incident photon is $E_0 = p_\gamma c = hc/\lambda_0$.

(a) Conserving momentum in the x direction gives

$$p_\gamma = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta, \quad \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta \qquad [1]$$

Conserving momentum in the y direction (with $\phi = \theta$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \quad \text{or} \quad p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \quad \text{or} \quad \lambda' = \frac{2hc}{E_0}\cos\theta \qquad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \frac{h}{m_e c}\left(1 - \cos\theta\right)$, $\qquad \frac{2hc}{E_0}\cos\theta - \frac{2hc}{E_0} = \frac{h}{m_e c}\left(1 - \cos\theta\right)$

which reduces to $\qquad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$

Thus, $\qquad \boxed{\phi = \theta = \cos^{-1}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$

(b) From Equation [3], $\qquad \lambda' = \frac{2hc}{E_0}\cos\theta = \frac{2hc}{E_0}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$

Therefore, $\qquad E'_\gamma = \frac{hc}{\lambda'} = \frac{hc}{\left(2hc/E_0\right)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)} = \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$,

and $\qquad p'_\gamma = \frac{E'_\gamma}{c} = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

(c) From conservation of energy, $\qquad K_e = E_0 - E'_\gamma = E_0 - \frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or $\qquad K_e = \frac{E_0}{2}\left(\frac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$

Finally, from Equation (2), $\qquad p_e = p'_\gamma = \boxed{\frac{E_0}{2c}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

40.31 (a) Thanks to Compton we have four equations in the unknowns ϕ, v, and λ':

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + \gamma m_e c^2 - m_e c^2 \qquad \text{(energy conservation)} \qquad [1]$$

$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\cos 2\phi + \gamma m_e v \cos\phi \qquad \text{(momentum in } x \text{ direction)} \qquad [2]$$

$$0 = \frac{h}{\lambda'}\sin 2\phi - \gamma m_e v \sin\phi \qquad \text{(momentum in } y \text{ direction)} \qquad [3]$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos 2\phi) \qquad \text{(Compton equation)} \qquad [4]$$

Using $\sin 2\phi = 2\sin\phi\cos\phi$ in Equation [3] gives $\gamma m_e v = \frac{2h}{\lambda'}\cos\phi$.

Substituting this into Equation [2] and using $\cos 2\phi = 2\cos^2\phi - 1$ yields

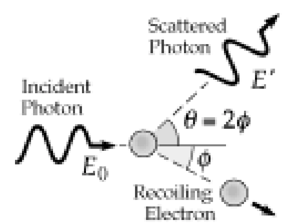
$$\frac{h}{\lambda_0} = \frac{h}{\lambda'}(2\cos^2\phi - 1) + \frac{2h}{\lambda'}\cos^2\phi = \frac{h}{\lambda'}(4\cos^2\phi - 1),$$

or $\quad \lambda' = 4\lambda_0\cos^2\phi - \lambda_0 \qquad [5]$

Substituting the last result into the Compton equation gives

$$4\lambda_0\cos^2\phi - 2\lambda_0 = \frac{h}{m_e c}\left[1 - \left(2\cos^2\phi - 1\right)\right] = 2\frac{hc}{m_e c^2}\left(1 - \cos^2\phi\right).$$

With the substitution $\lambda_0 = hc/E_0$, this reduces to



$$\cos^2\phi = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{1+x}{2+x} \quad \text{where} \quad x \equiv \frac{E_0}{m_e c^2}.$$

For $x = \frac{0.700 \text{ MeV}}{0.511 \text{ MeV}} = 1.37$, this gives $\phi = \cos^{-1}\sqrt{\frac{1+x}{2+x}} = \boxed{33.0°}$

(b) From Equation [5], $\lambda' = \lambda_0\left(4\cos^2\phi - 1\right) = \lambda_0\left[4\left(\frac{1+x}{2+x}\right) - 1\right] = \lambda_0\left(\frac{2+3x}{2+x}\right)$.

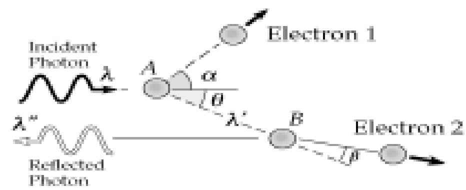
Then, Equation [1] becomes

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda_0}\left(\frac{2+x}{2+3x}\right) + \gamma m_e c^2 - m_e c^2 \quad \text{or} \quad \frac{E_0}{m_e c^2} - \frac{E_0}{m_e c^2}\left(\frac{2+x}{2+3x}\right) + 1 = \gamma.$$

Thus, $\gamma = 1 + x - x\left(\frac{2+x}{2+3x}\right)$, and with $x = 1.37$ we get $\gamma = 1.614$.

Therefore, $\frac{v}{c} = \sqrt{1 - \gamma^{-2}} = \sqrt{1 - 0.384} = 0.785$ or $v = \boxed{0.785c}$.

40.32 $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$



$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1 - \cos(\pi - \theta)\right]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi - \theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi - \theta) = -\cos\theta$, so $\lambda'' - \lambda = 2\frac{h}{m_e c} = \boxed{0.00486 \text{ nm}}$

40.33 (a) $K = \frac{1}{2}m_e v^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.40\times10^{6}\text{ m/s}\right)^2 = 8.93\times10^{-19}\text{ J} = 5.58\text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1240\text{ eV}\cdot\text{nm}}{0.800\text{ nm}} = 1550\text{ eV}$$

$E' = E_0 - K$, and $\lambda' = \frac{hc}{E'} = \frac{1240\text{ eV}\cdot\text{nm}}{1550\text{ eV} - 5.58\text{ eV}} = 0.803\text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.00288\text{ nm} = \boxed{2.88\text{ pm}}$$

(b) $\Delta\lambda = \lambda_C(1 - \cos\theta) \Rightarrow \cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.00288\text{ nm}}{0.00243\text{ nm}} = -0.189$, so $\boxed{\theta = 101°}$

***40.34** Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = (1 - \cos 180°)(h/mc) = 2h/mc$ where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = hc/E_0$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{0.511\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{938\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.00109}$$

40.35 Start with Balmer's equation, $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, or $\lambda = \frac{(4n^2 / R_H)}{(n^2 - 4)}$.

Substituting $R_H = 1.0973732 \times 10^7 \text{ m}^{-1}$, we obtain

$$\lambda = \frac{(3.645 \times 10^{-7}\text{ m})n^2}{n^2 - 4} = \frac{364.5n^2}{n^2 - 4}\text{ nm}, \quad \text{where } n = 3, 4, 5, \ldots$$

40.36 (a) Using $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, for $n_f = 2$, and $n_i \geq 3$, we get:

$$\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(2.00 \times 10^7\text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(200.0)n^2}{n^2 - 4}\text{ nm}$$

This says that $200\text{ nm} \leq \lambda \leq 360\text{ nm}$, which is $\boxed{\text{ultraviolet}}$.

(b) Using $n \geq 3$, $\lambda = \frac{4n^2}{R_H\left(n^2 - 4\right)} = \frac{4n^2}{\left(0.500 \times 10^7\text{ m}^{-1}\right)\left(n^2 - 4\right)} = \frac{(800.0)n^2}{n^2 - 4}\text{ nm}$

This says that $800\text{ nm} \leq \lambda \leq 1440\text{ nm}$, which is in the $\boxed{\text{infrared}}$.

40.37 (a) Lyman series: $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$ $n = 2, 3, 4, \ldots$

$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = (1.097 \times 10^7)\left(1 - \frac{1}{n^2}\right)$ $\boxed{n = 5}$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ $n = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n = \infty$ for ionization

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{9} - \frac{1}{n^2}\right)$ For $n = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$

Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$ $n = 5, 6, 7, \ldots$

$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{16} - \frac{1}{n^2}\right)$ $n = \infty$ for ionization $\lambda_{min} = 1458$ nm

Once again this wavelength $\boxed{\text{cannot be associated with the Brackett series}}$

40.38 (a) $\lambda_{\min} = \dfrac{hc}{E_{\max}}$

Lyman ($n_f = 1$): $\lambda_{\min} = \dfrac{hc}{|E_1|} = \dfrac{1240 \text{ eV}\cdot\text{nm}}{13.6 \text{ eV}} = \boxed{91.2 \text{ nm}}$ (Ultraviolet)

Balmer ($n_f = 2$): $\lambda_{\min} = \dfrac{hc}{|E_2|} = \dfrac{1240 \text{ eV}\cdot\text{nm}}{\left(\frac{1}{4}\right)13.6 \text{ eV}} = \boxed{365 \text{ nm}}$ (UV)

Paschen ($n_f = 3$): $\lambda_{\min} = \;.\;.\;. = 3^2(91.2 \text{ nm}) = \boxed{821 \text{ nm}}$ (Infrared)

Bracket ($n_f = 4$): $\lambda_{\min} = \;.\;.\;. = 4^2(91.2 \text{ nm}) = \boxed{1460 \text{ nm}}$ (IR)

(b) $E_{\max} = \dfrac{hc}{\lambda_{\min}}$

Lyman: $E_{\max} = \boxed{13.6 \text{ eV}}$ $\left(= |E_1|\right)$

Balmer: $E_{\max} = \boxed{3.40 \text{ eV}}$ $\left(= |E_2|\right)$

Paschen: $E_{\max} = \boxed{1.51 \text{ eV}}$ $\left(= |E_3|\right)$

Brackett: $E_{\max} = \boxed{0.850 \text{ eV}}$ $\left(= |E_4|\right)$

40.39 Liquid O_2 $\lambda_{\text{abs}} = 1269$ nm

$E = \dfrac{hc}{\lambda} = \dfrac{1.2398\times10^{-6}}{1.269\times10^{-6}} = 0.977 \text{ eV}$ for each molecule.

For two molecules, $\lambda = \dfrac{hc}{2E} = \boxed{634 \text{ nm, red}}$

By absorbing the red photons, the liquid O_2 appears to be blue.

***40.40** (a) $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$ where $r_1 = (1)^2 a_0 = 0.00529 \text{ nm} = 5.29\times10^{-11} \text{ m}$

$$v_1 = \sqrt{\frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{\left(9.11\times10^{-31} \text{ kg}\right)\left(5.29\times10^{-11} \text{ m}\right)}} = \boxed{2.19\times10^6 \text{ m/s}}$$

(b) $K_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}\left(9.11\times10^{-31} \text{ kg}\right)\left(2.19\times10^6 \text{ m/s}\right)^2 = 2.18\times10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$

(c) $U_1 = -\dfrac{k_e e^2}{r_1} = -\dfrac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{5.29\times10^{-11} \text{ m}} = -4.35\times10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$

40.41 (a) $r_2^2 = (0.0529\text{ nm})(2)^2 = \boxed{0.212\text{ nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{(9.11\times10^{-31}\text{ kg})(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{0.212\times10^{-9}\text{ m}}} = \boxed{9.95\times10^{-25}\text{ kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95\times10^{-25}\text{ kg}\cdot\text{m/s})(0.212\times10^{-9}\text{ m}) = \boxed{2.11\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \frac{1}{2}m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{(9.95\times10^{-25}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})} = 5.43\times10^{-19}\text{ J} = \boxed{3.40\text{ eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{0.212\times10^{-9}\text{ m}} = -1.09\times10^{-18}\text{ J} = \boxed{-6.80\text{ eV}}$

(f) $E_2 = K_2 + U_2 = 3.40\text{ eV} - 6.80\text{ eV} = \boxed{-3.40\text{ eV}}$

40.42 $$\Delta E = (13.6\text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(A) for $n_i = 2$ and $n_f = 5$ $\Delta E = 2.86$ eV (absorption)

(B) for $n_i = 5$ and $n_f = 3$ $\Delta E = -0.967$ eV (emission)

(C) for $n_i = 7$ and $n_f = 4$ $\Delta E = -0.572$ eV (emission)

(D) for $n_i = 4$ and $n_f = 7$ $\Delta E = 0.572$ eV (absorption)

(a) $E = \dfrac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{\text{B}}$.

(b) The atom gains most energy in transition $\boxed{\text{A}}$.

(c) The atom loses energy in transitions $\boxed{\text{B and C}}$.

40.43 (b) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = (1.097\times10^7\text{ m}^{-1})\left(\dfrac{1}{2^2} - \dfrac{1}{6^2}\right)$ so $\boxed{\lambda = 410\text{ nm}}$

(a) $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m / s})}{410\times10^{-9}\text{ m}} = 4.85\times10^{-19}\text{ J} = \boxed{3.03\text{ eV}}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8}{410\times10^{-9}} = \boxed{7.32\times10^{14}\text{ Hz}}$

***40.44** We use $E_n = \dfrac{-13.6\text{ eV}}{n^2}$

To ionize the atom when the electron is in the n^{th} level, it is necessary to add an amount of energy given by

$$E = -E_n = \frac{13.6\text{ eV}}{n^2}$$

(a) Thus, in the ground state where $n = 1$, we have $\boxed{E = 13.6\text{ eV}}$

(b) In the $n = 3$ level, $E = \dfrac{13.6\text{ eV}}{9} = \boxed{1.51\text{ eV}}$

***40.45** Starting with $\frac{1}{2}m_e v^2 = \dfrac{k_e e^2}{2r}$, we have $v^2 = \dfrac{k_e e^2}{m_e r}$

and using $r_n = \dfrac{n^2\hbar^2}{m_e k_e e^2}$

gives $v_n^2 = \dfrac{k_e e^2}{m_e \dfrac{n^2\hbar^2}{m_e k_e e^2}}$ or $v_n = \dfrac{k_e e^2}{n\hbar}$

***40.46** (a) The velocity of the moon in its orbit is $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(3.84\times10^8\text{ m})}{2.36\times10^6\text{ s}} = 1.02\times10^3\text{ m/s}$

So, $L = mvr = (7.36\times10^{22}\text{ kg})(1.02\times10^3\text{ m/s})(3.84\times10^8\text{ m}) = \boxed{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) We have $L = n\hbar$

or $n = \dfrac{L}{\hbar} = \dfrac{2.89\times10^{34}\text{ kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{2.74\times10^{68}}$

(c) We have $n\hbar = L = mvr = m(GM_e/r)^{1/2}r$,

so $r = \dfrac{\hbar^2}{m^2 GM_e}n^2 = Rn^2$ and $\dfrac{\Delta r}{r} = \dfrac{(n+1)^2R - n^2R}{n^2R} = \dfrac{2n+1}{n^2}$

which is approximately equal to $\dfrac{2}{n} = \boxed{7.30\times10^{-69}}$

40.47 The batch of excited atoms must make these six transitions to get back to state one: $2 \to 1$, and also $3 \to 2$ and $3 \to 1$, and also $4 \to 3$ and $4 \to 2$ and $4 \to 1$. Thus, the incoming light must have just enough energy to produce the $1 \to 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6\text{ eV}}{1^2} = -13.6\text{ eV} \quad \text{to} \quad E_f = -\frac{13.6\text{ eV}}{4^2} = -0.850\text{ eV},$$

so the incoming photons have wavelength

$$\lambda = \frac{hc}{E_f - E_i} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{-0.850\text{ eV} - (-13.6\text{ eV})}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 9.75\times10^{-8}\text{ m} = \boxed{97.5\text{ nm}}$$

40.48 Each atom gives up its kinetic energy in emitting a photon, so

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.216\times10^{-7}\text{ m})} = 1.63\times10^{-18}\text{ J}$$

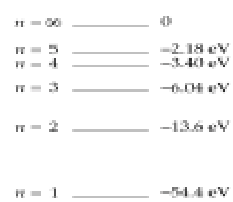
$$v = \boxed{4.42\times10^{4}\text{ m/s}}$$

40.49 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2}$$

Thus for He lium $(Z = 2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4\text{ eV}}{n^2} \qquad n = 1, 2, 3, \ldots}$$



(b) For He^{+}, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n = 1$ to the $n = \infty$ state is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\text{ eV})(2)^2}{(1)^2} = \boxed{54.4\text{ eV}}$$

40.50 $r = \dfrac{n^2\hbar^2}{Zm_ek_ee^2} = \dfrac{n^2}{Z}\left(\dfrac{\hbar^2}{m_ek_ee^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{(9.11\times10^{-31}\ \text{kg})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}\right] = \frac{5.29\times10^{-11}\ \text{m}}{Z}$$

(a) For He^{+}, $Z = 2$ $r = \dfrac{5.29\times10^{-11}\ \text{m}}{2} = 2.65\times10^{-11}\ \text{m} = \boxed{0.0265\ \text{nm}}$

(b) For Li^{2+}, $Z = 3$ $r = \dfrac{5.29\times10^{-11}\ \text{m}}{3} = 1.77\times10^{-11}\ \text{m} = \boxed{0.0177\ \text{nm}}$

(c) For Be^{3+}, $Z = 4$ $r = \dfrac{5.29\times10^{-11}\ \text{m}}{4} = 1.32\times10^{-11}\ \text{m} = \boxed{0.0132\ \text{nm}}$

40.51 Since $F = qvB = \dfrac{mv^2}{r}$ we have $qrB = mv$,

or $qr^2B = mvr = n\hbar$ so $\boxed{r_n = \sqrt{\dfrac{n\hbar}{qB}}}$

40.52 (a) The time for one complete orbit is: $T = \dfrac{2\pi r}{v}$

From Bohr's quantization postulate, $L = m_evr = n\hbar$, we see that $v = \dfrac{n\hbar}{m_er}$

Thus, the orbital period becomes:

$$T = \frac{2\pi m_e r^2}{n\hbar} = \frac{2\pi m_e(a_0n^2)^2}{n\hbar} = \frac{2\pi m_e a_0^2}{\hbar}n^3 \quad \text{or} \quad T = t_0n^3 \quad \text{where}$$

$$t_0 = \frac{2\pi m_e a_0^2}{\hbar} = \frac{2\pi(9.11\times10^{-31}\ \text{kg})(0.0529\times10^{-9}\ \text{m})^2}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})} = \boxed{1.52\times10^{-16}\ \text{s}}$$

(b) With $n = 2$, we have $T = 8t_0 = 8(1.52\times10^{-16}\ \text{s}) = 1.21\times10^{-15}\ \text{s}$

Thus, if the electrons stay in the $n = 2$ state for 10 μs, it will make

$$\frac{10.0\times10^{-6}\ \text{s}}{1.21\times10^{-15}\ \text{s/rev}} = \boxed{8.23\times10^{9}\ \text{revolutions}} \text{ of the nucleus}$$

(c) $\boxed{\text{Yes, for } 8.23\times10^{9} \text{ "electron years"}}$

***40.53** $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(1.67 \times 10^{-27}\ \text{kg})(1.00 \times 10^{6}\ \text{m/s})} = \boxed{3.97 \times 10^{-13}\ \text{m}}$

40.54 (a) $\dfrac{p^2}{2m} = (50.0)(1.60 \times 10^{-19}\ \text{J})$

$p = 3.81 \times 10^{-24}\ \text{kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{0.174\ \text{nm}}$

(b) $\dfrac{p^2}{2m} = (50.0 \times 10^{3})(1.60 \times 10^{-19}\ \text{J})$

$p = 1.20 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$

$\lambda = \dfrac{h}{p} = \boxed{5.49 \times 10^{-12}\ \text{m}}$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(mc^2 + K)^2 - m^2c^4\right]^{1/2}} = 5.37 \times 10^{-12}\ \text{m}$$

***40.55** (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \frac{1}{2}m_e v^2 = \dfrac{m_e^2 v^2}{2m_e} = \dfrac{p^2}{2m_e}$

so $p = \sqrt{2m_e K}$

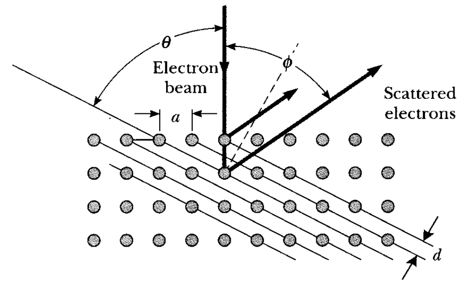
and $\lambda = \dfrac{h}{\sqrt{2m_e K}} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11 \times 10^{-31}\ \text{kg}\right)(3.00)\left(1.60 \times 10^{-19}\ \text{J}\right)}}$

$\lambda = 7.09 \times 10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$

(b) Photon: $\lambda = c/f$ and $E = hf$ so $f = E/h$ and

$$\lambda = \frac{hc}{E} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{(3.00)(1.60 \times 10^{-19}\ \text{J})} = 4.14 \times 10^{-7}\ \text{m} = \boxed{414\ \text{nm}}$$

40.56 From the Bragg condition (Eq. 38.13),



$$m\lambda = 2d\sin\theta = 2d\cos(\phi/2)$$

But, $d = a\sin(\phi/2)$ where a is the lattice spacing.
Thus, with $m = 1$,

$$\lambda = 2a\sin(\phi/2)\cos(\phi/2) = a\sin\phi$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\text{ kg})(54.0\times1.60\times10^{-19}\text{ J})}} = 1.67\times10^{-10}\text{ m}$$

Therefore, the lattice spacing is

$$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\text{ m}}{\sin 50.0^\circ} = 2.18\times10^{-10}\text{ m} = \boxed{0.218\text{ nm}}$$

***40.57** (a) $\lambda \sim 10^{-14}$ m or less.

$$p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\text{ J}\cdot\text{s}}{10^{-14}\text{ m}} = 10^{-19}\text{ kg}\cdot\text{m/s or more.}$$

The energy of the electron is

$$E = \sqrt{p^2c^2 + m_e^2c^4} \sim \left[(10^{-19})^2(3\times10^8)^2 + (9\times10^{-31})^2(3\times10^8)^4\right]^{1/2} \sim 10^{-11}\text{ J} \sim 10^8\text{ eV or more,}$$

so that $K = E - m_ec^2 \sim 10^8\text{ eV} - (0.5\times10^6\text{ eV})$ $\boxed{\sim 10^8\text{ eV}}$ or more.

(b) The electric potential energy of the electron would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10^{-19}\text{ C})(-e)}{10^{-14}\text{ m}} \sim -10^5\text{ eV}$$

With its kinetic energy much larger than its negative potential energy, **[the electron would immediately escape the nucleus]** .

Goal Solution

The nucleus of an atom is on the order of 10^{-14} m in diameter. For an electron to be confined to a nucleus, its de Broglie wavelength would have to be of this order of magnitude or smaller. (a) What would be the kinetic energy of an electron confined to this region? (b) On the basis of this result, would you expect to find an electron in a nucleus? Explain.

G: The de Broglie wavelength of a normal ground-state orbiting electron is on the order 10^{-10} m (the diameter of a hydrogen atom), so with a shorter wavelength, the electron would have more kinetic energy if confined inside the nucleus. If the kinetic energy is much greater than the potential energy from its attraction with the positive nucleus, then the electron will escape from its electrostatic potential well.

O: If we try to calculate the velocity of the electron from the de Broglie wavelength, we find that

$$v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(10^{-14}\ \text{m}\right)} = 7.27\times10^{10}\ \text{m / s}$$

which is not possible since it exceeds the speed of light. Therefore, we must use the relativistic energy expression to find the kinetic energy of this fast-moving electron.

A: (a) The relativistic kinetic energy of a particle is $K = E - mc^2$, where $E^2 = (pc)^2 + \left(mc^2\right)^2$, and the momentum is $p = h/\lambda$:

$$p = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 6.63\times10^{-20}\ \text{N}\cdot\text{s}$$

$$E = \sqrt{\left(1.99\times10^{-11}\ \text{J}\right)^2 + \left(8.19\times10^{-14}\ \text{J}\right)^2} = 1.99\times10^{-11}\ \text{J}$$

$$K = E - mc^2 = \frac{1.99\times10^{-11}\ \text{J} - 8.19\times10^{-14}\ \text{J}}{1.60\times10^{-19}\ \text{J / eV}} = 124\ \text{MeV} \sim 100\ \text{MeV}$$

(b) The electrostatic potential energy of the electron 10^{-14} m away from a positive proton is :

$$U = -k_e e^2/r = -\frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{10^{-14}\ \text{m}} = -2.30\times10^{-14}\ \text{J} \sim -0.1\ \text{MeV}$$

L: Since the kinetic energy is nearly 1000 times greater than the potential energy, the electron would immediately escape the proton's attraction and would not be confined to the nucleus.

It is also interesting to notice in the above calculations that the rest energy of the electron is negligible compared to the momentum contribution to the total energy.

40.58 (a) From $E = \gamma m_e c^2$ $\qquad \gamma = \dfrac{20.0 \times 10^3 \text{ MeV}}{0.511 \text{ MeV}} = \boxed{3.91 \times 10^4}$

(b) $p \approx \dfrac{E}{c}$ $\qquad$ (for $m_e c^2 << pc$)

$$p = \frac{(2.00 \times 10^4 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}}$$

(c) $$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}} = \boxed{6.22 \times 10^{-17} \text{ m}}$$

Since the size of a nucleus is on the order of 10^{-14} m, the 20-GeV electrons would be small enough to go through the nucleus.

40.59 (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $\qquad p = \dfrac{h}{\lambda}$, $\qquad$ and $\qquad mc = \dfrac{h}{\lambda_C}$

so $\qquad h^2f^2 = \dfrac{h^2c^2}{\lambda^2} + \dfrac{h^2c^2}{\lambda_C^2}$ $\qquad$ and $\qquad \left(\dfrac{f}{c}\right)^2 = \dfrac{1}{\lambda^2} + \dfrac{1}{\lambda_C^2}$ $\qquad$ (Eq. 1)

(b) For a photon $\qquad f/c = 1/\lambda$.

The third term $1/\lambda_C$ in Equation 1 for electrons and other massive particles shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$

40.60 (a) The wavelength of the student is $\lambda = h/p = h/mv$. If w is the width of the diffraction aperture, then we need $w \le 10.0\lambda = 10.0(h/mv)$, so that

$$v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(80.0 \text{ kg})(0.750 \text{ m})}\right) = \boxed{1.10 \times 10^{-34} \text{ m/s}}$$

(b) Using $t = \dfrac{d}{v}$ we get: $\qquad t \ge \dfrac{0.150 \text{ m}}{1.10 \times 10^{-34} \text{ m/s}} = \boxed{1.36 \times 10^{33} \text{ s}}$

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

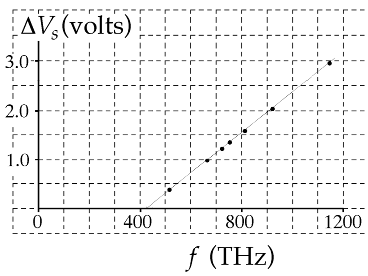
40.61 The de Broglie wavelength is: $\lambda = \dfrac{h}{\gamma m_e v}$

The Compton wavelength is: $\lambda_C = \dfrac{h}{m_e c}$

Therefore, we see that to have $\lambda = \lambda_C$, it is necessary that $\gamma v = c$.

This gives: $\dfrac{v}{\sqrt{1 - v^2 / c^2}} = c$, or $\left(\dfrac{v}{c}\right)^2 = 1 - \left(\dfrac{v}{c}\right)^2$, yielding $v = \boxed{\dfrac{c}{\sqrt{2}}}$.

40.62 $\Delta V_S = \left(\dfrac{h}{e}\right) f - \dfrac{\phi}{e}$



From two points on the graph $0 = \left(\dfrac{h}{e}\right)\left(4.1 \times 10^{14}\ \text{Hz}\right) - \dfrac{\phi}{e}$

and $3.3\ \text{V} = \left(\dfrac{h}{e}\right)\left(12 \times 10^{14}\ \text{Hz}\right) - \dfrac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\ \text{eV}}$

(b) $\dfrac{h}{e} = \boxed{4.2 \times 10^{-15}\ \text{V} \cdot \text{s}}$

(c) At the cutoff wavelength $\dfrac{hc}{\lambda_c} = \phi = \left(\dfrac{h}{e}\right)\dfrac{ec}{\lambda_c}$

$$\lambda_c = (4.2 \times 10^{-15}\ \text{V} \cdot \text{s})(1.6 \times 10^{-19}\ \text{C}) \frac{(3.0 \times 10^8\ \text{m/s})}{(1.7\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})} = \boxed{730\ \text{nm}}$$

40.63 $$K_{\max} = \frac{q^2 B^2 R^2}{2 m_e} = \frac{(1.60 \times 10^{-19}\ \text{C})^2 (2.00 \times 10^{-5}\ \text{T})^2 (0.200\ \text{m})^2}{2(9.11 \times 10^{-31}\ \text{kg})} = 2.25 \times 10^{-19}\ \text{J} = 1.40\ \text{eV} = hf - \phi$$

$$\phi = hf - K_{\max} = \frac{hc}{\lambda} - K_{\max} = \frac{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{450 \times 10^{-9}\ \text{m}} - 1.40\ \text{eV} = \boxed{1.36\ \text{eV}}$$

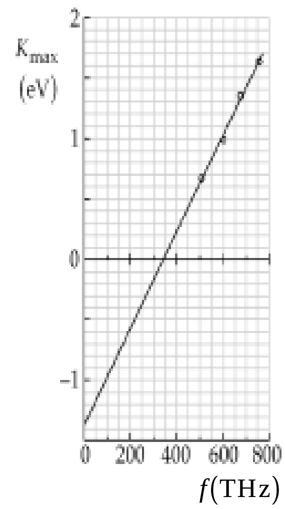
40.64 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

$$K_{\max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$

Thus, the work function is $\phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$

40.65 We want an Einstein plot of $K_{\max}$ versus f



λ, nm	f, 10^{14} Hz	$K_{\max}$, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63

(a) slope $= \frac{0.402 \text{ eV}}{10^{14} \text{ Hz}} \pm 8\%$

(b) $e(\Delta V_S) = hf - \phi$

$$h = (0.402)\frac{1.60 \times 10^{-19} \text{ J} \cdot \text{s}}{10^{14}} = \boxed{6.4 \times 10^{-34} \text{ J} \cdot \text{s} \pm 8\%}$$

(c) $K_{\max} = 0$ at $f \approx 344 \times 10^{12}$ Hz

$\phi = hf = 2.32 \times 10^{-19}$ J $= \boxed{1.4 \text{ eV}}$

40.66 $$\Delta\lambda = \frac{h}{m_p c}(1 - \cos\theta) = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m / s})}(0.234) = 3.09 \times 10^{-16} \text{ m}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m / s})}{(200 \text{ MeV})(1.60 \times 10^{-13} \text{ J / MeV})} = 6.20 \times 10^{-15} \text{ m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 6.51 \times 10^{-15} \text{ m}$$

(a) $E_\gamma = \frac{hc}{\lambda'} = \boxed{191 \text{ MeV}}$

(b) $K_p = \boxed{9.20 \text{ MeV}}$

40.67 M is the mass of the positron which equals m_e, the mass of the electron.

So $$\mu \equiv \text{reduced mass} = \frac{m_e M}{m_e + M} = \frac{m_e}{2}$$

$$r_{\text{pos}} = \frac{n^2 \hbar^2}{Z\mu k_e e^2} = \frac{n^2\hbar^2}{Z(m_e/2)k_e e^2} = \frac{2n^2\hbar^2}{Z m_e k_e e^2} \qquad \text{or} \qquad r_{\text{pos}} = 2r_{\text{Hyd}} = \boxed{\left(1.06\times 10^{-10}\text{ m}\right)n^2}$$

This is the separation of the two particles.

$$E_{\text{pos}} = -\frac{\mu k_e{}^2 e^4}{2\hbar^2}\frac{1}{n^2} = -\frac{m_e k_e{}^2 e^4}{4\hbar^2}\left(\frac{1}{n^2}\right);\quad n = 1, 2, 3, \ldots \qquad \text{or} \qquad E_{\text{pos}} = \frac{E_{\text{Hyd}}}{2} = \boxed{\frac{-6.80\text{ eV}}{n^2}}$$

Goal Solution

Positronium is a hydrogen-like atom consisting of a positron (a positively charged electron) and an electron revolving around each other. Using the Bohr model, find the allowed radii (relative to the center of mass of the two particles) and the allowed energies of the system.

G: Since we are told that positronium is like hydrogen, we might expect the allowed radii and energy levels to be about the same as for hydrogen: $r = a_0 n^2 = \left(5.29\times 10^{-11}\text{ m}\right)n^2$ and $E_n = (-13.6\text{ eV})/n^2$.

O: Similar to the textbook calculations for hydrogen, we can use the quantization of angular momentum of positronium to find the allowed radii and energy levels.

A: Let r represent the distance between the electron and the positron. The two move in a circle of radius $r/2$ around their center of mass with opposite velocities. The total angular momentum is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar, \quad \text{where} \quad n = 1, 2, 3, \ldots$$

For each particle, $\Sigma F = ma$ expands to $$\frac{k_e e^2}{r^2} = \frac{mv^2}{r/2}$$

We can eliminate $v = \frac{n\hbar}{mr}$ to find $$\frac{k_e e^2}{r} = \frac{2mn^2\hbar}{m^2 r^2}$$

So the separation distances are $$r = \frac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \left(1.06\times 10^{-10}\text{ m}\right)n^2$$

The orbital radii are $r/2 = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 - \frac{k_e e^2}{r}$$

Since $mv^2 = \frac{k_e e^2}{2r}$, $$E = \frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} = \frac{-k_e e^2}{4a_0 n^2} = -\frac{6.80\text{ eV}}{n^2}$$

L: It appears that the allowed radii for positronium are twice as large as for hydrogen, while the energy levels are half as big. One way to explain this is that in a hydrogen atom, the proton is much more massive than the electron, so the proton remains nearly stationary with essentially no kinetic energy. However, in positronium, the positron and electron have the same mass and therefore both have kinetic energy that separates them from each other and reduces their total energy compared with hydrogen.

40.68 Isolate the terms involving ϕ in Equations 40.12 and 40.13. Square and add to eliminate ϕ.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 v^2$$

Solve for $\frac{v^2}{c^2}=\frac{b}{\left(b+c^2\right)}$:

$$b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

Substitute into Eq. 40.11:

$$1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\sqrt{1-\frac{b}{b+c^2}}$$

Square each side:

$$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

From this we get Eq. 40.10:

$$\lambda'-\lambda_0=\left(h/m_e c\right)\left[1-\cos\theta\right]$$

40.69 $hf=\Delta E=\frac{4\pi^2 m_e k_e^2 e^4}{2h^2}\left(\frac{1}{(n-1)^2}-\frac{1}{n^2}\right)$ so $f=\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\frac{2n-1}{(n-1)^2 n^2}\right)$

As n approaches infinity, we have f approaching $\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3}$

The classical frequency is $f=\frac{v}{2\pi r}=\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e}}\frac{1}{r^{3/2}}$ where $r=\frac{n^2h^2}{4\pi m_e k_e e^2}$

Using this equation to eliminate r from the expression for f, $f=\frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3}$

40.70 Show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy: $\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+K_e=m_e c^2(\gamma-1)$ if $\frac{hc}{\lambda'}=0$ (1)

Momentum: $\frac{h}{\lambda_0}=\frac{h}{\lambda'}+\gamma m_e v=\gamma m_e v$ if $\lambda'=\infty$ (2)

From (1), $\gamma=\frac{h}{\lambda_0 m_e c}+1$ (3)

$$v=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \qquad (4)$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h(h+2\lambda_0 m_e c)}{(h+\lambda_0 m_e c)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

40.71 Begin with momentum expressions: $p=\frac{h}{\lambda}$, and $p=\gamma mv=\gamma mc\left(\frac{v}{c}\right)$.

Equating these expressions, $\gamma\left(\frac{v}{c}\right)=\left(\frac{h}{mc}\right)\frac{1}{\lambda}=\frac{\lambda_C}{\lambda}$

Thus, $\frac{(v/c)^2}{1-(v/c)^2}=\left(\frac{\lambda_C}{\lambda}\right)^2$

or $\left(\frac{v}{c}\right)^2=\left(\frac{\lambda_C}{\lambda}\right)^2-\left(\frac{\lambda_C}{\lambda}\right)^2\left(\frac{v}{c}\right)^2=\frac{(\lambda_C/\lambda)^2}{1+(\lambda_C/\lambda)^2}=\frac{1}{(\lambda/\lambda_C)^2+1}$

giving $\boxed{v=\frac{c}{\sqrt{1+(\lambda\ /\ \lambda_C)^2}}}$

40.72 (a) The energy of the ground state is: $E_1=-\frac{hc}{\lambda_{\text{series limit}}}=-\frac{1240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}}=\boxed{-8.16\text{ eV}}$

From the wavelength of the L_α line, we see: $E_2-E_1=\frac{hc}{\lambda}=\frac{1240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}}=6.12\text{ eV}$

$$E_2=E_1+6.12\text{ eV}=\boxed{-2.04\text{ eV}}$$

Using the wavelength of the L_β line gives: $E_3-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}}=7.26\text{ eV}$

so $E_3=\boxed{-0.902\text{ eV}}$

Next, using the L_γ line gives: $E_4-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}}=7.65\text{ eV}$

and $E_4=\boxed{-0.508\text{ eV}}$

From the L_δ line, $E_5-E_1=\frac{1240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}}=7.83\text{ eV}$

so $E_5=\boxed{-0.325\text{ eV}}$

(b) For the Balmer series, $\frac{hc}{\lambda}=E_i-E_2$, or $\lambda=\frac{1240\text{ nm}\cdot\text{eV}}{E_i-E_2}$

For the α line, $E_i=E_3$ and so $\lambda_\alpha=\frac{1240\text{ nm}\cdot\text{eV}}{(-0.902\text{ eV})-(-2.04\text{ eV})}=\boxed{1090\text{ nm}}$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811\text{ nm}}$, $\boxed{724\text{ nm}}$, and $\boxed{609\text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$0.600(202.6\text{ nm}) = \boxed{122\text{ nm}}$, $0.600(170.9\text{ nm}) = \boxed{103\text{ nm}}$, $0.600(162.1\text{ nm}) = \boxed{97.3\text{ nm}}$, $0.600(158.3\text{ nm}) = \boxed{95.0\text{ nm}}$, and $0.600(152.0\text{ nm}) = \boxed{91.2\text{ nm}}$.

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{1-(v/c)}{1+(v/c)}} = 0.600 \qquad \text{yielding} \qquad \boxed{v = 0.471c}$$

40.73 (a) Starting with Planck's law,

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}$$

the total power radiated per unit area

$$\int_0^\infty I(\lambda, T)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}\,d\lambda.$$

Change variables by letting

$$x = \frac{hc}{\lambda k_B T}$$

and

$$dx = -\frac{hc\,d\lambda}{k_B T\lambda^2}$$

Note that as λ varies from $0 \to \infty$, x varies from $\infty \to 0$.

Then

$$\int_0^\infty I(\lambda, T)\,d\lambda = -\frac{2\pi k_B^4 T^4}{h^3c^2}\int_\infty^0 \frac{x^3}{(e^x - 1)}\,dx = \frac{2\pi k_B^4 T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$$

Therefore,

$$\boxed{\int_0^\infty I(\lambda, T)\,d\lambda = \left(\frac{2\pi^5 k_B^4}{15h^3c^2}\right)T^4 = \sigma T^4}$$

(b) From part (a),

$$\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2} = \frac{2\pi^5\left(1.38\times10^{-23}\text{ J/K}\right)^4}{15\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3\left(3.00\times10^8\text{ m/s}\right)^2}$$

$$\sigma = \boxed{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}$$

***40.74** Planck's law states $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]} = 2\pi hc^2 \lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-1}$

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda} = 2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_B T} - 1\right]^{-1} - \lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-2} e^{hc/\lambda k_B T}\left(-\frac{hc}{\lambda^2 k_B T}\right)\right\} = 0$$

$$\frac{dI}{d\lambda} = \frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_B T} - 1\right]}\left\{-5 + \frac{hc}{\lambda k_B T}\frac{e^{hc/\lambda k_B T}}{\left[e^{hc/\lambda k_B T} - 1\right]}\right\} = 0$$

Letting $x = \dfrac{hc}{\lambda k_B T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x - 1} = 5.$

We zero in on the solution to this transcendental equation by iterations as shown in the table below. The solution is found to be

x	$xe^x/(e^x - 1)$
4.00000	4.0746294
4.50000	4.5505521
5.00000	5.0339183
4.90000	4.9367620
4.95000	4.9853130
4.97500	5.0096090
4.96300	4.9979452
4.96900	5.0037767
4.96600	5.0008609
4.96450	4.9994030
4.96550	5.0003749
4.96500	4.9998890
4.96525	5.0001320
4.96513	5.0000153
4.96507	4.9999570
4.96510	4.9999862
4.965115	5.0000008

$$x = \frac{hc}{\lambda_{\max} k_B T} = 4.965115 \qquad \text{and} \qquad \lambda_{\max} T = \frac{hc}{4.965115 k_B}$$

Thus, $\lambda_{\max} T = \dfrac{\left(6.626075 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997925 \times 10^{8}\ \text{m/s}\right)}{4.965115\left(1.380658 \times 10^{-23}\ \text{J/K}\right)} = \boxed{2.897755 \times 10^{-3}\ \text{m}\cdot\text{K}}$

This result is very close to Wien's experimental value of $\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m}\cdot\text{K}$

for this constant.

40.75

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1}$$

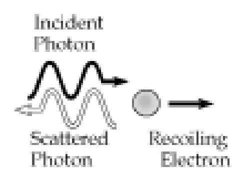
$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

40.76

$$r_1 = \frac{(1)^2\hbar^2}{Z\mu k_e e^2} = \frac{\hbar^2}{(82)(207 m_e)k_e e^2} = \frac{a_0}{(82)(207)} = \frac{0.0529 \text{ nm}}{(82)(207)} = \boxed{3.12 \text{ fm}}$$

$$E_1 = \frac{-13.6 \text{ eV}}{(1)^2}\left(\frac{207}{1}\right)\left(\frac{82}{1}\right)^2 = \boxed{-18.9 \text{ MeV}}$$

40.77 This is a case of Compton scattering with a scattering angle of 180°.



$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}\left(1 - \cos 180°\right) = \frac{2h}{m_e c}$$

$$E_0 = \frac{hc}{\lambda_0}, \quad \text{so} \quad \lambda_0 = \frac{hc}{E_0} \quad \text{and} \quad \lambda' = \lambda_0 + \Delta\lambda = \frac{hc}{E_0} + \frac{2h}{m_e c} = \frac{hc}{E_0}\left(1 + \frac{2E_0}{m_e c^2}\right)$$

The kinetic energy of the recoiling electron is then

$$K = E_0 - \frac{hc}{\lambda'} = E_0 - \frac{E_0}{\left(1 + 2E_0/m_e c^2\right)} = E_0\left(\frac{1 + 2E_0/m_e c^2 - 1}{1 + 2E_0/m_e c^2}\right) = \frac{2E_0^2/m_e c^2}{1 + 2E_0/m_e c^2}$$

Defining $a \equiv E_0/m_e c^2$, the kinetic energy can be written as

$$K = \frac{2E_0 a}{1+2a} = \frac{2(hf)a}{1+2a} = \boxed{2hfa(1+2a)^{-1}}$$

where f is the frequency of the incident photon.

40.78 (a) Planck's radiation law predicts maximum intensity at a wavelength $\lambda_{\max}$ we find from

$$\frac{dI}{d\lambda}=0=\frac{d}{d\lambda}\left\{2\pi hc^2\lambda^{-5}\left[e^{(hc/\lambda k_B T)}-1\right]^{-1}\right\}$$

$$0=2\pi hc^2\lambda^{-5}(-1)\left[e^{(hc/\lambda k_B T)}-1\right]^{-2}e^{(hc/\lambda k_B T)}\left(-hc/\lambda^2 k_B T\right)+2\pi hc^2(-5)\lambda^{-6}\left[e^{(hc/\lambda k_B T)}-1\right]^{-1}$$

or
$$\frac{-hc\,e^{(hc/\lambda k_B T)}}{\lambda^7 k_B T\left[e^{(hc/\lambda k_B T)}-1\right]^2}+\frac{5}{\lambda^6\left[e^{(hc/\lambda k_B T)}-1\right]}=0$$

which reduces to
$$5\left(\lambda k_B T/hc\right)\left[e^{(hc/\lambda k_B T)}-1\right]=e^{(hc/\lambda k_B T)}$$

Define $x=hc/\lambda k_B T$. Then we require $5e^x-5=xe^x$.

Numerical solution of this transcendental equation gives $x=4.965$ to four digits. So $\lambda_{\max}=hc/4.965k_B T$, in agreement with Wien's law.

The intensity radiated over all wavelengths is
$$\int_0^\infty I(\lambda,T)d\lambda=A+B=\int_0^\infty\frac{2\pi hc^2\,d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)}-1\right]}$$

Again, define $x=hc/\lambda k_B T$ so $\lambda=hc/xk_B T$ and $d\lambda=-\left(hc/x^2k_B T\right)dx$

Then,
$$A+B=\int_{x=\infty}^{0}\frac{-2\pi hc^2x^5k_B^5T^5\,hc\,dx}{h^5c^5x^2\,k_B T\left(e^x-1\right)}=\frac{2\pi k_B^4T^4}{h^3c^2}\int_0^\infty\frac{x^3dx}{\left(e^x-1\right)}$$

The integral is tabulated as $\pi^4/15$, so (in agreement with Stefan's law)
$$A+B=\frac{2\pi^5k_B^4T^4}{15h^3c^2}$$

The intensity radiated over wavelengths shorter than $\lambda_{\max}$ is

$$\int_0^{\lambda_{\max}}I(\lambda,T)d\lambda=A=\int_0^{\lambda_{\max}}\frac{2\pi hc^2\,d\lambda}{\lambda^5\left[e^{(hc/\lambda k_B T)}-1\right]}$$

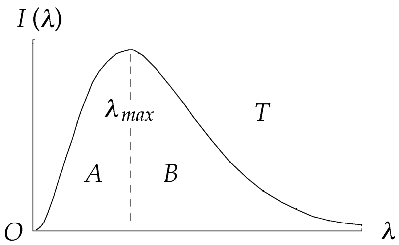
With $x=hc/\lambda k_B T$, this similarly becomes
$$A=\frac{2\pi k_B^4T^4}{h^3c^2}\int_{4.965}^\infty\frac{x^3\,dx}{e^x-1}$$

So the fraction of power or of intensity radiated at wavelengths shorter than $\lambda_{\max}$ is

$$\frac{A}{A+B}=\frac{\dfrac{2\pi k_B^4T^4}{h^3c^2}\left(\dfrac{\pi^4}{15}-\displaystyle\int_0^{4.965}\frac{x^3\,dx}{e^x-1}\right)}{\dfrac{2\pi^5k_B^4T^4}{15h^3c^2}}=\boxed{1-\frac{15}{\pi^4}\int_0^{4.965}\frac{x^3\,dx}{e^x-1}}$$

(b) Here are some sample values of the integrand, along with a sketch of the curve:

x	$x^3\left(e^x-1\right)^{-1}$
0.000	0.00
0.100	9.51×10^{-3}
0.200	3.61×10^{-2}
1.00	0.582
2.00	1.25
3.00	1.42
4.00	1.19
4.90	0.883
4.965	0.860



Approximating the integral by trapezoids gives $\dfrac{A}{A+B}\approx1-\dfrac{15}{\pi^4}(4.870)=\boxed{0.2501}$

40.79 $\lambda_C=\dfrac{h}{m_ec}$ and $\lambda=\dfrac{h}{p}$: $\dfrac{\lambda_C}{\lambda}=\dfrac{h/m_ec}{h/p}=\dfrac{p}{m_ec}$;

$E^2=c^2p^2+(m_ec^2)^2$: $p=\sqrt{\dfrac{E^2}{c^2}-(m_ec)^2}$

$$\frac{\lambda_C}{\lambda}=\frac{1}{m_ec}\sqrt{\frac{E^2}{c^2}-(m_ec)^2}=\sqrt{\frac{1}{(m_ec)^2}\left[\frac{E^2}{c^2}-(m_ec)^2\right]}=\sqrt{\left(\frac{E}{m_ec^2}\right)^2-1}$$

40.80 $p=mv=\sqrt{2mE}=\sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)(0.0400\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$

$\lambda=\dfrac{h}{mv}=1.43\times10^{-10}\text{ m}=\boxed{0.143\text{ nm}}$

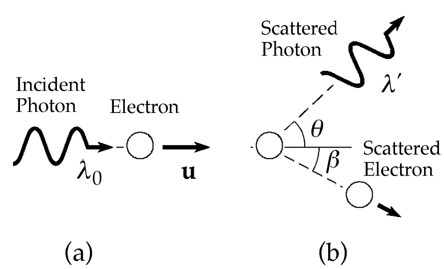
This is of the same order of magnitude as the spacing between atoms in a crystal so diffraction should appear.

40.81 Let u' represent the final speed of the electron and let $\gamma'=\left(1-u'^2/c^2\right)^{-1/2}$. We must eliminate β and u' from the three conservation equations:

$$\frac{hc}{\lambda_0}+\gamma m_ec^2=\frac{hc}{\lambda'}+\gamma' m_ec^2 \qquad [1]$$

$$\frac{h}{\lambda_0}+\gamma m_eu-\frac{h}{\lambda'}\cos\theta=\gamma' m_eu'\cos\beta \qquad [2]$$

$$\frac{h}{\lambda'}\sin\theta=\gamma' m_eu'\sin\beta \qquad [3]$$



Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2}+\gamma^2 m_e^2 u^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}=\gamma'^2 m_e^2 u'^2$$

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2 u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}=\frac{m_e^2 u'^2}{1-u'^2/c^2}$$

Call the left-hand side b. Then $b-\frac{bu'^2}{c^2}=m_e^2u'^2$ and $u'^2=\frac{b}{m_e^2+b/c^2}=\frac{c^2b}{m_e^2c^2+b}$

Now square Equation [1] and substitute to eliminate γ':

$$\frac{h^2}{\lambda^2}+\gamma^2 m_e^2 c^2+\frac{h^2}{\lambda'^2}+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h^2}{\lambda_0\lambda'}-\frac{2h\gamma m_e c}{\lambda'}=\frac{m_e^2c^2}{1-u'^2/c^2}=m_e^2c^2+b$$

So we have

$$\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2 c^2+\frac{2h\gamma m_e c}{\lambda_0}-\frac{2h\gamma m_e c}{\lambda'}-\frac{2h^2}{\lambda_0\lambda'}$$
$$=m_ec^2+\frac{h^2}{\lambda_0^2}+\frac{h^2}{\lambda'^2}+\gamma^2 m_e^2 u^2+\frac{2h\gamma m_e u}{\lambda_0}-\frac{2h\gamma m_e u\cos\theta}{\lambda'}-\frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\lambda_0\lambda'/m_e^2c^2$

$$\lambda_0\lambda'\gamma^2+\frac{2h\lambda'\gamma}{m_ec}-\frac{2h\lambda_0\gamma}{m_ec}-\frac{2h^2}{m_e^2c^2}=\lambda_0\lambda'+\frac{\lambda_0\lambda'\gamma^2u^2}{c^2}+\frac{2h\lambda' u\gamma}{m_ec^2}-\frac{2h\gamma\lambda_0 u\cos\theta}{m_ec^2}-\frac{2h^2\cos\theta}{m_e^2c^2}$$

$$\lambda_0\lambda'\left(\gamma^2-1-\frac{\gamma^2u^2}{c^2}\right)+\frac{2h\gamma\lambda'}{m_ec}\left(1-\frac{u}{c}\right)=\frac{2h\gamma\lambda_0}{m_ec}\left(1-\frac{u\cos\theta}{c}\right)+\frac{2h^2}{m_e^2c^2}(1-\cos\theta)$$

The first term is zero. Then $\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h\gamma^{-1}}{m_ec}\left(\frac{1}{1-u/c}\right)(1-\cos\theta)$

Since $\gamma^{-1}=\sqrt{1-(u/c)^2}=\sqrt{(1-u/c)(1+u/c)}$

this result may be written as $\boxed{\lambda'=\lambda_0\left(\frac{1-(u\cos\theta)/c}{1-u/c}\right)+\frac{h}{m_ec}\sqrt{\frac{1+u/c}{1-u/c}}(1-\cos\theta)}$

Chapter 41 Solutions

41.1 (a) $$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(0.400\ \text{m/s})} = \boxed{9.92\times10^{-7}\ \text{m}}$$

(b) For destructive interference in a multiple-slit experiment,

$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$$

with $m = 0$ for the first minimum. Then,

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.0284°$$

$$\frac{y}{L} = \tan\theta \qquad \text{so} \qquad y = L\tan\theta = (10.0\ \text{m})(\tan\ 0.0284°) = \boxed{4.96\ \text{mm}}$$

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

41.2 Consider the first bright band away from the center: $d\sin\theta = m\lambda$

$$(6.00\times10^{-8}\ \text{m})\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = 1\lambda = 1.20\times10^{-10}\ \text{m}$$

$$\lambda = \frac{h}{m_e v} \qquad \text{so} \qquad m_e v = \frac{h}{\lambda} \qquad \text{and}$$

$$K = \tfrac{1}{2}m_e v^2 = \frac{m_e^{\,2}v^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e(\Delta V)$$

$$\Delta V = \frac{h^2}{2em_e\lambda^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})(9.11\times10^{-31}\ \text{kg})(1.20\times10^{-10}\ \text{m})^2} = \boxed{105\ \text{V}}$$

41.3 (a) The wavelength of a non-relativistic particle of mass m is given by $\lambda = h/p = h/\sqrt{2mK}$ where the kinetic energy K is in joules. If the neutron kinetic energy K_n is given in electron volts, its kinetic energy in joules is $K = (1.60\times10^{-19}\ \text{J/eV})K_n$ and the equation for the wavelength becomes

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(1.67\times10^{-27}\ \text{kg})(1.60\times10^{-19}\ \text{J/eV})K_n}} = \boxed{\frac{2.87\times10^{-11}}{\sqrt{K_n}}\ \text{m}}$$

where K_n is expressed in electron volts.

(b) If $K_n = 1.00\ \text{keV} = 1000\ \text{eV}$, then

41.4 $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}, \quad \text{so} \quad K = \frac{h^2}{2m\lambda^2}$$

If the particles are electrons and $\lambda \sim 0.1 \text{ nm} = 10^{-10} \text{ m}$, the kinetic energy in electron volts is

$$K = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(10^{-10} \text{ m}\right)^2}\left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}\right) = \boxed{\sim 10^2 \text{ eV}}$$

41.5 $$\lambda = \frac{h}{p} \qquad p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.00 \times 10^{-11} \text{ m}} = 6.63 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

(a) electrons: $$K_e = \frac{p^2}{2m_e} = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(9.11 \times 10^{-31}\right)} \text{ J} = \boxed{15.1 \text{ keV}}$$

The relativistic answer is more precisely correct:

$$K_e = \left(p^2c^2 + m_e{}^2c^4\right)^{1/2} - m_ec^2 = 14/.9 \text{ keV}$$

(b) photons: $$E_\gamma = pc = \left(6.63 \times 10^{-23}\right)\left(3.00 \times 10^8\right) = \boxed{124 \text{ keV}}$$

41.6 The theoretical limit of the electron microscope is the wavelength of the electrons. If $K_e = 40.0 \text{ keV}$, then $E = K_e + m_ec^2 = 551 \text{ keV}$ and

$$p = \frac{1}{c}\sqrt{E^2 - m_e{}^2c^4} = \frac{\sqrt{(551 \text{ keV})^2 - (511 \text{ keV})^2}}{3.00 \times 10^8 \text{ m/s}}\left(\frac{1.60 \times 10^{-16} \text{ J}}{1.00 \text{ keV}}\right) = 1.10 \times 10^{-22} \text{ kg} \cdot \text{m/s}$$

The electron wavelength, and hence the theoretical limit of the microscope, is then

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.10 \times 10^{-22} \text{ kg} \cdot \text{m/s}} = 6.03 \times 10^{-12} \text{ m} = \boxed{6.03 \text{ pm}}$$

41.7 $$E = K + m_ec^2 = 1.00 \text{ MeV} + 0.511 \text{ MeV} = 1.51 \text{ MeV}$$

$$p^2c^2 = \sqrt{E^2 - m_e{}^2c^4} = \sqrt{(1.51 \text{ MeV})^2 - (0.511 \text{ MeV})^2} \quad \text{so}$$
$$p = 1.42 \text{ MeV}/c$$

$$\lambda = \frac{h}{p} = \frac{hc}{1.42 \text{ MeV}} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(1.42 \times 10^6\right)\left(1.60 \times 10^{-19} \text{ J}\right)} = 8.74 \times 10^{-13} \text{ m}$$

Suppose the array is like a flat diffraction grating with openings 0.250 nm apart:

$$d \sin\theta = m\lambda$$

41.8 (a) $\Delta p \Delta x = m\Delta v \Delta x \geq \hbar / 2$ so

$$\Delta v \geq \frac{h}{4\pi m \Delta x} = \frac{2\pi \text{ J}\cdot\text{s}}{4\pi (2.00 \text{ kg})(1.00 \text{ m})} = \boxed{0.250 \text{ m/s}}$$

(b) The duck might move by $(0.25 \text{ m/s})(5 \text{ s}) = 1.25 \text{ m}$. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00 \text{ m} + 1.25 \text{ m} = \boxed{2.25 \text{ m}}$

41.9 For the electron,

$$\Delta p = m_e \Delta v = \left(9.11\times10^{-31} \text{ kg}\right)(500 \text{ m/s})\left(1.00\times10^{-4}\right) = 4.56\times10^{-32} \text{ kg}\cdot\text{m/s}$$

$$\Delta x = \frac{h}{4\pi \Delta p} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{4\pi\left(4.56\times10^{-32} \text{ kg}\cdot\text{m/s}\right)} = \boxed{1.16 \text{ mm}}$$

For the bullet, $\Delta p = m\Delta v = (0.0200 \text{ kg})(500 \text{ m/s})\left(1.00\times10^{-4}\right) = 1.00\times10^{-3} \text{ kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi \Delta p} = \boxed{5.28\times10^{-32} \text{ m}}$$

Goal Solution

An electron ($m_e = 9.11\times10^{-31}$ kg) and a bullet ($m = 0.0200$ kg) each have a speed of 500 m/s, accurate to within 0.0100%. Within what limits could we determine the position of the objects?

G: It seems reasonable that a tiny particle like an electron could be located within a more narrow region than a bigger object like a bullet, but we often find that the realm of the very small does not obey common sense.

O: Heisenberg's uncertainty principle can be used to find the uncertainty in position from the uncertainty in the momentum.

A: The uncertainty principle states: $\Delta x \Delta p_x \geq \hbar / 2$ where $\Delta p_x = m\Delta v$ and $h = \hbar / 2\pi$.

Both the electron and bullet have a velocity uncertainty,

$$\Delta v = (0.000100)(500 \text{ m/s}) = 0.0500 \text{ m/s}$$

For the electron, the minimum uncertainty in position is

$$\Delta x = \frac{h}{4\pi m \Delta v} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31} \text{ kg}\right)(0.0500 \text{ m/s})} = 1.16 \text{ mm}$$

For the bullet,

$$\Delta x = \frac{h}{4\pi m \Delta v} = \frac{6.63\times10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.0200 \text{ kg})(0.0500 \text{ m/s})} = 5.28\times10^{-32} \text{ m}$$

41.10 $\dfrac{\Delta y}{x} = \dfrac{\Delta p_y}{p_x}$ and $d\Delta p_y \ge h/4\pi$ Eliminate Δp_y and solve for x.

$$x = 4\pi p_x(\Delta y)\frac{d}{h} = 4\pi\left(1.00\times10^{-3}\text{ kg}\right)(100\text{ m/s})\left(1.00\times10^{-2}\text{ m}\right)\frac{\left(2.00\times10^{-3}\text{ m}\right)}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} =$$

$\boxed{3.79\times10^{28}\text{ m}}$

This is 190 times greater than the diameter of the Universe!

41.11 $\Delta p\Delta x \ge \dfrac{\hbar}{2}$ so $\Delta p = m_e\Delta v \ge \dfrac{h}{4\pi\,\Delta x}$

$$\Delta v \ge \frac{h}{4\pi\, m_e\Delta x} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\text{ kg}\right)\left(5.00\times10^{-11}\text{ m}\right)} = \boxed{1.16\times10^{6}\text{ m/s}}$$

41.12 With $\Delta x = 2\times10^{-15}$ m, the uncertainty principle requires $\Delta p_x \ge \dfrac{\hbar}{2\Delta x} = 2.6\times10^{-20}\text{ kg}\cdot\text{m/s}$

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} = 3\times10^{-20}\text{ kg}\cdot\text{m/s}$.

For an electron, the non-relativistic approximation $p = m_e v$ would predict $v = 3\times10^{10}$ m/s, while v cannot be greater than c.

Thus, a better solution would be $E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} = 56\text{ MeV} = \gamma\, m_e c^2$

$$\gamma \approx 110 = \frac{1}{\sqrt{1 - v^2/c^2}}$$ so

$$v \approx 0.99996c$$

For a proton, $v = p/m$ gives $v = 1.8\times10^{7}$ m / s, less than one-tenth the speed of light.

41.13 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \hbar/2\Delta x_i$. It falls to the floor in time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{2H/g}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i},$$ where

$$A = \frac{\hbar}{\ldots}\sqrt{\frac{2H}{\ldots}}$$

so $\Delta x_i = \sqrt{A}$, and the minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\left(\frac{2\hbar}{m}\right)^{1/2} \left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.0546\times10^{-34}\text{ J}\cdot\text{s}\right)}{5.00\times10^{-4}\text{ kg}}\right]^{1/2} \left[\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}\right]^{1/4} = \boxed{5.19\times10^{-16}\text{ m}}$$

41.14 Probability $P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi(x^2+a^2)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\frac{x}{a}\Big|_{-a}^{a}$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{1/2}$$

41.15 (a) $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin\left(5.00\times10^{10}x\right)$

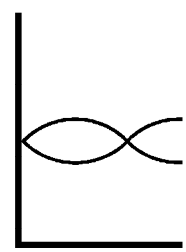
so $\frac{2\pi}{\lambda} = 5.00\times10^{10}\text{ m}^{-1}$ $\lambda = \frac{2\pi}{\left(5.00\times10^{10}\right)} = \boxed{1.26\times10^{-10}\text{ m}}$

(b) $p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{1.26\times10^{-10}\text{ m}} = \boxed{5.27\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(c) $m = 9.11\times10^{-31}$ kg

$$K = \frac{p^2}{2m} = \frac{\left(5.27\times10^{-24}\text{ kg}\cdot\text{m/s}\right)^2}{\left(2\times9.11\times10^{-31}\text{ kg}\right)} = 1.52\times10^{-17}\text{ J} = \frac{1.52\times10^{-17}\text{ J}}{1.602\times10^{-19}\text{ J/eV}} = \boxed{95.5\text{ eV}}$$

41.16 For an electron to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.



$\frac{n\lambda}{2} = 1.00\times10^{-9}\text{ m}$ so

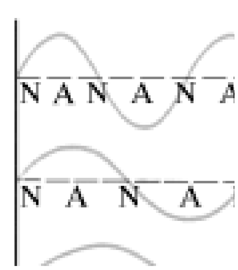
$$\lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

(a) Since $K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{\left(2\times10^{-9}\right)^2} = \left(0.377n^2\right)\text{ eV}$

For $K \approx 6\text{ eV}$, $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\text{ eV}}$

41.17 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:



Since $d_{N\text{ to N}} = \frac{\lambda}{2}$ and

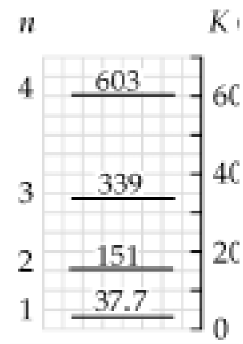
h

Next,

$$K=\frac{p^2}{2m_e}=\frac{h^2}{8m_e d}=\frac{1}{d^2}\left[\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\ \text{kg}\right)}\right]$$

Evaluating, $$K=\frac{6.02\times10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$$

$$K=\frac{3.77\times10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$$



In state 1, $d=1.00\times10^{-10}\ \text{m}$
$K_1=37.7\ \text{eV}$

In state 2, $d=5.00\times10^{-11}\ \text{m}$
$K_2=151\ \text{eV}$

In state 3, $d=3.33\times10^{-11}\ \text{m}$
$K_3=339\ \text{eV}$

In state 4, $d=2.50\times10^{-11}\ \text{m}$
$K_4=603\ \text{eV}$

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E=151\ \text{eV}-37.7\ \text{eV}=113\ \text{eV}=hf=\frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda=\frac{hc}{E}=\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}=11.0\ \text{nm}$$

The wavelengths of the other spectral lines we find similarly:

Transition						
$E(\text{eV})$						
$\lambda(\text{nm})$						

41.18 $E_1 = 2.00\text{ eV} = 3.20\times10^{-19}\text{ J}$

For the ground-state, $E_1 = \dfrac{h^2}{8m_eL^2}$

(a) $L = \dfrac{h}{\sqrt{8m_eE_1}} = 4.34\times10^{-10}\text{ m} = \boxed{0.434\text{ nm}}$

(b) $\Delta E = E_2 - E_1 = 4\left(\dfrac{h^2}{8m_eL^2}\right) - \left(\dfrac{h^2}{8m_eL^2}\right) = \boxed{6.00\text{ eV}}$

41.19 $\Delta E = \dfrac{hc}{\lambda} = \left(\dfrac{h^2}{8m_eL^2}\right)\left[2^2 - 1^2\right] = \dfrac{3h^2}{8m_eL^2}$

$L = \sqrt{\dfrac{3h\lambda}{8m_ec}} = 7.93\times10^{-10}\text{ m} = \boxed{0.793\text{ nm}}$

41.20 $\Delta E = \dfrac{hc}{\lambda} = \left(\dfrac{h^2}{8m_eL^2}\right)\left[2^2 - 1^2\right] = \dfrac{3h^2}{8m_eL^2}$

so $L = \boxed{\sqrt{\dfrac{3h\lambda}{8m_ec}}}$

41.21 $E_n = \dfrac{n^2h^2}{8mL^2}$

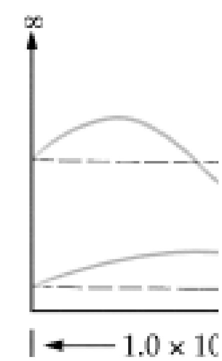
so $\Delta E = E_2 - E_1 = \dfrac{3h^2}{8mL^2} = \dfrac{3(hc)^2}{8mc^2L^2}$

and $\Delta E = hf = \dfrac{hc}{\lambda}$

Hence, $\lambda = \dfrac{8mc^2L^2}{3hc} = \dfrac{8\left(938\times10^6\text{ eV}\right)\left(1.00\times10^{-5}\text{ nm}\right)^2}{3(1240\text{ eV}\cdot\text{nm})}$

$\lambda = \boxed{2.02\times10^{-4}\text{ nm (gamma ray)}}$

$E = \dfrac{hc}{\lambda} = \boxed{6.15\text{ MeV}}$



Fi gu re fo r G oa l So lu ti

Goal Solution

The nuclear potential energy that binds protons and neutrons in a nucleus is often approximated by a square well. Imagine a proton confined in an infinitely high square well of width 10.0 fm, a typical nuclear diameter. Calculate the wavelength and energy associated with the photon emitted when the proton moves from the $n=2$ state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong?

G: Nuclear radiation from nucleon transitions is usually in the form of high energy gamma rays with short wavelengths.

O: The energy of the particle can be obtained from the wavelengths of the standing waves corresponding to each level. The transition between energy levels will result in the emission of a photon with this energy difference.

A: At level 1, the node-to-node distance of the standing wave is $1.00\times10^{-14}\text{ m}$, so the wavelength is twice this distance: $h/p = 2.00\times10^{-14}\text{ m}$. The proton's kinetic energy is

$$K=\tfrac{1}{2}mv^2=\frac{p^2}{2m}=\frac{h^2}{2m\lambda^2}=\frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-14}\text{ m}\right)^2}=\frac{3.29\times10^{-13}\text{ J}}{1.60\times10^{-19}\text{ J/eV}}=2.06\text{ MeV}$$

In the first excited state, level 2, the node-to-node distance is two times smaller than in state 1. The momentum is two times larger and the energy is four times larger: $K=8.23\text{ MeV}$.

The proton has mass, has charge, moves slowly compared to light in a standing-wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$$2.06\text{ MeV}-8.23\text{ MeV}=-6.17\text{ MeV}$$

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $+6.17\text{ MeV}$.

Its frequency is
$$f=\frac{E}{h}=\frac{\left(6.17\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{6.63\times10^{-34}\text{ J}\cdot\text{s}}=1.49\times10^{21}\text{ Hz}$$

and its wavelength is
$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\text{ m/s}}{1.49\times10^{21}\text{ s}^{-1}}=2.02\times10^{-13}\text{ m}$$

This is a gamma ray, according to Figure 34.17.

L: The radiated photons are energetic gamma rays as we expected for a nuclear transition. In the above calculations, we assumed that the proton was not relativistic ($v<0.1c$), but we should check this assumption for the highest energy state we examined ($n=2$):

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2\left(8.23\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}}=3.97\times10^{7}\text{ m/s}=0.133c$$

This appears to be a borderline case where we should probably use relativistic equations, but our classical treatment should give reasonable results, within $(0.133)^2=1\%$ accuracy.

41.22 $\lambda = 2D$ for the lowest energy state

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{8mD} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})^2}{8\left[4\left(1.66\times 10^{-27}\ \text{kg}\right)\right]\left(1.00\times 10^{-14}\ \text{m}\right)^2} = 8.27\times 10^{-14}\ \text{J} =$$
$\boxed{0.517\ \text{MeV}}$

$$p = \frac{h}{\lambda} = \frac{h}{2D} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{2\left(1.00\times 10^{-14}\ \text{m}\right)} = \boxed{3.31\times 10^{-20}\ \text{kg}\cdot\text{m/s}}$$

41.23 $E_n = \left(\frac{h^2}{8mL^2}\right)n^2$

$$E_1 = \frac{\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times 10^{-27}\ \text{kg}\right)\left(2.00\times 10^{-14}\ \text{m}\right)^2} = 8.21\times 10^{-14}\ \text{J}$$

$E_1 = \boxed{0.513\ \text{MeV}}$ $E_2 = 4E_1 = \boxed{2.05\ \text{MeV}}$ $E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$

41.24 (a) $$< x >= \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\frac{1}{2} - \frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$< x >= \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{L/2}$$

(b) $$\text{Probability} = \int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.20 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times 10^{-5}}$$

(c) $$\text{Probability} = \left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times 10^{-2}}$$

(d) In the $n = 2$ graph in Figure 41.11 (b), it is more probable to find the particle either near

$x = \frac{L}{4}$ or $x = \frac{3L}{4}$ than at the center, where the probability density is zero.

$$\int_{x=0}^{L} A^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

41.26 The desired probability is

$$P = \int_{x=0}^{x=L/4} |\psi|^2 dx = \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{2\pi x}{L}\right)dx$$

where

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Thus,

$$P = \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Big|_0^{L/4} = \left(\frac{1}{4} - 0 - 0 + 0\right) = \boxed{0.250}$$

41.27 In $0 \le x \le L$, the argument $2\pi x / L$ of the sine function ranges from 0 to 2π. The probability density $(2/L)\sin^2(2\pi x/L)$ reaches maxima at $\sin\theta = 1$ and $\sin\theta = -1$ at

$$\frac{2\pi x}{L} = \frac{\pi}{2} \qquad \text{and} \qquad \frac{2\pi x}{L} = \frac{3\pi}{2}$$

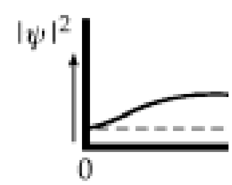
∴ The most probable positions of the particle are at $\boxed{x = \frac{L}{4} \text{ and } x = \frac{3L}{4}}$

41.28 (a) The probability is

$$P = \int_0^{L/3} |\psi|^2 dx = \int_0^{L/3} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$$

$$P = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Big|_0^{L/3} = \left(\frac{1}{3} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = L/2$. Thus, the probability of finding the particle between $x = 2L/3$ and $x = L$ is the same 0.196. Therefore, the probability of finding it in the range $L/3 \le x \le 2L/3$ is $P = 1.00 - 2(0.196) = 0.609$.



(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}} = \boxed{1/3}$.

41.29 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67\times10^{-27}\text{ kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{-10}\text{ m}\right)^2} = 8.22\times10^{-22}\text{ J} = \boxed{5.13\times10^{-3}\text{ eV}}$$

(b) For an electron $\left(m = 9.11\times10^{-31}\text{ kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(9.11\times10^{-31}\text{ kg}\right)\left(2.00\times10^{-10}\text{ m}\right)^2} = 1.51\times10^{-18}\text{ J} = \boxed{9.41\text{ eV}}$$

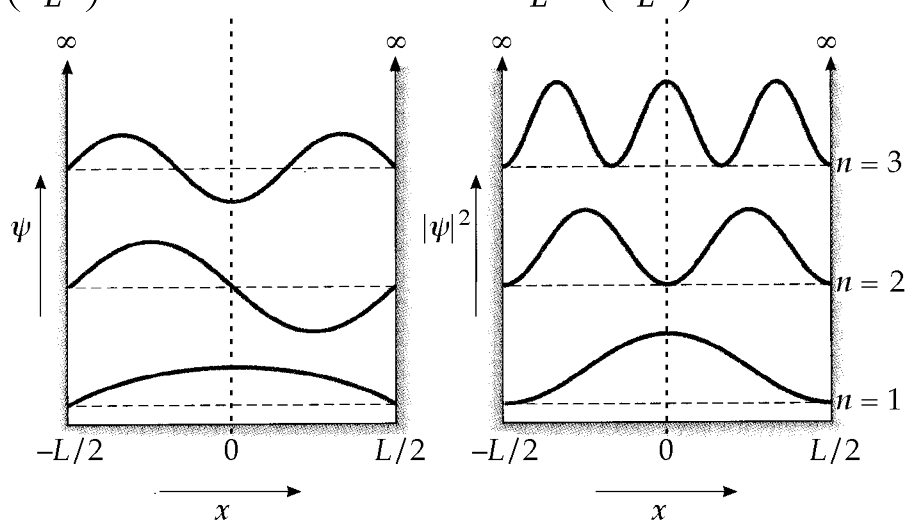
(c) The electron has a much higher energy because it is much less massive.

41.30 (a)

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right) \qquad P_1(x) = \left|\psi_1(x)\right|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right) \qquad P_2(x) = \left|\psi_2(x)\right|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$$

$$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right) \qquad P_3(x) = \left|\psi_3(x)\right|^2 = \frac{2}{L}\cos\left(\frac{3\pi x}{L}\right)$$



41.31 We have $\psi = Ae^{i(kx-\omega t)}$ and $\dfrac{\partial^2\psi}{\partial x^2} = -k^2\psi$

Schrödinger's equation: $$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = \frac{2m}{h^2}(E-U)\psi$$

Since $k^2 = \dfrac{(2\pi)^2}{\lambda^2} = \dfrac{(2\pi p)^2}{h^2} = \dfrac{p^2}{h^2}$ and $(E-U) = p^2/2m$

41.32 $\psi(x) = A\cos kx + B\sin kx$ $\qquad \dfrac{\partial \psi}{\partial x} = -kA\sin kx + kB\cos kx$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx \qquad -\frac{2m}{\mathrm{h}^2}(E-U)\psi = -\frac{2mE}{\mathrm{h}^2}(A\cos kx + B\sin kx)$$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2 \psi}{\partial x^2} = \left(-\frac{2m}{\mathrm{h}^2}\right)(E-U)\psi \qquad \text{or}$$

$$-k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\mathrm{h}^2}\right)(A\cos kx + B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E = \dfrac{\mathrm{h}^2 k^2}{2m}}$

41.33 Problem 45 in Ch. 16 helps students to understand how to draw conclusions from an identity.

(a) $\psi(x) = A\left(1 - \dfrac{x^2}{L^2}\right)$ $\qquad \dfrac{d\psi}{dx} = -\dfrac{2Ax}{L^2}$ $\qquad \dfrac{\partial^2 \psi}{\partial x^2} = -\dfrac{2A}{L^2}$

Schrödinger's equation $\qquad \dfrac{\partial^2 \psi}{\partial x^2} = -\dfrac{2m}{\mathrm{h}^2}(E-U)\psi$

becomes

$$-\frac{2A}{L^2} = \frac{2m}{\mathrm{h}^2}EA\left(1 - \frac{x^2}{L^2}\right) + \frac{2m}{\mathrm{h}^2}\frac{\left(-\mathrm{h}^2 x^2\right)A\left(1 - \frac{x^2}{L^2}\right)}{mL^2\left(L^2 - x^2\right)}$$

$$-\frac{1}{L^2} = -\frac{mE}{\mathrm{h}^2} + \frac{mEx^2}{\mathrm{h}^2 L^2} - \frac{x^2}{L^4}$$

This will be true for all x if both $\qquad \dfrac{1}{L^2} = \dfrac{mE}{\mathrm{h}^2}$ $\qquad$ and $\qquad \dfrac{mE}{\mathrm{h}^2 L^2} - \dfrac{1}{L^4} = 0$

Both of these conditions are satisfied for a particle of energy $\boxed{E = \dfrac{\mathrm{h}^2}{L^2 m}}$.

(b) For normalization,

$$1 = \int_{-L}^{L} A^2\left(1 - \frac{x^2}{L^2}\right)^2 dx = A^2\int_{-L}^{L}\left(1 - \frac{2x^2}{L^2} + \frac{x^4}{L^4}\right)dx$$

$$1 = A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_{-L}^{L} = A^2\left[L - \frac{2}{3}L + \frac{L}{5} + L - \frac{2}{3}L + \frac{L}{5}\right] = A^2\,\frac{16L}{15} \qquad \boxed{A = \sqrt{\frac{15}{16L}}}$$

(c)

$$P = \frac{47}{81} = \boxed{0.580}$$

41.34 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x)=\left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x)=Axe^{-x^2/L^2}$ Then $\dfrac{d^2\psi}{dx^2}=\left(4Ax^3-6AxL^2\right)\dfrac{e^{-x^2/L^2}}{L^4}$

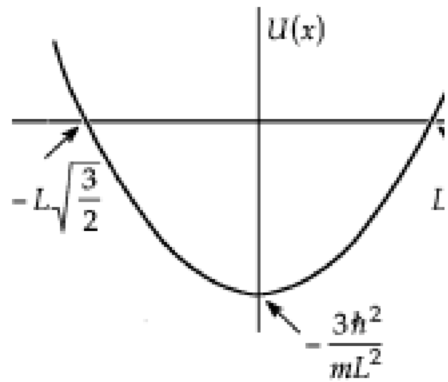
or

$$\frac{d^2\psi}{dx^2}=\frac{\left(4x^2-6L^2\right)}{L^4}\psi(x)$$

and

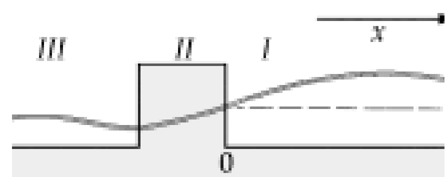
$$U(x)=\frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2}-6\right)$$

See figure to the right .



41.35 (a) See figure to the right .

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $2L$.



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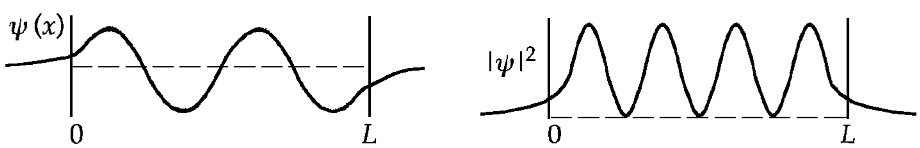
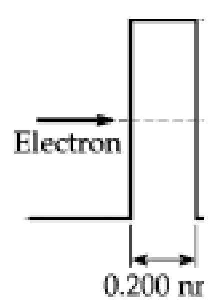


41.37 $T=e^{-2CL}$ (Use Equation 41.17)



$$2CL=\frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right)=4.58$$

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Goal Solution

An electron with kinetic energy $E = 5.00\ \text{eV}$ is incident on a barrier with thickness $L = 0.100\ \text{nm}$ and height $U = 10.0\ \text{eV}$ (Fig. P41.37). What is the probability that the electron (a) will tunnel through the barrier and (b) will be reflected?

G: Since the barrier energy is higher than the kinetic energy of the electron, transmission is not likely, but should be possible since the barrier is not infinitely high or thick.

O: The probability of transmission is found from the transmission coefficient equation 41.18.

A: The transmission coefficient is

$$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(10.0\ \text{eV} - 5.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}/2\pi} = 1.14\times10^{10}\ \text{m}^{-1}$$

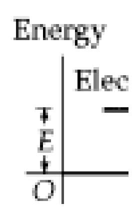
(a) The probability of transmission is

$$T = e^{-2CL} = e^{-2\left(1.14\times10^{10}\ \text{m}^{-1}\right)\left(2.00\times10^{-10}\ \text{m}\right)} = e^{-4.58} = 0.0103$$

(b) If the electron does not tunnel, it is reflected, with probability $1 - 0.0103 = 0.990$

L: Our expectation was correct: there is only a 1% chance that the electron will penetrate the barrier. This tunneling probability would be greater if the barrier were thinner, shorter, or if the kinetic energy of the electron were greater.

41.38 $$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$



$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

41.39 From problem 38, $C = 3.62\times10^{9}\ \text{m}^{-1}$

$$10^{-6} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right]$$

Taking logarithms, $-13.816 = -2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$

New $L = 1.91\ \text{nm}$

‘41.40 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-CL}.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0\ /\text{nm})(0.500\text{ nm})}}{e^{-2(10.0\ /\text{nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}$$

‘41.41 With transmission coefficient e^{-CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0\ /\text{nm})L} - e^{-2(10.0\ /\text{nm})(L+0.00200\text{ nm})}}{e^{-2(10.0\ /\text{nm})L}} = 1 - e^{29.0(0.00200)} = 0.0392 = \boxed{3.92\%}$$

41.42 $\psi = Be^{-(m\omega/2\hbar)x^2}$ so $\dfrac{d\psi}{dx} = -\left(\dfrac{m\omega}{\hbar}\right)x\psi$ and $\dfrac{d^2\psi}{dx^2} = \left(\dfrac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\dfrac{m\omega}{\hbar}\right)\psi$

Substituting into Equation 41.19 gives

$$\left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi = \left(\frac{2mE}{\hbar^2}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$$

which is satisfied provided that $E = \dfrac{\hbar\omega}{2}$.

41.43 Problem 45 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi = Axe^{-bx^2}$ so $\dfrac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2Ae^{-bx^2}$

and

$$\frac{d^2\psi}{dx^2} = -2bxAe^{-bx^2} - 4bxAe^{-bx^2} + 4b^2x^3Ae^{-bx^2} = -6b\psi + 4b^2x^2\psi$$

Substituting into Equation 41.19, $$-6b\psi + 4b^2x^2\psi = -\left(\frac{2mE}{\hbar}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$$

For this to be true as an identity, it must be true for all values of x.

So we must have both $-6b = -\dfrac{2mE}{\hbar^2}$ and $4b^2 = \left(\dfrac{m\omega}{\hbar}\right)^2$

(a) Therefore $\boxed{b = \dfrac{m\omega}{2\hbar}}$

(b) and $E = \dfrac{3b\hbar^2}{m} = \boxed{\dfrac{3}{2}\hbar\omega}$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

41.44 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda}=\hbar\omega=\hbar\sqrt{\frac{k}{m}} \qquad \text{so}$$

$$\lambda=2\pi c\sqrt{\frac{m}{k}}=2\pi\left(3.00\times10^{8}\text{ m/s}\right)\left(\frac{9.11\times10^{-31}\text{ kg}}{8.99\text{ N/m}}\right)^{1/2}=\boxed{600\text{ nm}}$$

41.45 (a) With $\psi=Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all}}|\psi|^2dx=1$

becomes
$$1=\int_{-\infty}^{\infty}B^2e^{-2(m\omega/2\hbar)x^2}dx=2B^2\int_0^{\infty}e^{-2(m\omega/2\hbar)x^2}dx=2B^2\frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1=B^2\sqrt{\frac{\pi\hbar}{m\omega}}$ and $\boxed{B=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$

(b) For small δ, the probability of finding the particle in the range $-\delta/2\le x\le\delta/2$ is

$$\int_{-\delta/2}^{\delta/2}|\psi|^2dx=\delta|\psi(0)|^2=\delta B^2e^{-0}=\boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

41.46 (a) With $<x>=0$ and $<p_x>=0$, the average value of x^2 is $(\Delta x)^2$ and the average value of $p_x^{\,2}$ is $(\Delta p_x)^2$. Then $\Delta x\ge\hbar/2\Delta p_x$ requires

$$E\ge\frac{p_x^{\,2}}{2m}+\frac{k}{2}\frac{\hbar^2}{4p_x^{\,2}}=\boxed{\frac{p_x^{\,2}}{2m}+\frac{k\hbar^2}{8p_x^{\,2}}}$$

(b) To minimize this as a function of $p_x^{\,2}$, we require
$$\frac{dE}{dp_x^{\,2}}=0=\frac{1}{2m}+\frac{k\hbar^2}{8}(-1)\frac{1}{p_x^{\,4}}$$

Then
$$\frac{k\hbar^2}{8p_x^{\,4}}=\frac{1}{2m}$$
$$p_x^{\,2}=\left(\frac{2mk\hbar^2}{8}\right)^{1/2}=\frac{\hbar\sqrt{mk}}{2}$$

and
$$E\ge\frac{\hbar\sqrt{mk}}{2(2m)}+\frac{k\hbar^2 3}{8\hbar\sqrt{mk}}=\frac{\hbar}{4}\sqrt{\frac{k}{m}}+\frac{\hbar}{4}\sqrt{\frac{k}{m}}.$$

41.47 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02 \text{ kg})(9.8 \text{ m/s}^2)(0.12 \text{ m}) = 0.0235 \text{ J}$$

and $$E = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.02 \text{ kg})(0.8 \text{ m/s})^2 = 0.0064 \text{ J}$$

Then $$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02 \text{ kg})(0.0171 \text{ J})}}{1.055 \times 10^{-34} \text{ J}\cdot\text{s}} = 2.5 \times 10^{32} \text{ m}^{-1}$$

and the transmission coefficient is

$$e^{-2CL} = e^{-2(2.5\times10^{32})(2\times10^{-3})} = e^{-10\times10^{29}} = e^{-2.30(4.3\times10^{29})} = 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

41.48 (a) $\lambda = 2L = \boxed{2.00 \times 10^{-10} \text{ m}}$

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{2.00 \times 10^{-10} \text{ m}} = \boxed{3.31 \times 10^{-24} \text{ kg}\cdot\text{m/s}}$

(c) $E = \dfrac{p^2}{2m} = \boxed{0.172 \text{ eV}}$

41.49 (a) See the first figure to the right.

(b) See the second figure to the right.

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$

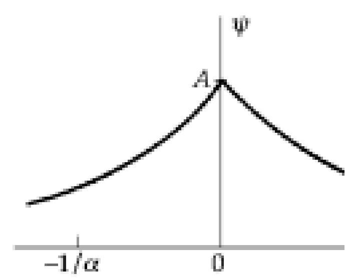
(d) Since ψ is symmetric,

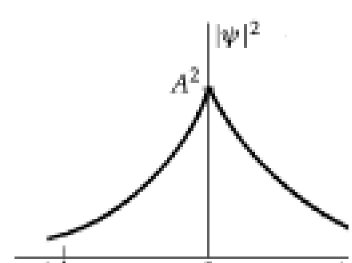
$$\int_{-\infty}^{\infty} |\psi|^2 dx = 2\int_0^{\infty} |\psi|^2 dx = 1$$

or

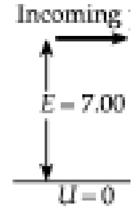
$$2A^2 \int_0^{\infty} e^{-2\alpha x} dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $A = \sqrt{\alpha}$





(a) Use Schrödinger's equation



$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

with solutions $\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$ [region *I*]

$$\psi_2 = Ce^{ik_2x}$$ [region *II*]

Where $k_1 = \frac{\sqrt{2mE}}{\hbar}$

and $k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$

Then, matching functions and derivatives at $x = 0$: $(\psi_1)_0 = (\psi_2)_0 \Rightarrow A + B = C$

and

$$\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0 \Rightarrow k_1(A-B) = k_2C$$

Then $B = \frac{1-k_2/k_1}{1+k_2/k_1}A$

$$C = \frac{2}{1+k_2/k_1}A$$

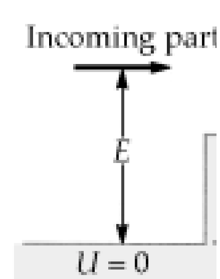
Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \frac{B^2}{A^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$

(b) With $E = 7.00$ eV and $U = 5.00$ eV, $\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535$

The reflection probability is $R = \frac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.0920}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$

41.51 $R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}$



$\frac{\hbar^2k^2}{2m} = E - U$ for constant U

$\frac{h^2k_1^{\,2}}{2m} = E$ since $U = 0$ (1)

Dividing (2) by (1), $\frac{k_2^{\,2}}{k_1^{\,2}} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$ so

$\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$

and therefore,

$$R = \frac{\left(1 - 1/\sqrt{2}\right)^2}{\left(1 + 1/\sqrt{2}\right)^2} = \frac{\left(\sqrt{2} - 1\right)^2}{\left(\sqrt{2} + 1\right)^2} = \boxed{0.0294}$$

41.52 (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.11 of the textbook.

(b)

$$P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2 dx = \left(\frac{2}{1.00\text{ nm}}\right)\int_{0.150\text{ nm}}^{0.350\text{ nm}} \sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right)dx = \frac{2.00}{\text{nm}}\left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

In the above result we used $\int \sin^2 ax dx = (x/2) - (1/4a)\sin(2ax)$

$$P_1 = \frac{1.00}{\text{nm}}\left(x - \frac{1.00\text{ nm}}{2\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right)_{0.150\text{ nm}}^{0.350\text{ nm}}$$

$$P_1 = \frac{1.00}{\text{nm}}\left\{0.350\text{ nm} - 0.150\text{ nm} - \frac{1.00\text{ nm}}{2\pi}\left[\sin(0.700\pi) - \sin(0.300\pi)\right]\right\} = \boxed{0.200}$$

(c) $$P_2 = \frac{2}{1.00}\int_{0.150}^{0.350}\sin^2\left(\frac{2\pi x}{1.00}\right)dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350} = 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}\left[\sin(1.40\pi) - \sin(0.600\pi)\right]\right\} = \boxed{0.351}$$

(d) Using $E_n = \dfrac{n^2h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377\text{ eV}}$ and $E_2 = \boxed{1.51\text{ eV}}$

41.53 (a) $mgy_i = \frac{1}{2}mv_f^2$ $\qquad v_f = \sqrt{2gy_i} = \sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = 31.3\text{ m/s}$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})} = \boxed{2.82\times10^{-37}\text{ m}}$$ (not observable)

(b) $\Delta E\Delta t \ge \hbar/2$ so $$\Delta E \ge \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3}\text{ s})} = \boxed{1.06\times10^{-32}\text{ J}}$$

(c) $$\frac{\Delta E}{E} = \frac{1.06\times10^{-32}\text{ J}}{(75.0\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})} = \boxed{2.87\times10^{-35}\text{ \%}}$$

41.54 From the uncertainty principle $\Delta E\Delta t - \hbar/2$ or $\Delta\left(mc^2\right)\Delta t = \hbar/2$. Therefore,

$$\frac{\Delta m}{m} = \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(8.70\times10^{-17}\text{ s}\right)(135\text{ MeV})}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) =$$

$$\boxed{2.81\times10^{-8}}$$

41.55 (a) $f = \dfrac{E}{h} = \dfrac{180\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1.00\text{ eV}}\right) = \boxed{4.34\times10^{14}\text{ Hz}}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.34\times10^{14}\text{ Hz}} = 6.91\times10^{-7}\text{ m} = \boxed{691\text{ nm}}$

(c) $\Delta E\Delta t \ge \dfrac{\hbar}{2}$ so

$$\Delta E \ge \frac{\hbar}{2\Delta t} = \frac{h}{4\pi\,\Delta t} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(2.00\times10^{-8}\text{ s}\right)} = 2.64\times10^{-29}\text{ J} = \boxed{1.65\times10^{-10}\text{ eV}}$$

41.56 (a) $f = \boxed{\dfrac{E}{h}}$

(b) $\lambda = \dfrac{c}{f} = \boxed{\dfrac{hc}{E}}$

(c) $\Delta E\Delta t \ge \dfrac{\hbar}{2}$ so $\Delta E \ge \dfrac{\hbar}{2\Delta t} = \boxed{\dfrac{h}{4\pi T}}$

41.57 $\left\langle x^2\right\rangle = \int_{-\infty}^{\infty} x^2\left|\psi\right|^2 dx$

For a one-dimensional box of width L, $\psi_n = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)$

Thus, $\left\langle x^2\right\rangle = \dfrac{2}{L}\int_0^L x^2\sin^2\left(\dfrac{n\pi x}{L}\right)dx = \boxed{\dfrac{L^2}{3} - \dfrac{L^2}{2n^2\pi^2}}$ (from integral tables)

41.58 (a) $\int_{-\infty}^{\infty}\left|\psi\right|^2 dx = 1$ becomes

$$A^2\int_{-L/4}^{L/4}\cos^2\left(\frac{2\pi x}{L}\right)dx = A^2\left(\frac{L}{2\pi}\right)\left[\frac{\pi x}{L} + \frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4} = A^2\left(\frac{L}{2\pi}\right)\left(\frac{\pi}{2}\right) = 1$$

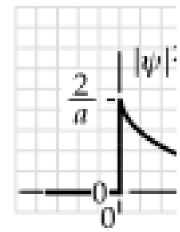
or $A^2 = \dfrac{4}{L}$ and $\boxed{A = \dfrac{2}{\sqrt{L}}}$

(b) The probability of finding the particle between 0 and $L/8$ is

$$\int_0^{L/8}\left|\psi\right|^2 dx = A^2\int_0^{L/8}\cos^2\left(\frac{2\pi x}{L}\right)dx = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

41.59 For a particle with wave function $\psi(x) = \sqrt{\frac{2}{a}}e^{-x/a}$ for $x > 0$ and 0 for $x < 0$

(a) $|\psi(x)|^2 = 0, \; x < 0$ and $|\psi^2(x)| = \frac{2}{a}e^{-2x/a}, \; x > 0$



(b) $\text{Prob}(x < 0) = \int_{-\infty}^{0} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} (0) \, dx = \boxed{0}$

(c) Normalization

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_{-\infty}^{0} |\psi|^2 dx + \int_{0}^{\infty} |\psi|^2 dx = 1$$

$$\int_{-\infty}^{0} 0 \, dx + \int_{0}^{\infty} (2/a) e^{-2x/a} dx = 0 - e^{-2x/a}\Big|_{0}^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0 < x < a) = \int_{0}^{a} |\psi|^2 \, dx = \int_{0}^{a} (2/a) e^{-2x/a} dx = e^{-2x/a}\Big|_{0}^{a} = 1 - e^{-2} =$$
$\boxed{0.865}$

41.60 (a) $\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m_e{}^2c^4}} = \frac{hc}{\sqrt{\left(m_ec^2 + K\right)^2 - \left(m_ec^2\right)^2}}$

$$\lambda = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}{\sqrt{(576 \text{ keV})^2 - (511 \text{ keV})^2}}\left(\frac{1 \text{ keV}}{1.60 \times 10^{-16} \text{ J}}\right) = \boxed{4.68 \times 10^{-12} \text{ m}}$$

(b) $50.0\lambda = \boxed{2.34 \times 10^{-10} \text{ m}}$

41.61 (a) $\Delta x \Delta p \geq \hbar/2$ so if $\Delta x = r$, $\Delta p \geq \boxed{\hbar/2r}$

(b) Choosing $\Delta p = \frac{\hbar}{r}$, $K = \frac{p^2}{2m_e} = \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_er^2}}$

$$U = -\frac{k_ee^2}{r}, \text{ so } E = K + U = \boxed{\frac{\hbar^2}{2m_er^2} - \frac{k_ee^2}{r}}$$

(c) To minimize E,

dE $\hbar$ k_ee^2 $\hbar^2$

Then, $E = \frac{\hbar^2}{2m_e}\left(\frac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\frac{m_e k_e e^2}{\hbar^2}\right) = -\left(\frac{m_e k_e^2 e^4}{2\hbar^2}\right) = \boxed{-13.6 \text{ eV}}$

41.62 (a) The requirement that $\frac{n\lambda}{2} = L$ so $p = \frac{h}{\lambda} = \frac{nh}{2L}$ is still valid.

$$E = \sqrt{(pc)^2 + (mc^2)^2} \quad \Rightarrow \quad E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2}$$

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2}$$

(b) Taking $L = 1.00 \times 10^{-12}$ m, $m = 9.11 \times 10^{-31}$ kg, and $n = 1$, we find $K_1 = \boxed{4.69 \times 10^{-14} \text{ J}}$

Nonrelativistic, $E_1 = \frac{h^2}{8mL^2} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^{-12} \text{ m})^2} = 6.02 \times 10^{-14}$ J

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

41.63 (a) $U = \frac{e^2}{4\pi \epsilon_0 d}\left[-1 + \frac{1}{2} - \frac{1}{3} + \left(-1 + \frac{1}{2}\right) + (-1)\right] = \frac{(-7/3)e^2}{4\pi \epsilon_0 d} = \boxed{-\frac{7k_e e^2}{3d}}$

(b) From Equation 41.9, $K = 2E_1 = \frac{2h^2}{8m_e(9d^2)} = \boxed{\frac{h^2}{36m_e d^2}}$

(c) $E = U + K$ and $\frac{dE}{dd} = 0$ for a minimum: $\frac{7k_e e^2}{3d^2} - \frac{h^2}{18m_e d^3} = 0$

$$d = \frac{3h^2}{(7)(18k_e e^2 m_e)} = \frac{h^2}{42m_e k_e e^2} = \frac{(6.626 \times 10^{-34})^2}{(42)(9.11 \times 10^{-31})(8.99 \times 10^9)(1.602 \times 10^{-19} \text{ C})^2} =$$

$\boxed{0.0499 \text{ nm}}$

(d) Since the lithium spacing is a, where $Na^3 = V$, and the density is Nm/V, where m is the mass of one atom, we get:

$$a = \left(\frac{Vm}{Nm}\right)^{1/3} = \left(\frac{m}{\text{density}}\right)^{1/3} = \left(\frac{1.66 \times 10^{-27} \text{ kg} \times 7}{530 \text{ kg}}\right)^{1/3} \text{ m} = 2.80 \times 10^{-10} \text{ m} = \boxed{0.280 \text{ nm}}$$

(5.62 times larger than c).

41.64 (a) $\psi = Bxe^{-(m\omega/2\hbar)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(\frac{-m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(\frac{-m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2\left(\frac{-m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting into the Schrödinger Equation (41.19), we have

$$-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)x^3e^{-(m\omega/2\hbar)x^2} = -\frac{2mE}{\hbar^2}Bxe^{-(m\omega/2\hbar)x^2} + \left(\frac{m\omega}{\hbar}\right)^2 x^2Bxe^{-(m\omega/2\hbar)x^2}$$

This is true if $-3\omega = -\frac{2E}{\hbar}$; it is true if $\boxed{E = \frac{3}{2}\hbar\omega}$

(b) We never find the particle at $\boxed{x=0}$ because $\psi = 0$ there.

(c) ψ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{\hbar}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$

(d) We require $\int_{-\infty}^{\infty}|\psi|^2 dx = 1$:

$$1 = \int_{-\infty}^{\infty} B^2x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\int_0^{\infty} x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\,\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}\hbar^{3/2}}{(m\omega)^{3/2}}$$

Then $$B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$$

(e) At $x = 2\sqrt{\hbar/m\omega}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(4\hbar/m\omega) = 2\hbar\omega$. This is larger than the total energy $3\hbar\omega/2$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2 dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/\hbar)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{\hbar}\right)^{3/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$

41.65 (a) $\int_0^L |\psi|^2 dx = 1$: $A^2 \int_0^L \left[\sin^2\left(\frac{\pi x}{L}\right) + 16\sin^2\left(\frac{2\pi x}{L}\right) + 8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right) \right] dx = 1$

$$A^2\left[\left(\frac{L}{2}\right) + 16\left(\frac{L}{2}\right) + 8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right] = 1$$

$$A^2\left[\frac{17L}{2} + 16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right] = A^2\left[\frac{17L}{2} + \frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Big|_{x=0}^{x=L}\right] = 1$$

$A^2 = \frac{2}{17L}$, so the normalization constant is $\boxed{A = \sqrt{2/17L}}$

(b) $\int_{-a}^{a} |\psi|^2 dx = 1$:

$$\int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right) + |B|^2\sin^2\left(\frac{\pi x}{a}\right) + 2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx = 1$$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx = 4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx = \frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Big|_{-a}^{a} = 0$$

so that $a\left(|A|^2 + |B|^2\right) = 1$, giving $\boxed{|A|^2 + |B|^2 = 1/a}$.

41.66 With one slit open $P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$

With both slits open, $P = |\psi_1 + \psi_2|^2$

At a maximum, the wave functions are in phase $P_{\max} = \left(|\psi_1| + |\psi_2|\right)^2$

At a minimum, the wave functions are out of phase $P_{\min} = \left(|\psi_1| - |\psi_2|\right)^2$

Now $\frac{P_1}{P_2} = \frac{|\psi_1|^2}{|\psi_2|^2} = 25.0$, so $\frac{|\psi_1|}{|\psi_2|} = 5.00$

‘41.67 (a) The light is unpolarized. It contains both horizontal and vertical field oscillations.

(b) The interference pattern appears, but with diminished overall intensity.

(c) The results are the same in each case.

(d) The interference pattern appears and disappears as the polarizer turns, with alternately increasing and decreasing contrast between the bright and dark fringes. The intensity on the screen is precisely zero at the center of a dark fringe four times in each revolution, when the filter axis has turned by 45°, 135°, 225°, and 315° from the vertical.

(e) Looking at the overall light energy arriving at the screen, we see a low-contrast interference pattern. After we sort out the individual photon runs into those for trial 1, those for trial 2, and those for trial 3, we have the original results replicated: The runs for trials 1 and 2 form the two blue graphs in Figure 41.3, and the runs for trial 3 build up the red graph.

Chapter 42 Solutions

42.1 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r} \quad \text{or} \quad r_{\min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\min} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\text{ C}\right)^2}{(4.00 \text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)} = \boxed{5.68\times10^{-14}\text{ m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\min}^2} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)158\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(5.68\times10^{-14}\text{ m}\right)^2} = \boxed{11.3\text{ N}} \text{ away from the nucleus}$$

42.2 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{T}}}{r} \quad \text{or} \quad r_{\min} = \frac{k_e(2e)(Ze)}{E} = \boxed{\frac{2Zk_e e^2}{E}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{T}}}{r_{\min}^2} = 2Zk_e e^2\left(\frac{E}{2Zk_e e^2}\right)^2 = \boxed{\frac{E^2}{2Zk_e e^2}} \text{ away from the target nucleus}$$

42.3 (a) The photon has energy 2.28 eV.

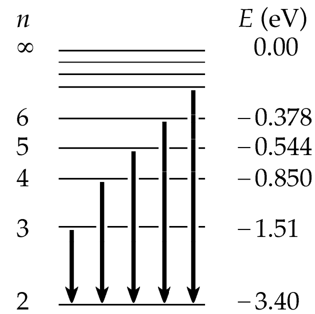
And $(13.6 \text{ eV})/2^2 = 3.40$ eV is required to ionize a hydrogen atom from state $n = 2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n = 2$, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy

$$-\frac{13.6 \text{ eV}}{3^2} = -1.51 \text{ eV}$$

(b) The electron thus freed can have kinetic energy $K_e = 2.28 \text{ eV} - 1.51 \text{ eV} = 0.769 \text{ eV} = \frac{1}{2}m_e v^2$

$$v = \sqrt{\frac{2(0.769)(1.60\times10^{-19})\text{J}}{(9.11\times10^{-31})\text{kg}}} = \boxed{520 \text{ km/s}}$$

***42.4** (a) Longest wavelength implies lowest frequency and smallest energy: the electron falls from $n = 3$ to $n = 2$, losing energy



Balmer Series

$$-\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$$

The photon frequency is $f = \Delta E/h$ and its wavelength is

$$\lambda = \frac{c}{f} = \frac{ch}{\Delta E} = \frac{(3.00\times10^8\text{ m/s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{1.89\text{ eV}}\left(\frac{\text{eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$\lambda = \boxed{656\text{ nm}}$$

(b) The biggest energy loss is for an electron to fall from ionization, $n = \infty$, to the $n = 2$ state.

It loses energy $$-\frac{13.6\text{ eV}}{\infty} + \frac{13.6\text{ eV}}{2^2} = \boxed{3.40\text{ eV}}$$

to emit light of wavelength $$\lambda = \frac{hc}{\Delta E} = \frac{(3.00\times10^8\text{ m/s})(6.626\times10^{-34}\text{ J}\cdot\text{s})}{3.40\text{ eV}(1.60\times10^{-19}\text{ J/eV})} = \boxed{365\text{ nm}}$$

42.5 (a) For positronium, $\mu = \frac{m_e}{2}$, so $\lambda_{32} = (656\text{ nm})2 = 1312\text{ nm} = \boxed{1.31\ \mu\text{m}}$ (infrared region).

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$, so $\lambda_{32} = (656/4)\text{ nm} = \boxed{164\text{ nm}}$ (ultraviolet region).

Goal Solution

A general expression for the energy levels of one-electron atoms and ions is

$$E_n = -\left(\frac{\mu k_e^2 q_1^2 q_2^2}{2\hbar^2 n^2}\right)$$

where k_e is the Coulomb constant, q_1 and q_2 are the charges of the two particles, and μ is the reduced mass, given by $\mu = m_1 m_2/(m_1 + m_2)$. In Problem 4 we found that the wavelength for the $n = 3$ to $n = 2$ transition of the hydrogen atom is 656.3 nm (visible red light). What are the wavelengths for this same transition in (a) positronium, which consists of an electron and a positron, and (b) singly ionized helium? (**Note**: A positron is a positively charged electron.)

G: The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

O: All the factors in the above equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

A: For hydrogen, $\mu = \dfrac{m_p m_e}{m_p + m_e} \approx m_e$

The photon energy is $\Delta E = E_3 - E_2$

Its wavelength is $\lambda = 656.3$ nm, where $\lambda = \dfrac{c}{f} = \dfrac{hc}{\Delta E}$

(a) For positronium, $\mu = \dfrac{m_e m_e}{m_e + m_e} = \dfrac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium." The photon energy is inversely proportional to its wavelength, so for positronium,

$\lambda_{32} = 2(656.3 \text{ nm}) = 1313 \text{ nm}$ (in the infrared region)

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$,

so the transition energy is $2^2 = 4$ times larger than hydrogen. Then,

$\lambda_{32} = \left(\dfrac{656}{4}\right) \text{nm} = 164 \text{ nm}$ (in the ultraviolet region)

L: As expected, the wavelengths for positronium and helium are respectively larger and smaller than for hydrogen. Other energy transitions should have wavelength shifts consistent with this pattern. It is important to remember that the reduced mass is not the total mass, but is generally close in magnitude to the smaller mass of the system (hence the name **reduced** mass).

***42.6** (a) For a particular transition from n_i to n_f,

$$\Delta E_H = -\frac{\mu_H k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_H}$$

and

$$\Delta E_D = -\frac{\mu_D k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_D}$$

where $\mu_H = \dfrac{m_e m_p}{m_e + m_p}$ and $\mu_D = \dfrac{m_e m_D}{m_e + m_D}$

By division, $\dfrac{\Delta E_H}{\Delta E_D} = \dfrac{\mu_H}{\mu_D} = \dfrac{\lambda_D}{\lambda_H}$ or $\lambda_D = \left(\dfrac{\mu_H}{\mu_D}\right)\lambda_H$

Then, $\boxed{\lambda_H - \lambda_D = \left(1 - \dfrac{\mu_H}{\mu_D}\right)\lambda_H}$

(b) $$\frac{\mu_H}{\mu_D} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_D}{m_e m_D}\right) = \frac{(1.007\ 276 \text{ u})(0.000\ 549 \text{ u} + 2.013\ 553 \text{ u})}{(0.000\ 549 \text{ u} + 1.007\ 276 \text{ u})(2.013\ 553 \text{ u})} = 0.999728$$

$\lambda_H - \lambda_D = (1 - 0.999\ 728)(656.3 \text{ nm}) = \boxed{0.179 \text{ nm}}$

42.7 (a) In the 3*d* subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	-1	-1	-2	-2
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 10 states)

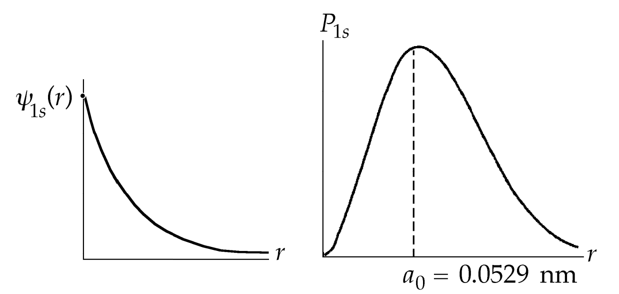
(b) In the 3*p* subshell, $n = 3$ and $\ell = 1$, we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	+0	+0	-1	-1
m_s	+1/2	-1/2	+1/2	-1/2	+1/2	-1/2

(A total of 6 states)

42.8 $$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \qquad \text{(Eq. 42.3)}$$

$$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0} \qquad \text{(Eq. 42.7)}$$



42.9 (a) $$\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2\, dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_0^\infty r^2 e^{-2r/a_0}\, dr$$

Using integral tables, $$\int |\psi|^2 dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{r=0}^{r=\infty} = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$$

so the wave function as given is normalized.

(b) $$P_{a_0/2 \to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2\, dr = 4\pi \left(\frac{1}{\pi a_0^3}\right) \int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0}\, dr$$

Again, using integral tables,

$$P_{a_0/2 \to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17 a_0^2}{4}\right) - e^{-1}\left(\frac{5 a_0^2}{4}\right)\right] = \boxed{0.497}$$

42.10 $\psi = \frac{1}{\sqrt{3}} \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$ so $P_r = 4\pi r^2 \left|\psi^2\right| = 4\pi r^2 \frac{r^2}{24a_0^5} e^{-r/a_0}$

Set $\frac{dP}{dr} = \frac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0} + r^4\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right] = 0$

Solving for r, this is a maximum at $\boxed{r = 4a_0}$

42.11 $\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ $\frac{2}{r}\frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^5}} e^{-r/a_0} = \frac{2}{ra_0}\psi$ $\frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \frac{1}{a_0^2}\psi$

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi$$

But $a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$, so $-\frac{e^2}{8\pi\epsilon_0 a_0} = E$ or $\boxed{E = -\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

42.12 The hydrogen ground-state radial probability density is

$$P(r) = 4\pi r^2\left|\psi_{1s}\right|^2 = \frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

The number of observations at $2a_0$ is, by proportion

$$N = 1000\frac{P(2a_0)}{P(a_0/2)} = 1000\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = 1000(16)e^{-3} = \boxed{797 \text{ times}}$$

42.13 (a) For the d state, $\ell = 2$, $L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}$ J · s

(b) For the f state, $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}$ J · s

***42.14** $L = \sqrt{\ell(\ell+1)}\hbar$ so $4.714\times10^{-34} = \sqrt{\ell(\ell+1)}\dfrac{6.626\times10^{-34}}{2\pi}$

$$\ell(\ell+1) = \frac{\left(4.714\times10^{-34}\right)^2(2\pi)^2}{\left(6.626\times10^{-34}\right)^2} = 1.998\times10^1 \approx 20 = 4(4+1) \quad \text{so} \quad \boxed{\ell = 4}$$

42.15 The 5th excited state has n = 6, energy $\dfrac{-13.6\text{ eV}}{36} = -0.378\text{ eV}$

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(1090\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = 1.14\text{ eV}$

to end up with energy $-0.378\text{ eV} - 1.14\text{ eV} = -1.52\text{ eV}$

which is the energy in state 3: $-\dfrac{13.6\text{ eV}}{3^3} = -1.51\text{ eV}$

While n = 3, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\,h}$

42.16 For a 3d state, n = 3 and ℓ = 2. Therefore, $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2(2+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58\times10^{-34}\text{ J}\cdot\text{s}$

m_ℓ can have the values $-2, -1, 0, 1$, and 2, so $\boxed{L_z \text{ can have the values} -2\hbar, -\hbar, 0, \text{ and } 2\hbar}$

Using the relation $\cos\theta = L_z / L$, we find that the possible values of θ are equal to

$\boxed{145°, 114°, 90.0°, 65.9°, \text{ and } 35.3°}$.

42.17 (a) $n = 1$: For $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = \pm\dfrac{1}{2}$, $\rightarrow$ 2 sets

n	ℓ	m_ℓ	m_s
1	0	0	–1/2
1	0	0	+1/2

$2n^2 = 1(1)^2 = \boxed{2}$

(b) For $n = 2$, we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	–1	±1/2
2	1	0	±1/2
2	1	1	±1/2

yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, ℓ can take on values ranging from 0 to $n-1$. So the general expression is

$$s = \sum_{0}^{n-1} 2(2\ell+1)$$

The series is an arithmetic progression: 2 + 6 + 10 + 14, the sum of which is

$s = \frac{n}{2}[2a+(n-1)d]$ where $a = 2,\ d = 4$: $\quad s = \frac{n}{2}[4+(n-1)4] = 2n^2$

(c) $n = 3$: $2(1)+2(3)+2(5) = 2+6+10 = 18$ $\quad 2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $2(1)+2(3)+2(5)+2(7) = 32$ $\quad 2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $32+2(9) = 32+18 = 50$ $\quad 2n^2 = 2(5)^2 = \boxed{50}$

42.18 $\mu_B = \frac{e\hbar}{2m_e}$ $\quad e = 1.60\times10^{-19}$ C $\quad \hbar = 1.055\times10^{-34}$ J · s $\quad m_e = 9.11\times10^{-31}$ kg

$$\mu_B = \boxed{9.27\times10^{-24}\ \text{J/T} = 5.79\times10^{-5}\ \text{eV/T}}$$

42.19 (a) Density of a proton: $\rho = \frac{m}{V} = \frac{1.67\times10^{-27}\ \text{kg}}{(4/3)\pi(1.00\times10^{-15}\ \text{m})^3} = \boxed{3.99\times10^{17}\ \text{kg/m}^3}$

(b) Size of model electron: $r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3\times9.11\times10^{-31}\ \text{kg}\cdot\text{m}^3}{4\pi\times3.99\times10^{17}\ \text{kg}}\right)^{1/3} = \boxed{8.17\times10^{-17}\ \text{m}}$

(c) Moment of inertia: $I = \frac{2}{5}mr^2 = \frac{2}{5}(9.11\times10^{-31}\ \text{kg})(8.17\times10^{-17}\ \text{m})^2 = 2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore, $v = \frac{hr}{2I} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(8.17\times10^{-17}\ \text{m})}{2\pi(2)(2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2)} = \boxed{1.77\times10^{12}\ \text{m/s}}$

(d) This is $\boxed{5.91\times10^{3}\ \text{times larger}}$ than the speed of light.

42.20 (a) $L = mvr = m\dfrac{2\pi r}{T}r = \sqrt{\ell(\ell+1)}\hbar = \sqrt{(\ell^2+\ell)}\hbar \approx \ell\hbar$

$$(5.98\times10^{24}\text{ kg})\,\frac{2\pi(1.496\times10^{11}\text{ m})^2}{3.156\times10^7\text{ s}} = \ell\hbar \quad\text{so}\quad \frac{2.66\times10^{40}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = \ell = \boxed{2.52\times10^{74}}$$

(b) $|E| = |-U+K| = |-K| = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{mr^2}{mr^2}mv^2 = \dfrac{1}{2}\dfrac{L^2}{mr^2} = \dfrac{1}{2}\dfrac{\ell(\ell+1)\hbar^2}{mr^2} \approx \dfrac{1}{2}\dfrac{\ell^2\hbar^2}{mr^2}$

$$\frac{dE}{d\ell} = \frac{1}{2}\frac{2\ell\hbar^2}{mr^2}\frac{\ell}{\ell} = 2\frac{E}{\ell}\text{ so}\qquad dE = 2\frac{E}{\ell}d\ell = 2\frac{\frac{1}{2}(5.98\times10^{24}\text{ kg})\left(\dfrac{2\pi\times1.496\times10^{11}\text{ m}}{3.156\times10^7\text{ s}}\right)^2 1}{2.52\times10^{74}}$$

$$\Delta E = \frac{5.30\times10^{33}\text{ J}}{2.52\times10^{74}} = \boxed{2.10\times10^{-41}\text{ J}}$$

***42.21** $\mu_n = \dfrac{e\hbar}{2m_p}$ $\quad e = 1.60\times10^{-19}$ C $\quad \hbar = 1.055\times10^{-34}$ J · s $\quad m_p = 1.67\times10^{-27}$ kg

(a) $\mu_n = \boxed{5.05\times10^{-27}\text{ J/T}} = \boxed{31.6\text{ neV/T}}$

(b) $\dfrac{\mu_n}{\mu_B} = \dfrac{1}{1836} = \dfrac{m_e}{m_p}$

Apparently it is harder to "spin up" a nucleus than a electron, because of its greater mass.

42.22 In the N shell, $n = 4$. For $n = 4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell\hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

42.23 The 3d subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.
Therefore, we can have $\boxed{n = 3;\ \ell = 2;\ m_\ell = -2, -1, 0, 1, 2;\ s = 1;\text{ and } m_s = -1, 0, 1}$, leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

42.24 (a) $1s^2 2s^2 2p^4$

(b) For the 1s electrons, $n = 1$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the two 2s electrons, $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +1/2$ and $-1/2$
For the four 2p electrons, $n = 2$; $\ell = 1$; $m_\ell = -1, 0,$ or 1; and $m_s = +1/2$ or $-1/2$

42.25 The 4s subshell fills first, for potassium and calcium, before the 3d subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\text{Ar}]3d^4 4s^2$ would have lower energy than $[\text{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\text{Ar}]3d^5 4s^1$ is the ground state for chromium.

***42.26** (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

electron one	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron two	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron one	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron two	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be 36 possible states, six for each electron independently.

42.27

Shell	K	L				M									N
n	1	2				3									4
ℓ	0	0	1			0	1			2					0
m_ℓ	0	0	1	0	–1	0	1	0	–1	2	1	0	–1	–2	0
m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓
count	1 2	3 4			10	12			18	21				30	20
	He	Be			Ne	Mg			Ar					Zn	Ca

(a) zinc or copper

(b) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10}$ or $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1 3d^{10}$

42.28 Listing subshells in the order of filling, we have for element 110,

$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6\,6s^2\,4f^{14}\,5d^{10}\,6p^6\,7s^2\,5f^{14}\,6d^8$

In order of increasing principal quantum number, this is

$$\boxed{1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^{10}\,4f^{14}\,5s^2\,5p^6\,5d^{10}\,5f^{14}\,6s^2\,6p^6\,6d^8\,7s^2}$$

42.29 (a)

$n + l$	1	2	3	4	5	6	7
subshell	$1s$	$2s$	$2p$, $3s$	$3p$, $4s$	$3d$, $4p$, $5s$	$4d$, $5p$, $6s$	$4f$, $5d$, $6p$, $7s$

(b) $Z = 15$:

Filled subshells: $1s$, $2s$, $2p$, $3s$ (12 electrons)
Valence subshell: 3 electrons in $3p$ subshell
Prediction: Valance = +3 or −5
Element is phosphorus Valence +3 or −5 (Prediction correct)

$Z = 47$:

Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$ (38 electrons)
Outer subshell: 9 electrons in $4d$ subshell
Prediction: Valence = −1
Element is silver, (Prediction fails) Valence is +1

$Z = 86$:

Filled subshells: $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, $4p$, $5s$, $4d$, $5p$, $6s$, $4f$, $5d$, $6p$ (86 electrons)
Prediction Outer subshell is full: inert gas
Element is radon, inert (Prediction correct)

42.30 Electronic configuration: Sodium to Argon

$[1s^2\,2s^2\,2p^6]\ +3s^1 \rightarrow Na^{11}$
$+3s^2 \rightarrow Mg^{12}$
$+3s^2\,3p^1 \rightarrow Al^{13}$
$+3s^2\,3p^2 \rightarrow Si^{14}$
$+3s^2\,3p^3 \rightarrow P^{15}$
$+3s^2\,3p^4 \rightarrow S^{16}$
$+3s^2\,3p^5 \rightarrow Cl^{17}$
$+3s^2\,3p^6 \rightarrow Ar^{18}$
$[1s^2\,2s^2\,2p^6\,3s^2\,3p^6]4s^1 \rightarrow K^{19}$

***42.31** $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

ψ_{300} corresponds to $E_{300} = -\dfrac{Z^2 E_0}{n^2} = -\dfrac{2^2(13.6)}{(3)^2} = \boxed{-6.05\text{ eV}}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1, 0, 1}$

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2, -1, 0, 1, 2}$

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

42.32 $E = \dfrac{hc}{\lambda} = e(\Delta V) \Rightarrow \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}{10.0\times 10^{-9}\text{ m}} = (1.60\times 10^{-19})(\Delta V)$

$\Delta V = \boxed{124\text{ V}}$

***42.33** $E_{\text{photon max}} = \dfrac{hc}{\lambda_{\text{min}}} = e(\Delta V) = 40.0\text{ keV}$

$$\lambda_{\text{min}} = \frac{hc}{E_{\text{max}}} = \frac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}{40.0\times 10^{3}\text{ eV}}\left(\frac{1.00\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = \boxed{0.031\ 0\text{ nm}}$$

42.34 Some electrons can give all their kinetic energy $K_e = e(\Delta V)$ to the creation of a single photon of x-radiation, with

$$hf = \frac{hc}{\lambda} = e(\Delta V)$$

$$\lambda = \frac{hc}{e(\Delta V)} = \frac{(6.6261\times 10^{-34}\text{ J}\cdot\text{s})(2.9979\times 10^{8}\text{ m/s})}{(1.6022\times 10^{-19}\text{ C})(\Delta V)} = \boxed{\frac{1240\text{ nm}\cdot\text{V}}{\Delta V}}$$

42.35 Following Example 42.7, $E_\gamma = \dfrac{3}{4}(42-1)^2(13.6\text{ eV}) = 1.71\times 10^{4}\text{ eV} = 2.74\times 10^{-15}\text{ J}$

$f = 4.14\times 10^{18}\text{ Hz}$ and $\lambda = \boxed{0.072\ 5\text{ nm}}$

42.36 The K_β x-rays are emitted when there is a vacancy in the (n = 1) K shell and an electron from the (n = 3) M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\text{ eV} + \frac{13.6(Z-1)^2}{1^2}\text{ eV}$$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(0.152\times10^{-9}\text{ m})(1.60\times10^{-19}\text{ J/eV})} = 13.6\text{ eV}\left(-\frac{Z^2}{9}+\frac{18Z}{9}-\frac{81}{9}+Z^2-2Z+1\right)$$

$$8.17\times10^{3}\text{ eV} = 13.6\text{ eV}\left(\frac{8Z^2}{9}-8\right) \quad \text{so} \quad 601 = \frac{8Z^2}{9}-8 \quad \text{and} \quad Z = 26 \ \boxed{\text{Iron}}$$

42.37 (a) Suppose the electron in the M shell is shielded from the nucleus by two K plus seven L electrons. Then its energy is

$$-\frac{13.6\text{ eV}(83-9)^2}{3^2} = -8.27\text{ keV}$$

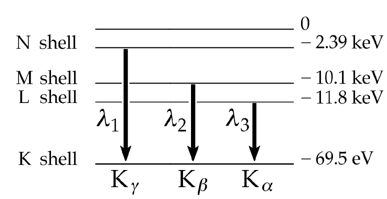
Suppose, after it has fallen into the vacancy in the L shell, it is shielded by just two K-shell electrons. Then its energy is

$$\frac{-13.6\text{ eV}(83-2)^2}{2^2} = -22.3\text{ keV}$$

Thus the electron's energy loss is the photon energy: $(22.3-8.27)\text{ keV} = \boxed{14.0\text{ keV}}$

(b) $\Delta E = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}\,(3.00\times10^{8}\text{ m/s})}{14.0\times10^{3}\times1.60\times10^{-19}\text{ J}} = \boxed{8.85\times10^{-11}\text{ m}}$

***42.38** $$E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{\lambda} = \frac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$$



for $\lambda_1 = 0.0185$ nm, $E = 67.11$ keV

$\lambda_2 = 0.0209$ nm, $E = 59.4$ keV

$\lambda_3 = 0.0215$ nm, $E = 57.7$ keV

The ionization energy for K shell = 69.5 keV, so, the ionization energies for the other shells are: $\boxed{\text{L shell} = 11.8\text{ keV}}$: $\boxed{\text{M shell} = 10.1\text{ keV}}$: $\boxed{\text{N shell} = 2.39\text{ keV}}$

***42.39** (a) The outermost electron in sodium has a 3*s* state for its ground state. The longest wavelength means minimum photon energy and smallest step on the energy level diagram. Since $n = 3$, n' must be 4. With $\ell = 0$, ℓ' must be $\boxed{1}$, since ℓ must change by 1 in a photon absorption process.

(b) $$\frac{1}{330 \times 10^{-9}\ \text{m}} = \left(1.097 \times 10^{7}\ \frac{1}{\text{m}}\right)\left[\frac{1}{(3 - 1.35)^2} - \frac{1}{(4 - \delta_1)^2}\right]$$

$$0.276 = \frac{1}{(1.65)^2} - \frac{1}{(4 - \delta_1)^2} = 0.367 - \frac{1}{(4 - \delta_1)^2} \quad \text{so} \quad (4 - \delta_1)^2 = 10.98 \quad \text{and} \quad \boxed{\delta_1 = 0.686}$$

42.40 $$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m / s})}{(2.10\ \text{eV})(1.60 \times 10^{19}\ \text{J / eV})} = \boxed{590\ \text{nm}}$$

***42.41** We require $A = u_f B = \dfrac{16\pi^2 \hbar}{\lambda^3} B$ or $$u_f = \frac{16\pi^2 \hbar}{\lambda^3} = \frac{16\pi^2\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)}{\left(645 \times 10^{-9}\ \text{m}\right)^3} = \boxed{6.21 \times 10^{-14}\ \frac{\text{J}\cdot\text{s}}{\text{m}^3}}$$

42.42 $$f = \frac{E}{h} = \boxed{2.82 \times 10^{13}\ \text{s}^{-1}}$$

$$\lambda = \frac{c}{f} = \boxed{10.6\ \mu\text{m}}, \quad \boxed{\text{infrared}}$$

42.43 $$E = \mathcal{P}t = (1.00 \times 10^{6}\ \text{W})(1.00 \times 10^{-8}\ \text{s}) = 0.0100\ \text{J}$$

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{694.3 \times 10^{-9}}\ \text{J} = 2.86 \times 10^{-19}\ \text{J}$$

$$N = \frac{E}{E_\gamma} = \frac{0.0100}{2.86 \times 10^{-19}} = \boxed{3.49 \times 10^{16}\ \text{photons}}$$

Goal Solution

A ruby laser delivers a 10.0-ns pulse of 1.00 MW average power. If the photons have a wavelength of 694.3 nm, how many are contained in the pulse?

G: Lasers generally produce concentrated beams that are bright (except for IR or UV lasers that produce invisible beams). Since our eyes can detect light levels as low as a few photons, there are probably at least 1000 photons in each pulse.

O: From the pulse width and average power, we can find the energy delivered by each pulse. The number of photons can then be found by dividing the pulse energy by the energy of each photon, which is determined from the photon wavelength.

A: The energy in each pulse is $E = \mathcal{P}t = (1.00\times10^{6}\text{ W})(1.00\times10^{-8}\text{ s}) = 1.00\times10^{-2}\text{ J}$

The energy of each photon is $E_\gamma = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34})(3.00\times10^{8})}{694.3\times10^{-9}}\text{ J} = 2.86\times10^{-19}\text{ J}$

So $N = \dfrac{E}{E_\gamma} = \dfrac{1.00\times10^{-2}\text{ J}}{2.86\times10^{-19}\text{ J / photon}} = 3.49\times10^{16}\text{ photons}$

L: With 10^{16} photons/pulse, this laser beam should produce a bright red spot when the light reflects from a surface, even though the time between pulses is generally much longer than the width of each pulse. For comparison, this laser produces more photons in a single ten-nanosecond pulse than a typical 5 mW helium-neon laser produces over a full second (about 1.6×10^{16} photons/second).

***42.44** In $G = e^{\sigma(n_u - n_\ell)L}$ we require $1.05 = e^{\left(1.00\times10^{-18}\text{ m}^2\right)(n_u - n_\ell)(0.500\text{ m})}$

Thus, $\ln(1.05) = \left(5.00\times10^{-19}\text{ m}^3\right)(n_u - n_\ell)$ so $n_u - n_\ell = \dfrac{\ln(1.05)}{5.00\times10^{-19}\text{ m}^3} = \boxed{9.76\times10^{16}\text{ m}^{-3}}$

42.45 (a) $\dfrac{N_3}{N_2} = \dfrac{N_g e^{-E_3/(k_B\cdot300\text{ K})}}{N_g e^{-E_2/(k_B\cdot300\text{ K})}} = e^{-(E_3 - E_2)/(k_B\cdot300\text{ K})} = e^{-hc/\lambda(k_B\cdot300\text{ K})}$

where λ is the wavelength of light radiated in the $3\to2$ transition:

$$\frac{N_3}{N_2} = e^{-\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)/\left(632.8\times10^{-9}\text{ m}\right)\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})} = e^{-75.8} = \boxed{1.22\times10^{-33}}$$

(b) $$\frac{N_3}{N_2} = e^{hc/\lambda k_B T} = e^{-\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)/\left(694.3\times10^{-9}\text{ m}\right)\left(1.38\times10^{-23}\text{ J/K}\right)(4.00\text{ K})} = e^{-5187}$$

To avoid overflowing your calculator, note that $10 = e^{\ln 10}$. Take

$$\frac{N_3}{N_2} = e^{\ln 10\times(-5187/\ln 10)} = \boxed{10^{-2253}}$$

***42.46** $N_u/N_l = e^{-(E_u - E_l)/k_B T}$ where the subscript u refers to an upper energy state and the subscript l to a lower energy state.

(a) Since $E_u - E_l = E_{\text{photon}} = hc/\lambda$, $N_u/N_l = e^{-hc/\lambda k_B T}$

Thus, we require $1.02 = e^{-hc/\lambda k_B T}$ or $\ln(1.02) = -\dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(694.3\times10^{-9}\ \text{m})(1.38\times10^{-23}\ \text{J/K})T}$

$$T = -\frac{2.07\times10^{4}\ \text{K}}{\ln(1.02)} = \boxed{-1.05\times10^{6}\ \text{K}}$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T = \infty$, for as $T \to \infty$ the populations of upper and lower states approach equality.

(b) Because $E_u - E_l > 0$, and in any real equilibrium state $T > 0$, $e^{-(E_u - E_l)/k_B T} < 1$ and $N_u < N_l$.

Thus, a population inversion cannot happen in thermal equilibrium.

42.47 (a) $I = \dfrac{(3.00\times10^{-3}\ \text{J})}{(1.00\times10^{-9}\ \text{s})\pi(15.0\times10^{-6}\ \text{m})^2} = \boxed{4.24\times10^{15}\ \text{W/m}^2}$

(b) $(3.00\times10^{-3}\ \text{J})\dfrac{(0.600\times10^{-9}\ \text{m})^2}{(30.0\times10^{-6}\ \text{m})^2} = \boxed{1.20\times10^{-12}\ \text{J}} = 7.50\ \text{MeV}$

***42.48** (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $E = E_2 - E_1 = \dfrac{(-13.6\ \text{eV})}{4} - \dfrac{(-13.6\ \text{eV})}{1} = 10.2\ \text{eV} = \boxed{1.63\times10^{-18}\ \text{J}}$

(b) We have $E = \dfrac{3}{2}k_B T$, or $T = \dfrac{2}{3k_B}E = \dfrac{2(1.63\times10^{-18}\ \text{J})}{3(1.38\times10^{-23}\ \text{J/K})} = \boxed{7.88\times10^{4}\ \text{K}}$

42.49 $r_{av} = \displaystyle\int_0^\infty rP(r)dr = \int_0^\infty \left(\frac{4r^3}{a_0^3}\right)(e^{-2r/a_0})dr$

Make a change of variables with $\dfrac{2r}{a_0} = x$ and $dr = \dfrac{a_0}{2}dx$.

Then $r_{av} = \dfrac{a_0}{4}\displaystyle\int_0^\infty x^3 e^{-x}dx = \frac{a_0}{4}\left[-x^3e^{-x} + 3\left(-x^2e^{-x} + 2e^{-x}(-x-1)\right)\right]\Big|_0^\infty = \boxed{\frac{3}{2}a_0}$

***42.50** $$\left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{4r^2}{a_0^3} e^{-2r/a_0} \frac{1}{r} dr = \frac{4}{a_0^3} \int_0^\infty r e^{-(2/a_0)r} dr = \frac{4}{a_0^3} \frac{1}{(2/a_0)^2} = \boxed{\frac{1}{a_0}}$$

We compare this to $\frac{1}{\langle r \rangle} = \frac{1}{3a_0/2} = \frac{2}{3a_0}$, and find that the average reciprocal value is NOT the reciprocal of the average value.

42.51 The wave equation for the $2s$ state is given by Eq. 42.7: $\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0} \right)^{3/2} \left[2 - \frac{r}{a_0} \right] e^{-r/2a_0}$

(a) Taking $r = a_0 = 0.529 \times 10^{-10}$ m, we find

$$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{0.529 \times 10^{-10}\ \text{m}} \right)^{3/2} [2-1] e^{-1/2} = \boxed{1.57 \times 10^{14}\ \text{m}^{-3/2}}$$

(b) $$\left| \psi_{2s}(a_0) \right|^2 = \left(1.57 \times 10^{14}\ \text{m}^{-3/2} \right)^2 = \boxed{2.47 \times 10^{28}\ \text{m}^{-3}}$$

(c) Using Equation 42.5 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2 \left| \psi_{2s}(a_0) \right|^2 = \boxed{8.69 \times 10^8\ \text{m}^{-1}}$

***42.52** We define the reduced mass to be μ, and the ground state energy to be E_1:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{207 m_e m_p}{207 m_e + m_p} = \frac{207\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(1.67 \times 10^{-27}\ \text{kg}\right)}{207\left(9.11 \times 10^{-31}\ \text{kg}\right) + \left(1.67 \times 10^{-27}\ \text{kg}\right)} = 1.69 \times 10^{-28}\ \text{kg}$$

$$E_1 = -\frac{\mu k_e^2 q_1^2 q_2^2}{2\hbar^2 (1)^2} = -\frac{\left(1.69 \times 10^{-28}\ \text{kg}\right)\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)^2 \left(1.60 \times 10^{-19}\ \text{C}\right)^3 e}{2\left(1.055 \times 10^{-34}\ \text{J} \cdot \text{s}\right)^2} = -2.52 \times 10^3\ \text{eV}$$

To ionize the muonium "atom" one must supply energy $\boxed{+2.52\ \text{keV}}$.

42.53 (a) $(3.00 \times 10^8\ \text{m/s})(14.0 \times 10^{-12}\ \text{s}) = \boxed{4.20\ \text{mm}}$

(b) $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19}$ J $\qquad N = \frac{3.00\ \text{J}}{2.86 \times 10^{-19}\ \text{J}} = \boxed{1.05 \times 10^{19}\ \text{photons}}$

(c) $V = (4.20\ \text{mm})\pi(3.00\ \text{mm})^2 = 119\ \text{mm}^3$ $\qquad n = \frac{1.05 \times 10^{19}}{119} = \boxed{8.82 \times 10^{16}\ \text{mm}^{-3}}$

42.54 (a) The length of the pulse is $\Delta L = \boxed{ct}$

(b) The energy of each photon is $E_\gamma = \dfrac{hc}{\lambda}$ so $N = \dfrac{E}{E_\gamma} = \boxed{\dfrac{E\lambda}{hc}}$

(c) $V = \Delta L\pi\dfrac{d^2}{4}$ $\quad n = \dfrac{N}{V} = \boxed{\left(\dfrac{4}{ct\pi d^2}\right)\left(\dfrac{E\lambda}{hc}\right)}$

42.55 We use $\psi_{2s}(r) = \dfrac{1}{4}(2\pi a_0^3)^{-1/2}\left(2 - \dfrac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 42.5, $P(r) = 4\pi r^2\psi^2 = \dfrac{1}{8}\left(\dfrac{r^2}{a_0^3}\right)\left(2 - \dfrac{r}{a_0}\right)^2 e^{-r/a_0}$

(a)

$$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2 - \frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2 - \frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or

$$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2 - \frac{r}{a_0}\right)\left[2\left(2 - \frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2 - \frac{r}{a_0}\right)\right]e^{-r/a_0} = 0$$

Therefore we require the roots of $\dfrac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$, and $r = \infty$ to be minima with $P(r) = 0$.

$[\,.\,.\,.\,.\,.\,] = 4 - (6r/a_0) + (r/a_0)^2 = 0$ with solutions $r = \left(3 \pm \sqrt{5}\right)a_0$.

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3 - \sqrt{5}\right)a_0 = 0.7639a_0$, then $P(r) = 0.0519/a_0$
When $r = \left(3 + \sqrt{5}\right)a_0 = 5.236a_0$, then $P(r) = 0.191/a_0$

Therefore, the most probable value of r is $\left(3 + \sqrt{5}\right)a_0 = \boxed{5.236a_0}$

(b) $\displaystyle\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0{}^3}\right)\left(2 - \frac{r}{a_0}\right)^2 e^{-r/a_0}\,dr$ Let $u = \dfrac{r}{a_0}$, $dr = a_0\,du$,

$$\int_0^\infty P(r)dr = \int_0^\infty \frac{1}{8}u^2(4 - 4u + u^2)e^{-u}\,dr = \int_0^\infty \frac{1}{8}(u^4 - 4u^3 + 4u^2)e^{-u}\,du = -\frac{1}{8}(u^4 + 4u^2 + 8u + 8)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as desired}}$.

***42.56** $\Delta z = \dfrac{at^2}{2} = \dfrac{1}{2}\left(\dfrac{F_z}{m_0}\right)t^2 = \dfrac{\mu_z(dB_z/dz)}{2m_0}\left(\dfrac{\Delta x}{v}\right)^2$ and $\mu_z = \dfrac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2 2m_e}{\Delta x^2 e\hbar} = \frac{2(108)(1.66\times10^{-27}\text{ kg})(10^4\text{ m}^2/\text{s}^2)2(9.11\times10^{-31}\text{ kg})(10^{-3}\text{ m})}{(1.00\text{ m}^2)(1.60\times10^{-19}\text{ C})(1.05\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{0.389\text{ T/m}}$$

42.57 With one vacancy in the K shell, excess energy $\Delta E \approx -(Z-1)^2(13.6 \text{ eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right) = 5.40 \text{ keV}$

We suppose the outermost 4*s* electron is shielded by 20 electrons inside its orbit:

$$E_{\text{ionization}} \approx \frac{2^2(13.6 \text{ eV})}{4^2} = 3.40 \text{ eV}$$

Note the experimental ionization energy is 6.76 eV. $K = \Delta E - E_{\text{ionization}} \approx \boxed{5.39 \text{ keV}}$

***42.58** $E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} = \Delta E$

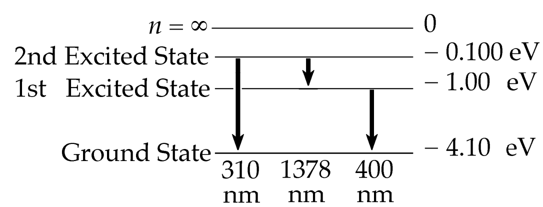
$\lambda_1 = 310$ nm, so $\Delta E_1 = 4.00$ eV

$\lambda_2 = 400$ nm, $\Delta E_2 = 3.10$ eV

$\lambda_3 = 1378$ nm, $\Delta E_3 = 0.900$ eV

and the ionization energy = 4.10 eV

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.



42.59 (a) One molecule's share of volume

Al: $V = \frac{\text{mass per molecule}}{\text{density}} = \left(\frac{27.0 \text{ g/mol}}{6.02\times 10^{23} \text{ molecules/mole}}\right)\left(\frac{1.00\times 10^{-6} \text{ m}^3}{2.70 \text{ g}}\right) = 1.66\times 10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.55\times 10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$

U: $V = \left(\frac{238 \text{ g}}{6.02\times 10^{23} \text{ molecules}}\right)\left(\frac{1.00\times 10^{-6} \text{ m}^3}{18.9 \text{ g}}\right) = 2.09\times 10^{-29} \text{ m}^3$

$\sqrt[3]{V} = \boxed{2.76\times 10^{-10} \text{ m} \sim 10^{-1} \text{ nm}}$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge, $+Ze-(Z-1)e = +e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

42.60 (a) No orbital magnetic moment to consider: higher energy for $\boxed{\begin{matrix}N\\S\end{matrix}}\ \boxed{\begin{matrix}N\\S\end{matrix}}$ parallel magnetic moments, for $\boxed{\text{antiparallel spins}}$ of the electron and proton.

(b) $E = \dfrac{hc}{\lambda} = 9.42 \times 10^{-25}\text{ J} = \boxed{5.89\ \mu\text{eV}}$

(c) $\Delta E \Delta t \approx \dfrac{\hbar}{2}$ so $\Delta E \approx \dfrac{1.055 \times 10^{-34}\text{ J}\cdot\text{s}}{2\left(10^7\text{ yr}\right)\left(3.16 \times 10^7\text{ s/yr}\right)}\left(\dfrac{1.00\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) = \boxed{1.04 \times 10^{-30}\text{ eV}}$

42.61 $P = \int_{2.50 a_0}^{\infty} \dfrac{4r^2}{a_0{}^3} e^{-2r/a_0}\, dr = \frac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z}\, dz$ where $z \equiv \dfrac{2r}{a_0}$

$$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_{5.00}^{\infty} = -\frac{1}{2}[0] + \frac{1}{2}(25.0 + 10.0 + 2.00)e^{-5} = \left(\frac{37}{2}\right)(0.00674) = \boxed{0.125}$$

Goal Solution

For hydrogen in the 1*s* state, what is the probability of finding the electron farther than $2.50 a_o$ from the nucleus?

G: From the graph shown in Figure 42.8, it appears that the probability of finding the electron beyond $2.5\ a_0$ is about 20%.

O: The precise probability can be found by integrating the 1*s* radial probability distribution function from $r = 2.50 a_o$ to ∞.

A: The general radial probability distribution function is $P(r) = 4\pi r^2 |\psi|^2$

With $\psi_{1s} = \left(\pi a_0{}^3\right)^{-1/2} e^{-r/a_0}$ it is $P(r) = 4r^2 a_0{}^{-3} e^{-2r/a_0}$

The required probability is then $P = \int_{2.50 a_0}^{\infty} P(r) dr = \int_{2.50 a_0}^{\infty} \dfrac{4r^2}{a_0{}^3} e^{-2r/a_0} dr$

Let $z = 2r/a_0$ and $dz = 2dr/a_0$: $P = \frac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z} dz$

Performing this integration by parts, $P = -\frac{1}{2}\left(z^2 + 2z + 2\right)e^{-z}\Big]_{5.00}^{\infty}$

$$P = -\tfrac{1}{2}(0) + \tfrac{1}{2}(25.0 + 10.0 + 2.00)e^{-5.00} = \left(\tfrac{37}{2}\right)(0.00674) = 0.125$$

L: The probability of 12.5% is less than the 20% we estimated, but close enough to be a reasonable result. In comparing the 1*s* probability density function with the others in Figure 42.8, it appears that the ground state is the most narrow, indicating that a 1*s* electron will probably be found in the narrow range of 0 to 4 Bohr radii, and most likely at $r = a_0$.

42.62 The probability, P, of finding the electron within the Bohr radius is

$$P = \int_{r=0}^{a_0} P_{1s}(r)\,dr = \int_{r=0}^{a_0} \left(\frac{4r^2}{a_0{}^3}\right) e^{-2r/a_0}\,dr$$

Defining $z \equiv 2r/a_0$, this becomes

$$P = -\frac{1}{2}(z^2 + 2z + 2)e^{-z}\Big|_0^2 = -\frac{1}{2}\left[(4+4+2)e^{-2} - (0+0+2)e^0\right] = \frac{1}{2}\left(2 - \frac{10}{e^2}\right) = \boxed{0.323}$$

The electron is likely to be within the Bohr radius about one-third of the time. The Bohr model indicates *none* of the time.

42.63 (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi \epsilon_0} \frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi\epsilon_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi\epsilon_0 r} \quad \text{so} \quad \frac{dE}{dt} = \frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi\epsilon_0}\frac{e^2 a^2}{c^3} = \frac{-e^2}{6\pi\epsilon_0 c^3}\left(\frac{e^2}{4\pi\epsilon_0 r^2 m_e}\right)^2$$

Therefore,
$$\frac{dr}{dt} = -\frac{e^4}{12\pi^2 \epsilon_0{}^2 r^2 m_e{}^2 c^3}$$

(b) $$-\int_{r=2.00\times 10^{-10}\text{ m}}^{r=0} 12\pi^2 \epsilon_0{}^2 r^2 m_e{}^2 c^3\,dr = e^4 \int_{t=0}^{T} dt$$

$$\frac{12\pi^2 \epsilon_0{}^2 m_e{}^2 c^3}{e^4} \frac{r^3}{3}\Bigg|_0^{2.00\times 10^{-10}} = T = \boxed{8.46 \times 10^{-10}\text{ s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

42.64 (a) $+3e - 0.85e - 0.85e = \boxed{1.30e}$

(b) The valence electron is in an $n = 2$ state, with energy

$$\frac{-13.6\text{ eV } Z_{\text{eff}}^2}{n^2} = \frac{-13.6\text{ eV}(1.30)^2}{2^2} = -5.75\text{ eV}$$

To ionize the atom you must put in $\boxed{+5.75\text{ eV}}$

This differs from the experimental value by 6%, so we could say the effective value of Z is accurate within 3%.

42.65 $\Delta E = 2\mu_B B = hf$ so $2\left(9.27\times10^{-24}\text{ J / T}\right)\left(0.350\text{ T}\right) = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)f$

and $f = \boxed{9.79\times10^{9}\text{ Hz}}$

42.66 The photon energy is $E_4 - E_3 = 20.66 - 18.70\text{ eV} = 1.96\text{ eV} = \dfrac{hc}{\lambda}$

$$\lambda = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{1.96\times1.60\times10^{-19}\text{ J}} = \boxed{633\text{ nm}}$$

42.67 (a) $$\frac{1}{\alpha} = \frac{\hbar c}{k_e e^2} = \frac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{2\pi\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2} = \boxed{137}$$

(b) $$\frac{\lambda_C}{r_e} = \frac{h}{mc}\frac{mc^2}{k_e e^2} = \frac{hc}{ke^2} = \boxed{\frac{2\pi}{\alpha}}$$

(c) $$\frac{a_0}{\lambda_C} = \frac{\hbar^2}{mk_e e^2}\frac{mc}{h} = \frac{1}{2\pi}\frac{\hbar c}{k_e e^2} = \frac{137}{2\pi} = \boxed{\frac{1}{2\pi\alpha}}$$

(d) $$\frac{1/R_H}{a_0} = \frac{1}{R_H a_0} = \frac{4\pi c\hbar^3}{mk_e^2 e^4}\frac{mk_e e^2}{\hbar^2} = 4\pi\frac{\hbar c}{k_e e^2} = \boxed{\frac{4\pi}{\alpha}}$$

42.68 $$\psi = \tfrac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$ $$\frac{\partial^2\psi}{\partial r^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2} - \frac{r}{4a_0}\right)$$

Substituting into Schrödinger's equation and dividing by ψ,

$$\frac{1}{a_0^2}\left(\frac{1}{2} - \frac{r}{4a_0}\right) = -\frac{2m}{\hbar^2}[E - U]\left(2 - \frac{r}{a_0}\right)$$

Now $$E - U = \frac{\hbar^2}{2m\,a_0^2}\left(\frac{1}{4}\right) - \frac{\left(ke^2/4a_0\right)\left(m/\hbar^2\right)}{\left(m/\hbar^2\right)} = -\frac{1}{4}\left(\frac{\hbar^2}{2m\,a_0^2}\right)$$

and $$\left(\frac{1}{a_0^2}\right)\left(\frac{1}{2} - \frac{r}{4a_0}\right) = \frac{1}{4a_0^2}\left(2 - \frac{r}{a_0}\right)$$ $\therefore \psi$ is a solution.

***42.69** The beam intensity is reduced by absorption of photons into atoms in the lower state. The number of transitions per time and per area is $-BN_l I(x) n\,dx/c$. The beam intensity is increased by stimulating emission in atoms in the upper state, with transition rate $+BN_u I(x) n\,dx/c$. The net rate of change in photon numbers per area is then $-B(N_l - N_u) I(x) n\,dx/c$.

Each photon has energy hf, so the net change in intensity is

$$dI(x) = -hfB(N_l - N_u) I(x) n\,dx/c = -hfB\Delta N\, I(x) n\,dx/c$$

Then, $\dfrac{dI(x)}{I(x)} = -\dfrac{hfB\Delta N\, n}{c}dx$ so $\displaystyle\int_{I_0}^{I(L)} \frac{dI(x)}{I(x)} = \int_{x=0}^{L}\left(-\frac{hfB\Delta N\, n}{c}\right)dx$

$$\ln[I(L)] - \ln[I_0] = \ln\left[\frac{I(L)}{I_0}\right] = -\frac{hfB\Delta N\, n}{c}(L-0)$$

$$I(L) = I_0\, e^{-hfB\Delta N\, nL/c} = I_0\, e^{-\alpha L}$$

This result is also expressed in problem 42.44 as $\dfrac{I(L)}{I_0} = G = e^{-\sigma(n_l - n_u)L} = e^{+\sigma(n_u - n_l)L}$

***42.70** (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\,\mathbf{i} + \frac{h}{\lambda}(-\mathbf{i}) = mv_f\,\mathbf{i} \qquad \text{so} \qquad v_f - v_i = -\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\ \text{s} = \Delta t$. Then,

$$a = \frac{v_f - v_i}{\Delta t} = -\frac{h}{m\lambda\,\Delta t} \sim -\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(10^{-25}\ \text{kg})(500\times10^{-9}\ \text{m})(10^{-8}\ \text{s})} \qquad \boxed{-10^6\ \text{m/s}^2}$$

(b) With constant average acceleration, $v_f^2 = v_i^2 + 2a(\Delta x)$

$$0 \sim (10^3\ \text{m/s})^2 + 2(-10^6\ \text{m/s}^2)\Delta x \quad \text{so} \quad \Delta x \sim \frac{(10^3\ \text{m/s})^2}{10^6\ \text{m/s}^2} \quad \boxed{\sim 1\ \text{m}}$$

Chapter 43 Solutions

43.1 (a) $F = \frac{q^2}{4\pi e_0 r^2} = \frac{(1.60\times10^{-19})^2(8.99\times10^9)}{(5.00\times10^{-10})^2}\text{ N} = \boxed{0.921\times10^{-9}\text{ N}}$ toward the other ion.

(b) $U = \frac{-q^2}{4\pi e_0 r} = -\frac{(1.60\times10^{-19})^2(8.99\times10^9)}{5.00\times10^{-10}}\text{ J} \approx \boxed{-2.88\text{ eV}}$

***43.2** We are told $\text{K} + \text{Cl} + 0.7\text{ eV} \rightarrow \text{K}^+ + \text{Cl}^-$

and $\text{Cl} + \text{e}^- \rightarrow \text{Cl}^- + 3.6\text{ eV}$

or $\text{Cl}^- \rightarrow \text{Cl} + \text{e}^- - 3.6\text{ eV}$

By substitution, $\text{K} + \text{Cl} + 0.7\text{ eV} \rightarrow \text{K}^+ + \text{Cl} + \text{e}^- - 3.6\text{ eV}$

$\text{K} + 4.3\text{ eV} \rightarrow \text{K}^+ + \text{e}^-$

or the ionization energy of potassium is $\boxed{4.3\text{ eV}}$

43.3 (a) Minimum energy of the molecule is found from

$$\frac{dU}{dr} = -12Ar^{-13} + 6Br^{-7} = 0, \quad \text{yielding } \boxed{r_0 = \left[\frac{2A}{B}\right]^{1/6}}$$

(b) $$E = U|_{r=\infty} - U|_{r=r_0} = 0 - \left[\frac{A}{4A^2/B^2} - \frac{B}{2A/B}\right] = -\left[\frac{1}{4} - \frac{1}{2}\right]\frac{B^2}{A} = \boxed{\frac{B^2}{4A}}$$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

(c) $$r_0 = \left[\frac{2\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\text{ eV}\cdot\text{m}^6}\right]^{1/6} = 7.42\times10^{-11}\text{ m} = \boxed{74.2\text{ pm}}$$

$$E = \frac{\left(1.488\times10^{-60}\text{ eV}\cdot\text{m}^6\right)^2}{4\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)} = \boxed{4.46\text{ eV}}$$

***43.4** At the boiling or condensation temperature, $k_BT \approx 10^{-3}\text{ eV} = 10^{-3}\left(1.6\times10^{-19}\text{ J}\right)$

$$T \approx \frac{1.6\times10^{-22}\text{ J}}{1.38\times10^{-23}\text{ J/K}} \quad \boxed{\sim 10\text{ K}}$$

43.5 $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{132.9(126.9)}{132.9 + 126.9}\left(1.66 \times 10^{-27} \text{ kg}\right) = 1.08 \times 10^{-25} \text{ kg}$$

$$I = \mu r^2 = (1.08 \times 10^{-25} \text{ kg})(0.127 \times 10^{-9} \text{ m})^2 = 1.74 \times 10^{-45} \text{ kg} \cdot \text{m}^2$$

(a) $$E = \tfrac{1}{2} I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{J(J+1)\hbar^2}{2I}$$

$J = 0$ gives $E = 0$

$J = 1$ gives $$E = \frac{\hbar^2}{I} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{4\pi^2 (1.74 \times 10^{-45} \text{ kg} \cdot \text{m}^2)} = 6.41 \times 10^{-24} \text{ J} = \boxed{40.0\ \mu\text{eV}}$$

$hf = 6.41 \times 10^{-24} \text{ J} - 0$ to $f = \boxed{9.66 \times 10^9 \text{ Hz}}$

(b) $$f = \frac{E_1}{h} = \frac{\hbar^2}{hI} = \frac{h}{4\pi^2 \mu r^2} \propto r^{-2}$$ $\boxed{\text{If } r \text{ is 10\% too small, } f \text{ is 20\% too large.}}$

43.6 $$hf = \Delta E = \frac{\hbar^2}{2I}\left[2(2+1)\right] - \frac{\hbar^2}{2I}\left[1(1+1)\right] = \frac{\hbar^2}{2I}(4)$$

$$I = \frac{4(h/2\pi)^2}{2hf} = \frac{h}{2\pi^2 f} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi^2 \left(2.30 \times 10^{11} \text{ Hz}\right)} = \boxed{1.46 \times 10^{-46} \text{ kg} \cdot \text{m}^2}$$

43.7 For the HCl molecule in the $J = 1$ rotational energy level, we are given $r_0 = 0.1275$ nm.

$$E_{\text{rot}} = \frac{\hbar^2}{2I} J(J+1)$$

Taking $J = 1$, we have $$E_{\text{rot}} = \frac{\hbar^2}{I} = \tfrac{1}{2} I\omega^2$$ or $$\omega = \sqrt{\frac{2\hbar^2}{I^2}} = \sqrt{2}\frac{\hbar}{I}$$

The moment of inertia of the molecule is given by Equation 43.3. $$I = \mu r_0^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r_0^2$$

$$I = \left[\frac{(1 \text{ u})(35 \text{ u})}{1 \text{ u} + 35 \text{ u}}\right] r_0^2 = (0.972 \text{ u})(1.66 \times 10^{-27} \text{ kg / u})(1.275 \times 10^{-10} \text{ m})^2 = 2.62 \times 10^{-47} \text{ kg} \cdot \text{m}^2$$

Therefore, $$\omega = \sqrt{2}\frac{\hbar}{I} = \sqrt{2}\,\frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2.62 \times 10^{-47} \text{ kg} \cdot \text{m}^2} = \boxed{5.69 \times 10^{12} \text{ rad / s}}$$

Goal Solution

An HCl molecule is excited to its first rotational-energy level, corresponding to $J = 1$. If the distance between its nuclei is 0.1275 nm, what is the angular speed of the molecule about its center of mass?

G: For a system as small as a molecule, we can expect the angular speed to be much faster than the few rad/s typical of everyday objects we encounter.

O: The rotational energy is given by the angular momentum quantum number, J. The angular speed can be calculated from this kinetic rotational energy and the moment of inertia of this one-dimensional molecule.

A: For the HCl molecule in the $J = 1$ rotational energy level, we are given $r_0 = 0.1275$ nm.

$$E_{rot} = \frac{\hbar}{2I} J(J+1) \quad \text{so with } J = 1, \quad E_{rot} = \frac{\hbar^2}{I} = \frac{1}{2} I\omega^2 \quad \text{and} \quad \omega = \sqrt{\frac{2\hbar^2}{I^2}} = \frac{\hbar\sqrt{2}}{I}$$

The moment of inertia of the molecule is given by: $I = \mu r_0^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r_0^2 = \left[\frac{(1\text{ u})(35\text{ u})}{1\text{ u} + 35\text{ u}}\right] r_0^2$

$$I = (0.972\text{ u})(1.66 \times 10^{-27}\text{ kg/u})(1.275 \times 10^{-10}\text{ m})^2 = 2.62 \times 10^{-47}\text{ kg}\cdot\text{m}^2$$

Therefore, $\omega = \sqrt{2}\frac{\hbar}{I} = \sqrt{2}\left(\frac{1.055 \times 10^{-34}\text{ J}\cdot\text{s}}{2.62 \times 10^{-47}\text{ kg}\cdot\text{m}^2}\right) = 5.69 \times 10^{12}\text{ rad/s}$

L: This angular speed is more than a billion times faster than the spin rate of a music CD, which rotates at 200 to 500 revolutions per minute, or $\omega = 20$ rad/s to 50 rad/s.

43.8 $I = m_1 r_1^2 + m_2 r_2^2$ where $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$

Then $r_1 = \frac{m_2 r_2}{m_1}$ so $\frac{m_2 r_2}{m_1} + r_2 = r$ and $r_2 = \frac{m_1 r}{m_1 + m_2}$

Also, $r_2 = \frac{m_1 r_1}{m_2}$ Thus, $r_1 + \frac{m_1 r_1}{m_2} = r$ and $r_1 = \frac{m_2 r}{m_1 + m_2}$

$$I = m_1 \frac{m_2^2 r^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2 r^2}{(m_1 + m_2)^2} = \frac{m_1 m_2 r^2 (m_2 + m_1)}{(m_1 + m_2)^2} = \frac{m_1 m_2 r^2}{m_1 + m_2} = \boxed{\mu r^2}$$

43.9 (a) $\mu = \frac{22.99(35.45)}{(22.99 + 35.45)}\left(1.66 \times 10^{-27}\text{ kg}\right) = 2.32 \times 10^{-26}\text{ kg}$

$$I = \mu r^2 = \left(2.32 \times 10^{-26}\text{ kg}\right)\left(0.280 \times 10^{-9}\text{ m}\right)^2 = \boxed{1.81 \times 10^{-45}\text{ kg}\cdot\text{m}^2}$$

(b) $\frac{hc}{\lambda} = \frac{\hbar^2}{2I} 2(2+1) - \frac{\hbar^2}{2I} 1(1+1) = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I} = \frac{2h^2}{4\pi^2 I}$

$$\lambda = \frac{c 4\pi^2 I}{2h} = \frac{\left(3.00 \times 10^8\text{ m/s}\right) 4\pi^2 \left(1.81 \times 10^{-45}\text{ kg}\cdot\text{m}^2\right)}{2\left(6.626 \times 10^{-34}\text{ J}\cdot\text{s}\right)} = \boxed{1.62\text{ cm}}$$

43.10 The energy of a rotational transition is $\Delta E = \left(\hbar^2/I\right)J$ where J is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know J from the data. However,

$$\Delta E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\lambda}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$

For each observed wavelength,

λ (mm)	ΔE (eV)
0.1204	0.01032
0.0964	0.01288
0.0804	0.01544
0.0690	0.01800
0.0604	0.02056

The $\Delta E's$ consistently increase by 0.00256 eV. $E_1 = \hbar^2/I = 0.00256\ \text{eV}$

and $$I = \frac{\hbar^2}{E_1} = \frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{(0.00256\ \text{eV})}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.72\times10^{-47}\ \text{kg}\cdot\text{m}^2}$$

For the HCl molecule, the internuclear radius is $r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{2.72\times10^{-47}}{1.62\times10^{-27}}}\ \text{m} = 0.130\ \text{nm}$

43.11 $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{35}{36}\times1.66\times10^{-27}\ \text{kg} = 1.61\times10^{-27}\ \text{kg}$$

$$\Delta E_{\text{vib}} = \hbar\sqrt{\frac{k}{\mu}} = \left(1.055\times10^{-34}\right)\sqrt{\frac{480}{1.61\times10^{-27}}} = 5.74\times10^{-20}\ \text{J} = \boxed{0.358\ \text{eV}}$$

43.12 (a) Minimum amplitude of vibration of HI is

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}hf:\ A = \sqrt{\frac{hf}{k}} = \sqrt{\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.69\times10^{13}\ /\ \text{s})}{320\ \text{N}\ /\ \text{m}}} = 1.18\times10^{-11}\ \text{m} = \boxed{0.0118\ \text{nm}}$$

(b) For HF, $$A = \sqrt{\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(8.72\times10^{13}\ /\ \text{s})}{970\ \text{N}\ /\ \text{m}}} = 7.72\times10^{-12}\ \text{m} = \boxed{0.00772\ \text{nm}}$$

Since HI has the smaller k, it is more weakly bound.

43.13 (a) The reduced mass of the O_2 is $\mu = \frac{(16\text{ u})(16\text{ u})}{(16\text{ u})+(16\text{ u})} = 8\text{ u} = 8\left(1.66\times10^{-27}\text{ kg}\right) = 1.33\times10^{-26}\text{ kg}$

The moment of inertia is then $I = \mu r^2 = \left(1.33\times10^{-26}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2 = 1.91\times10^{-46}\text{ kg}\cdot\text{m}^2$

The rotational energies are $E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.91\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}J(J+1)$

Thus $E_{\text{rot}} = \left(2.91\times10^{-23}\text{ J}\right)J(J+1)$

And for $J = 0, 1, 2,$ $E_{\text{rot}} = \boxed{0,\ 3.64\times10^{-4}\text{ eV},\ 1.09\times10^{-3}\text{ eV}}$

(b) $E_{\text{vib}} = \left(v+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{\mu}} = \left(v+\frac{1}{2}\right)\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\sqrt{\frac{1177\text{ N / m}}{8(1.66\times10^{-27}\text{ kg})}}$

$E_{\text{vib}} = \left(v+\frac{1}{2}\right)\left(3.14\times10^{-20}\text{ J}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \left(v+\frac{1}{2}\right)(0.196\text{ eV})$

For $v = 0, 1, 2,$ $\boxed{E_{\text{vib}} = 0.0982\text{ eV},\ 0.295\text{ eV},\ 0.491\text{ eV}}$

43.14 In Benzene, the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is $(0.110+0.100\text{ nm}) = 0.210\text{ nm}$ from the axis. Thus, $I = \Sigma mr^2$:

$$I = 6\left(1.99\times10^{-26}\text{ kg}\right)\left(0.110\times10^{-9}\text{ m}\right)^2 + 6\left(1.67\times10^{-27}\text{ kg}\right)\left(0.210\times10^{-9}\text{ m}\right)^2 = 1.89\times10^{-45}\text{ kg}\cdot\text{m}^2$$

The allowed rotational energies are then

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}J(J+1) = \left(2.95\times10^{-24}\text{ J}\right)J(J+1) = \left(18.4\times10^{-6}\text{ eV}\right)J(J+1)$$

$E_{\text{rot}} = \boxed{(18.4\ \mu\text{eV})J(J+1)\quad\text{where } J = 0, 1, 2, 3, \ldots}$

The first five of these allowed energies are: $E_{\text{rot}} = 0,\ 36.9\ \mu\text{eV},\ 111\ \mu\text{eV},\ 221\ \mu\text{eV},\text{ and } 369\ \mu\text{eV}$

43.15 $hf = \frac{h^2}{4\pi^2 I}J$ where the rotational transition is from $J-1$ to J,

where $f = 6.42\times10^{13}\text{ Hz}$ and $I = 1.46\times10^{-46}\text{ kg}\cdot\text{m}^2$ from Example 43.1.

$$J = \frac{4\pi^2 If}{h} = \frac{4\pi^2(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2)(6.42\times10^{13}\text{ / s})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{558}$$

***43.16** The emission energies are the same as the absorption energies, but the final state must be below $(v=1,\ J=0)$. The transition must satisfy $\Delta J = \pm 1$, so it must end with $J=1$. To be lower in energy, it must be $(v=0,\ J=1)$. The emitted photon energy is therefore

$$hf_{\text{photon}} = \left(E_{\text{vib}}\big|_{v=1} + E_{\text{rot}}\big|_{J=0}\right) - \left(E_{\text{vib}}\big|_{v=0} + E_{\text{rot}}\big|_{J=1}\right) = \left(E_{\text{vib}}\big|_{v=1} - E_{\text{vib}}\big|_{v=0}\right) - \left(E_{\text{rot}}\big|_{J=1} - E_{\text{rot}}\big|_{J=0}\right)$$

$$hf_{\text{photon}} = hf_{\text{vib}} - hf_{\text{rot}}$$

Thus, $f_{\text{photon}} = f_{\text{vib}} - f_{\text{rot}} = 6.42\times10^{13}\ \text{Hz} - 1.15\times10^{11}\ \text{Hz} = \boxed{6.41\times10^{13}\ \text{Hz}}$

***43.17** The moment of inertia about the molecular axis is $I_x = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 = \frac{4}{5}m\left(2.00\times10^{-15}\ \text{m}\right)^2$

The moment of inertia about a perpendicular axis is $I_y = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{m}{2}\left(2.00\times10^{-10}\ \text{m}\right)^2$

The allowed rotational energies are $E_{\text{rot}} = \left(\hbar^2/2I\right)J(J+1)$, so the energy of the first excited state is $E_1 = \hbar^2/I$. The ratio is therefore

$$\frac{E_{1,x}}{E_{1,y}} = \frac{\left(\hbar^2/I_x\right)}{\left(\hbar^2/I_y\right)} = \frac{I_y}{I_x} = \frac{\frac{1}{2}m\left(2.00\times10^{-10}\ \text{m}\right)^2}{\frac{4}{5}m\left(2.00\times10^{-15}\ \text{m}\right)} = \frac{5}{8}\left(10^5\right)^2 = \boxed{6.25\times10^9}$$

***43.18** Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\dfrac{10^{-4}\ \text{m}}{0.261\times10^{-9}\ \text{m}}\right)^3$ $\boxed{\sim 10^{17}}$

This number of salt crystals would have volume $\left(10^{-4}\ \text{m}\right)^3\left(\dfrac{10^{-4}\ \text{m}}{0.261\times10^{-9}\ \text{m}}\right)^3$ $\boxed{\sim 10^5\ \text{m}^3}$

If it is cubic, it has edge length 40 m.

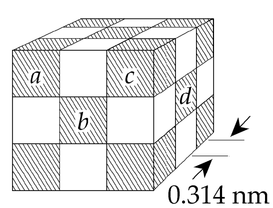
43.19 $$U = -\frac{\alpha k_e e^2}{r_0}\left(1-\frac{1}{m}\right) = -(1.7476)(8.99\times10^9)\frac{(1.60\times10^{-19})^2}{(0.281\times10^{-9})}\left(1-\frac{1}{8}\right) = -1.25\times10^{-18}\ \text{J} = \boxed{-7.84\ \text{eV}}$$

43.20 Visualize a K^+ ion at the center of each shaded cube, a Cl^- ion at the center of each white one.

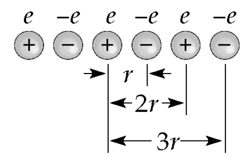
The distance ab is $\sqrt{2}(0.314\ \text{nm}) = \boxed{0.444\ \text{nm}}$

Distance ac is $2(0.314\ \text{nm}) = \boxed{0.628\ \text{nm}}$

Distance ad is $\sqrt{2^2 + (\sqrt{2})^2}(0.314\ \text{nm}) = \boxed{0.769\ \text{nm}}$



43.21 $U = -\dfrac{k_e e^2}{r} - \dfrac{k_e e^2}{r} + \dfrac{k_e e^2}{2r} + \dfrac{k_e e^2}{2r} - \dfrac{k_e e^2}{3r} - \dfrac{k_e e^2}{3r} + \dfrac{k_e e^2}{4r} + \dfrac{k_e e^2}{4r} - \ \ldots \ = -\dfrac{2k_e e^2}{r}\left(1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ \ldots\right)$



But, $\ln(1+x) = 1 - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ \ldots$

so, $U = -\dfrac{2k_e e^2}{r}\ln 2$, or $\boxed{U = -k_e \alpha \dfrac{e^2}{r} \quad \text{where} \quad \alpha = 2\ln 2}$

43.22 $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n_e}{8\pi}\right)^{2/3} = \left[\dfrac{(6.625\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{J / eV})}\right](3/8\pi)^{2/3} n^{2/3}$

$E_F = (3.65\times10^{-19})n^{2/3}$ eV with n measured in electrons/m^3

43.23 The density of conduction electrons n is given by $E_F = \dfrac{h^2}{2m}\left(\dfrac{3n_e}{8\pi}\right)^{2/3}$

or $n_e = \dfrac{8\pi}{3}\left(\dfrac{2mE_F}{h^2}\right)^{3/2} = \dfrac{8\pi}{3}\dfrac{\left[2(9.11\times10^{-31}\ \text{kg})(5.48)(1.60\times10^{-19}\ \text{J})\right]^{3/2}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^3} = 5.80\times10^{28}\ \text{m}^{-3}$

The number-density of silver atoms is

$$n_{Ag} = \left(10.6\times10^3\ \text{kg / m}^3\right)\left(\dfrac{1\ \text{atom}}{108\ \text{u}}\right)\left(\dfrac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = 5.91\times10^{28}\ \text{m}^{-3}$$

So an average atom contributes $\dfrac{5.80}{5.91} =$ $\boxed{0.981 \text{ electron to the conduction band}}$

43.24 (a) $\frac{1}{2}mv^2 = 7.05$ eV

$$v = \sqrt{\dfrac{2(7.05\ \text{eV})(1.60\times10^{-19}\ \text{J / eV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.57\times10^6\ \text{m/s}}$$

(b) $\boxed{\text{Larger than } 10^{-4}\ \text{m/s by ten orders of magnitude.}}$ However, the energy of an electron at room temperature is typically $k_B T = \frac{1}{40}$ eV.

43.25 For sodium, $M = 23.0\ \text{g/mol}$ and $\rho = 0.971\ \text{g/cm}^3$.

(a) $$n_e = \frac{N_A \rho}{M} = \frac{\left(6.02\times10^{23}\ \text{electrons/mol}\right)\left(0.971\ \text{g/cm}^3\right)}{23.0\ \text{g/mol}}$$

$$n_e = 2.54\times10^{22}\ \text{electrons/cm}^3 = \boxed{2.54\times10^{28}\ \text{electrons/m}^3}$$

(b) $$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)}\left[\frac{3\left(2.54\times10^{28}\ \text{m}^{-3}\right)}{8\pi}\right]^{2/3} = 5.05\times10^{-19}\ \text{J} = \boxed{3.15\ \text{eV}}$$

(c) $$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2\left(5.05\times10^{-19}\ \text{J}\right)}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.05\times10^6\ \text{m/s}}$$

***43.26** The melting point of silver is 1234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_B T}{E_F} = \frac{\left(1.38\times10^{-23}\ \text{J/K}\right)(1234\ \text{K})}{(5.48\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} \approx \boxed{2\%}$$

43.27 Taking $E_F = 5.48\ \text{eV}$ for sodium at 800 K,

$$f = \left[e^{(E-E_F)/k_B T} + 1\right]^{-1} = 0.950$$

$$e^{(E-E_F)/k_B T} = (1 / 0.950) - 1 = 0.0526$$

$$\frac{E-E_F}{k_B T} = \ln(0.0526) = -2.94$$

$$E - E_F = -2.94\frac{(1.38\times10^{-23})(800)\text{J}}{1.60\times10^{-19}\ \text{J / eV}} = -0.203\ \text{eV} \qquad \text{or} \qquad \boxed{E = 5.28\ \text{eV}}$$

Goal Solution

Calculate the energy of a conduction electron in silver at 800 K if the probability of finding an electron in that state is 0.950. The Fermi energy is 5.48 eV at this temperature.

G: Since there is a 95% probability of finding the electron in this state, its energy should be slightly less than the Fermi energy, as indicated by the graph in Figure 43.21.

O: The electron energy can be found from the Fermi-Dirac distribution function.

A: Taking $E_F = 5.48$ eV for silver at 800 K, and given $f(E) = 0.950$, we find

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} = 0.950 \qquad \text{or} \qquad e^{(E-E_F)/k_BT} = \frac{1}{0.950} - 1 = 0.05263$$

$$\frac{E - E_F}{k_BT} = \ln(0.05263) = -2.944 \qquad \text{so} \qquad E - E_F = -2.944 k_BT = -2.944\left(1.38\times10^{-23}\text{ J / K}\right)(800\text{ K})$$

$$E = E_F - 3.25\times10^{-20}\text{ J} = 5.48\text{ eV} - 0.203\text{ eV} = 5.28\text{ eV}$$

L: As expected, the energy of the electron is slightly less than the Fermi energy, which is about 5 eV for most metals. There is very little probability of finding an electron significantly above the Fermi energy in a metal.

43.28 $d = 1.00$ mm, so $V = \left(1.00\times10^{-3}\text{ m}\right)^3 = 1.00\times10^{-9}\text{ m}^3$

The **density of states** is $g(E) = CE^{1/2} = \dfrac{8\sqrt{2}\,\pi m^{3/2}}{h^3}E^{1/2}$

or

$$g(E) = \frac{8\sqrt{2}\,\pi\left(9.11\times10^{-31}\text{ kg}\right)^{3/2}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3}\sqrt{(4.00\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$g(E) = 8.50\times10^{46}\text{ m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\text{ m}^{-3}\cdot\text{ eV}^{-1}$$

So, the total number of electrons is

$$N = [g(E)](\Delta E)V = \left(1.36\times10^{28}\text{ m}^{-3}\cdot\text{ eV}^{-1}\right)(0.0250\text{ eV})\left(1.00\times10^{-9}\text{ m}^3\right) = \boxed{3.40\times10^{17}\text{ electrons}}$$

43.29 $E_{\text{av}} = \dfrac{1}{n_e}\displaystyle\int_0^\infty EN(E)\,dE$

At $T = 0$, $N(E) = 0$ for $E > E_{\text{F}}$;

Since $f(E) = 1$ for $E < E_{\text{F}}$ and $f(E) = 0$ for $E > E_{\text{F}}$, we can take $N(E) = CE^{1/2}$

$$E_{\text{av}} = \frac{1}{n_e}\int_0^{E_{\text{F}}} CE^{3/2}\,dE = \frac{C}{n_e}\int_0^{E_{\text{F}}} E^{3/2}\,dE = \frac{2C}{5n_e}E^{5/2}$$

But from Equation 43.24, $\dfrac{C}{n_e} = \dfrac{3}{2}E_{\text{F}}^{-3/2}$, so that $E_{\text{av}} = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{2}E_{\text{F}}^{-3/2}\right)E_{\text{F}}^{5/2} = \boxed{\dfrac{3}{5}E_{\text{F}}}$

43.30 Consider first the wave function in x. At $x = 0$ and $x = L$, $\psi = 0$.

Therefore, $\sin k_x L = 0$ and $k_x L = \pi, 2\pi, 3\pi, \ldots$
Similarly, $\sin k_y L = 0$ and $k_y L = \pi, 2\pi, 3\pi, \ldots$
$\sin k_z L = 0$ and $k_z L = \pi, 2\pi, 3\pi, \ldots$

$$\psi = A\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)\sin\left(\frac{n_z \pi z}{L}\right)$$

From $\dfrac{d^2\psi}{dx^2}+\dfrac{d^2\psi}{dy^2}+\dfrac{d^2\psi}{dz^2}=\dfrac{2m_e}{\hbar^2}(U-E)\psi$, we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2}-\frac{n_y^2\pi^2}{L^2}-\frac{n_z^2\pi^2}{L^2}\right)\psi=\frac{2m_e}{\hbar^2}(-E)\psi$$

$$\boxed{E=\frac{\hbar^2\pi^2}{2m_eL^2}(n_x^2+n_y^2+n_z^2) \qquad n_x, n_y, n_z = 1, 2, 3, \ldots}$$

Outside the box we require $\psi = 0$.

The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E=\dfrac{3\hbar^2\pi^2}{2m_eL^2}$

43.31 (a) The density of states at energy E is $g(E)=CE^{1/2}$

Hence, the required ratio is $\dfrac{g(8.50\text{ eV})}{g(7.00\text{ eV})}=\dfrac{C(8.50)^{1/2}}{C(7.00)^{1/2}}=\boxed{1.10}$

(b) From Eq. 43.22, the number of occupied states having energy E is $N(E)=\dfrac{CE^{1/2}}{e^{(E-E_F)/k_BT}+1}$

Hence, the required ratio is $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})}=\dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{e^{(7.00-7.00)/k_BT}+1}{e^{(8.50-7.00)/k_BT}+1}\right]$

At $T = 300$ K, $k_BT = 4.14\times10^{-21}\text{ J} = 0.0259\text{ eV}$, $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})}=\dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{2.00}{e^{(1.50)/0.0259}+1}\right]$

And $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})}=\boxed{1.55\times10^{-25}}$

Comparing this result with that from part (a), we conclude that very few states with $E > E_F$ are occupied.

43.32 (a) $E_g = 1.14$ eV for Si

$hf = 1.14\text{ eV} = (1.14\text{ eV})(1.60\times10^{-19}\text{ J/eV}) = 1.82\times10^{-19}\text{ J}$ so $f \geq \boxed{2.75\times10^{14}\text{ Hz}}$

(b) $c = \lambda f$; $\lambda=\dfrac{c}{f}=\dfrac{3.00\times10^{8}\text{ m/s}}{2.75\times10^{14}\text{ Hz}}=1.09\times10^{-6}\text{ m} = \boxed{1.09\ \mu\text{m}}$ (in the infrared region)

43.33 Photons of energy greater than 2.42 eV will be absorbed. This means wavelength shorter than

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{2.42\times1.60\times10^{-19}\ \text{J}} = 514\ \text{nm}$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

43.34 $$E_g = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m / s})}{650\times10^{-9}\ \text{m}}\ \text{J} \approx \boxed{1.91\ \text{eV}}$$

43.35 If $\lambda \le 1.00\times10^{-6}\ \text{m}$, then photons of sunlight have energy

$$E \ge \frac{hc}{\lambda_{\text{max}}} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{1.00\times10^{-6}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 1.24\ \text{eV}$$

Thus, the energy gap for the collector material should be $\boxed{E_g \le 1.24\ \text{eV}}$. Since Si has an energy gap $E_g \approx 1.14\ \text{eV}$, it will absorb radiation of this energy and greater. Therefore, $\boxed{\text{Si is acceptable}}$ as a material for a solar collector.

Goal Solution

Most solar radiation has a wavelength of 1 μm or less. What energy gap should the material in a solar cell have in order to absorb this radiation? Is silicon appropriate (see Table 43.5)?

G: Since most photovoltaic solar cells are made of silicon, this semiconductor seems to be an appropriate material for these devices.

O: To absorb the longest-wavelength photons, the energy gap should be no larger than the photon energy.

A: The minimum photon energy is

$$hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m / s}\right)}{10^{-6}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 1.24\ \text{eV}$$

Therefore, the energy gap in the absorbing material should be smaller than 1.24 eV.

L: So silicon, with gap of 1.14 eV < 1.24 eV, is an appropriate material for absorbing solar radiation.

***43.36** If the photon energy is 5.5 eV or higher, the diamond window will absorb. Here,

$$(hf)_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = 5.50\ \text{eV}: \qquad \lambda_{\text{min}} = \frac{hc}{5.5\ \text{eV}} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(5.5\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}$$

$$\lambda_{\text{min}} = 2.26\times10^{-7}\ \text{m} = \boxed{226\ \text{nm}}$$

43.37 $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$ Thus, $e^{e(\Delta V)/k_BT} = 1 + I/I_0$

and $\Delta V = \frac{k_BT}{e}\ln\left(1 + I/I_0\right)$

At $T = 300$ K, $\Delta V = \frac{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}{1.60\times10^{-19}\text{ C}}\ln\left(1+\frac{I}{I_0}\right) = (25.9\text{ mV})\ln\left(1+\frac{I}{I_0}\right)$

(a) If $I = 9.00I_0$, $\Delta V = (25.9\text{ mV})\ln(10.0) = \boxed{59.5\text{ mV}}$

(b) If $I = -0.900I_0$, $\Delta V = (25.9\text{ mV})\ln(0.100) = \boxed{-59.5\text{ mV}}$

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

43.38 The voltage across the diode is about 0.6 V. The voltage drop across the resistor is $(0.025\text{ A})(150\ \Omega) = 3.75$ V. Thus, $\mathcal{E} - 0.6\text{ V} - 3.8\text{ V} = 0$ and $\mathcal{E} = \boxed{4.4\text{ V}}$

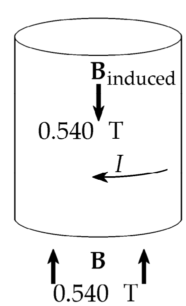
***43.39** First, we evaluate I_0 in $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$, given that $I = 200$ mA when $\Delta V = 100$ mV and $T = 300$ K.

$$\frac{e(\Delta V)}{k_BT} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.100\text{ V})}{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})} = 3.86 \text{ so} \qquad I_0 = \frac{I}{e^{e(\Delta V)/k_BT} - 1} = \frac{200\text{ mA}}{e^{3.86} - 1} = 4.28\text{ mA}$$

If $\Delta V = -100$ mV, $\frac{e(\Delta V)}{k_BT} = -3.86$; and the current will be

$$I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right) = (4.28\text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19\text{ mA}}$$

43.40 (a) $\boxed{\text{See the figure at right.}}$



(b) For a surface current around the outside of the cylinder as shown,

$$B = \frac{N\mu_0 I}{\ell} \quad\text{or}\quad NI = \frac{B\ell}{\mu_0} = \frac{(0.540\text{ T})(2.50\times10^{-2}\text{ m})}{(4\pi\times10^{-7})\text{ T}\cdot\text{m / A}} = \boxed{10.7\text{ kA}}$$

43.41 By Faraday's law (Equation 32.1), $\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} = A\frac{\Delta B}{\Delta t}$.

Thus, $\Delta I = \frac{A(\Delta B)}{L} = \frac{\pi(0.0100 \text{ m})^2(0.0200 \text{ T})}{3.10\times10^{-8} \text{ H}} = \boxed{203 \text{ A}}$

The direction of the induced current is such as to maintain the B–field through the ring.

Goal Solution

Determine the current generated in a superconducting ring of niobium metal 2.00 cm in diameter if a 0.0200-T magnetic field in a direction perpendicular to the ring is suddenly decreased to zero. The inductance of the ring is 3.10×10^{-8} H.

G: The resistance of a superconductor is zero, so the current is limited only by the change in magnetic flux and self-inductance. Therefore, unusually large currents (greater than 100 A) are possible.

O: The change in magnetic field through the ring will induce an emf according to Faraday's law of induction. Since we do not know how fast the magnetic field is changing, we must use the ring's inductance and the geometry of the ring to calculate the magnetic flux, which can then be used to find the current.

A: From Faraday's law (Eq. 31.1), we have

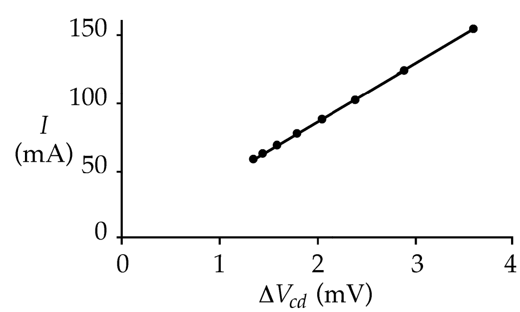
$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = L\frac{\Delta I}{\Delta t} \quad \text{or} \quad \Delta I = \frac{A\Delta B}{L} = \frac{\pi(0.0100 \text{ m})^2(0.0200 \text{ T})}{3.10\times10^{-8} \text{ H}} = 203 \text{ A}$$

The current is directed so as to produce its own magnetic field in the direction of the original field.

L: This induced current should remain constant as long as the ring is superconducting. If the ring failed to be a superconductor (e.g. if it warmed above the critical temperature), the metal would have a non-zero resistance, and the current would quickly drop to zero. It is interesting to note that we were able to calculate the current in the ring without knowing the emf. In order to calculate the emf, we would need to know how quickly the magnetic field goes to zero.

43.42 (a) $\Delta V = IR$

If $R = 0$, then $\Delta V = 0$, even when $I \neq 0$.



(b) The graph shows a direct proportionality.

$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155-57.8) \text{ mA}}{(3.61-1.356) \text{ mV}} = 43.1\ \Omega^{-1}$$

$R = \boxed{0.0232\ \Omega}$

(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

***43.43** (a) Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ where $\mu_{12} = \frac{(12\text{ u})(16\text{ u})}{12\text{ u} + 16\text{ u}} = 6.86\text{ u}$ and $\mu_{14} = \frac{(14\text{ u})(16\text{ u})}{14\text{ u} + 16\text{ u}} = 7.47\text{ u}$

Therefore,

$$f_{14} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)} = f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}} = \left(6.42\times10^{13}\text{ Hz}\right)\sqrt{\frac{6.86\text{ u}}{7.47\text{ u}}} = \boxed{6.15\times10^{13}\text{ Hz}}$$

(b) The equilibrium distance is the same for both molecules.

$$I_{14} = \mu_{14}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)I_{12}$$

$$I_{14} = \left(\frac{7.47\text{ u}}{6.86\text{ u}}\right)\left(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2\right) = \boxed{1.59\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(c) The molecule can move to the $(v=1, J=9)$ state or to the $(v=1, J=11)$ state. The energy it can absorb is either

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 9(9+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right],$$

or $$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 11(11+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right].$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} - 10\hbar/(2\pi I_{14})} \quad \text{or} \quad \lambda = \frac{c}{f_{14} + 11\hbar/(2\pi I_{14})}$$

These are: $$\lambda = \frac{3.00\times10^8\text{ m/s}}{6.15\times10^{13}\text{ Hz} - \frac{10\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.96\ \mu\text{m}}$$

and $$\lambda = \frac{3.00\times10^8\text{ m/s}}{6.15\times10^{13}\text{ Hz} + \frac{11\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.79\ \mu\text{m}}$$

43.44 For the N_2 molecule, $k = 2297\ \text{N/m}$, $m = 2.32 \times 10^{-26}$ kg, $r = 1.20 \times 10^{-10}$ m, $\mu = m/2$

$\omega = \sqrt{k/\mu} = 4.45 \times 10^{14}$ rad/s, $I = \mu r^2 = (1.16 \times 10^{-26}\ \text{kg})(1.20 \times 10^{-10}\ \text{m})^2 = 1.67 \times 10^{-46}\ \text{kg} \cdot \text{m}^2$

For a rotational state sufficient to allow a transition to the first exited vibrational state,

$$\frac{\hbar^2}{2I} J(J+1) = \hbar\omega \qquad \text{so} \qquad J(J+1) = \frac{2I\omega}{\hbar} = \frac{2(1.67 \times 10^{-46})(4.45 \times 10^{14})}{1.055 \times 10^{-34}} = 1410$$

Thus $\boxed{J = 37}$

43.45 $\Delta E_{\text{max}} = 4.5\ \text{eV} = \left(v + \frac{1}{2}\right)\hbar\omega$ so $\dfrac{(4.5\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})}{(1.055 \times 10^{-34}\ \text{J} \cdot \text{s})(8.28 \times 10^{14}\ \text{s}^{-1})} \geq \left(v + \frac{1}{2}\right)$

$8.25 > 7.5$ $\boxed{v = 7}$

43.46 With 4 van der Waal bonds per atom pair or 2 electrons per atom, the total energy of the solid is

$$E = 2(1.74 \times 10^{-23}\ \text{J/atom})\left(\frac{6.02 \times 10^{23}\ \text{atoms}}{4.00\ \text{g}}\right) = \boxed{5.23\ \text{J/g}}$$

43.47 The total potential energy is given by Equation 43.16: $U_{\text{total}} = -\alpha \dfrac{k_e e^2}{r} + \dfrac{B}{r^m}$

The total potential energy has its minimum value U_0 at the equilibrium spacing, $r = r_0$. At this point, $\left.dU/dr\right|_{r=r_0} = 0$,

or $$\left.\frac{dU}{dr}\right|_{r=r_0} = \left.\frac{d}{dr}\left(-\alpha \frac{k_e e^2}{r} + \frac{B}{r^m}\right)\right|_{r=r_0} = \alpha \frac{k_e e^2}{r_0^2} - \frac{mB}{r_0^{m+1}} = 0$$

Thus, $$B = \alpha \frac{k_e e^2}{m} r_0^{m-1}$$

Substituting this value of B into U_{total}, $$U_0 = -\alpha \frac{k_e e^2}{r_0} + \alpha \frac{k_e e^2}{m} r_0^{m-1}\left(\frac{1}{r_0^m}\right) = \boxed{-\alpha \frac{k_e e^2}{r_0}\left(1 - \frac{1}{m}\right)}$$

***43.48** Suppose it is a harmonic-oscillator potential well. Then, $\frac{1}{2}hf + 4.48\ \text{eV} = \frac{3}{2}hf + 3.96\ \text{eV}$ is the depth of the well below the dissociation point. We see $hf = 0.520$ eV, so the depth of the well is

$$\tfrac{1}{2}hf + 4.48\ \text{eV} = \tfrac{1}{2}(0.520\ \text{eV}) + 4.48\ \text{eV} = \boxed{4.74\ \text{eV}}$$

***43.49** (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at $3Ax_0^{-2}=B$.

$$x_0=\sqrt{\frac{3A}{B}}=\sqrt{\frac{3\left(0.150\text{ eV}\cdot\text{nm}^3\right)}{3.68\text{ eV}\cdot\text{nm}}}=\boxed{0.350\text{ nm}}$$

(b) The depth of the well is given by $U_0=U|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}$

$$U_0=U|_{x=x_0}=-\frac{2B^{3/2}}{3^{3/2}A^{1/2}}=-\frac{2(3.68\text{ eV}\cdot\text{nm})^{3/2}}{3^{3/2}\left(0.150\text{ eV}\cdot\text{nm}^3\right)^{1/2}}=\boxed{-7.02\text{ eV}}$$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that $\left.\frac{dF_x}{dx}\right|_{x=x_m}=0$

Thus, $\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ so that $x_m=\left(\frac{6A}{B}\right)^{1/2}$

Then $$F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=-\frac{B^2}{12A}=-\frac{(3.68\text{ eV}\cdot\text{nm})^2}{12\left(0.150\text{ eV}\cdot\text{nm}^3\right)}$$

or $$F_{\max}=-7.52\ \frac{\text{eV}}{\text{nm}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)=-1.20\times10^{-9}\text{ N}=\boxed{-1.20\text{ nN}}$$

43.50 (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at

$3Ax_0^{-2}=B$ or $\boxed{x_0=\sqrt{3A/B}}$

(b) The depth of the well is given by $U_0=U|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}=\boxed{-2\sqrt{\frac{B^3}{27A}}}$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that

$\left.\frac{dF_x}{dx}\right|_{x=x_m}=\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ then $F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=\boxed{-\frac{B^2}{12A}}$

***43.51** (a) At equilibrium separation, $r = r_e$, $\left.\frac{dU}{dr}\right|_{r=r_e} = -2aB\left[e^{-a(r_e - r_0)} - 1\right]e^{-a(r_e - r_0)} = 0$

We have neutral equilibrium as $r_e \to \infty$ and stable equilibrium at $e^{-a(r_e - r_0)} = 1$,

or $r_e = \boxed{r_0}$

(b) At $r = r_0$, $U = 0$. As $r \to \infty$, $U \to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point:

$$U(r) \approx U(r_0) + \left.\frac{dU}{dr}\right|_{r=r_0}(r - r_0) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}(r - r_0)^2$$

$$U(r) \approx 0 + 0 + \tfrac{1}{2}(-2Ba)\left[-ae^{-2(r-r_0)} - ae^{-(r-r_0)}\left(e^{-2(r-r_0)} - 1\right)\right]_{r=r_0}(r - r_0)^2 \approx Ba^2(r - r_0)^2$$

This is of the form $\tfrac{1}{2}kx^2 = \tfrac{1}{2}k(r - r_0)^2$

for a simple harmonic oscillator with $k = 2Ba^2$

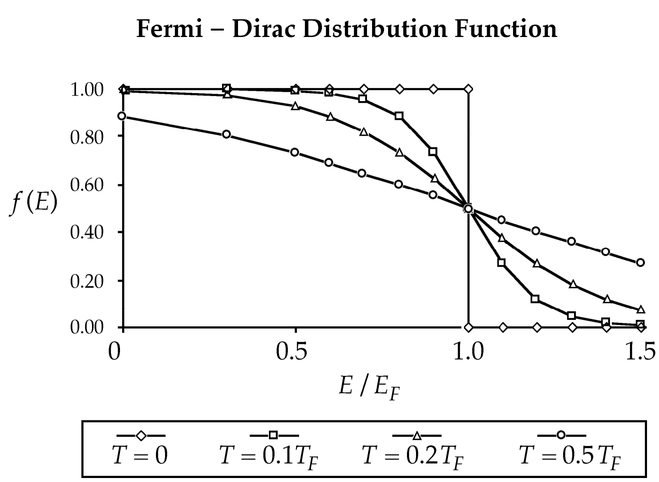
Then the molecule vibrates with frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{a}{2\pi}\sqrt{\frac{2B}{\mu}} = \boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$

(d) The zero-point energy is $\tfrac{1}{2}\hbar\omega = \tfrac{1}{2}hf = \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$

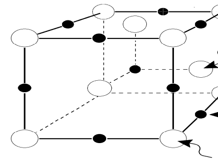
Therefore, to dissociate the molecule in its ground state requires energy $\boxed{B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$

43.52

	$T=0$		$T=0.1T_F$		$T=0.2T_F$		$T=0.5T_F$	
E/E_F	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$
0	$e^{-\infty}$	1.00	$e^{-10.0}$	1.000	$e^{-5.00}$	0.993	$e^{-2.00}$	0.881
0.500	$e^{-\infty}$	1.00	$e^{-5.00}$	0.993	$e^{-2.50}$	0.924	$e^{-1.00}$	0.731
0.600	$e^{-\infty}$	1.00	$e^{-4.00}$	0.982	$e^{-2.00}$	0.881	$e^{-0.800}$	0.690
0.700	$e^{-\infty}$	1.00	$e^{-3.00}$	0.953	$e^{-1.50}$	0.818	$e^{-0.600}$	0.646
0.800	$e^{-\infty}$	1.00	$e^{-2.00}$	0.881	$e^{-1.00}$	0.731	$e^{-0.400}$	0.599
0.900	$e^{-\infty}$	1.00	$e^{-1.00}$	0.731	$e^{-0.500}$	0.622	$e^{-0.200}$	0.550
1.00	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500
1.10	$e^{+\infty}$	0.00	$e^{1.00}$	0.269	$e^{0.500}$	0.378	$e^{0.200}$	0.450
1.20	$e^{+\infty}$	0.00	$e^{2.00}$	0.119	$e^{1.00}$	0.269	$e^{0.400}$	0.401
1.30	$e^{+\infty}$	0.00	$e^{3.00}$	0.0474	$e^{1.50}$	0.182	$e^{0.600}$	0.354
1.40	$e^{+\infty}$	0.00	$e^{4.00}$	0.0180	$e^{2.00}$	0.119	$e^{0.800}$	0.310
1.50	$e^{+\infty}$	0.00	$e^{5.00}$	0.00669	$e^{2.50}$	0.0759	$e^{1.00}$	0.269



43.53 (a) There are 6 Cl^- ions at distance $r = r_0$. The contribution of these ions to the electrostatic potential energy is $-6k_e e^2/r_0$.



There are 12 Na^+ ions at distance $r = \sqrt{2}\,r_0$. Their contribution to the electrostatic potential energy is $+12k_e e^2/\sqrt{2}\,r_0$. Next, there are 8 Cl^- ions at distance $r = \sqrt{3}\,r_0$. These contribute a term of $-8k_e e^2/\sqrt{3}\,r_0$ to the electrostatic potential energy.

To three terms, the electrostatic potential energy is:

$$U = \left(-6 + \frac{12}{\sqrt{2}} - \frac{8}{\sqrt{3}}\right)\frac{k_e e^2}{r_0} = -2.13\,\frac{k_e e^2}{r_0} \quad \text{or} \quad \boxed{U = -\alpha\frac{k_e e^2}{r_0} \text{ with } \alpha = 2.13}$$

(b) The fourth term consists of 6 Na^+ at distance $r = 2r_0$. Thus, to four terms,

$$U = (-2.13 + 3)\frac{k_e e^2}{r_0} = 0.866\frac{k_e e^2}{r_0}$$

So we see that the electrostatic potential energy is not even attractive to 4 terms, and that the infinite series $\boxed{\text{does not converge rapidly}}$ when groups of atoms corresponding to nearest neighbors, next-nearest neighbors, etc. are added together.

Chapter 44 Solutions

***44.1** An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim 10^{28}\text{ protons}}\quad\text{and}\quad\boxed{\sim 10^{28}\text{ neutrons}}$$

The electron number is precisely equal to the proton number, $\boxed{\sim 10^{28}\text{ electrons}}$

44.2 $\frac{1}{2}mv^2 = q(\Delta V)$ and $\frac{mv^2}{r} = qvB \;\Rightarrow\; 2m(\Delta V) = qr^2B^2$

$$r = \sqrt{\frac{2m(\Delta V)}{qB^2}} = \left[\frac{2(1000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}\right]^{1/2}\sqrt{m}$$

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{m}$$

(a) For ^{12}C: $m = 12$ u and

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{12(1.66\times10^{-27}\text{ kg})} = 0.0789\text{ m} = \boxed{7.89\text{ cm}}$$

For ^{13}C: $$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{13(1.66\times10^{-27}\text{ kg})} = 0.0821\text{ m} = \boxed{8.21\text{ cm}}$$

(b) With $r_1 = \sqrt{\frac{2m_1(\Delta V)}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2(\Delta V)}{qB^2}}$,

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$.

$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$ and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$ so they do agree.

***44.3** (a) $F = k_e \frac{Q_1 Q_2}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2)(6)(1.60 \times 10^{-19} \text{ C})^2}{(1.00 \times 10^{-14} \text{ m})^2} = \boxed{27.6 \text{ N}}$

(b) $a = \frac{F}{m} = \frac{27.6 \text{ N}}{6.64 \times 10^{-27} \text{ kg}} = \boxed{4.17 \times 10^{27} \text{ m/s}^2}$ away from the nucleus..

(c) $U = k_e \frac{Q_1 Q_2}{r} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2)(6)(1.60 \times 10^{-19} \text{ C})^2}{(1.00 \times 10^{-14} \text{ m})} = 2.76 \times 10^{-13} \text{ J} = \boxed{1.73 \text{ MeV}}$

44.4 $E_\alpha = 7.70$ MeV

(a) $d_{\min} = \frac{4k_e Ze^2}{mv^2} = \frac{2k_e Ze^2}{E_\alpha} = \frac{2(8.99 \times 10^9)(79)(1.60 \times 10^{-19})^2}{7.70(1.60 \times 10^{-13})} = 29.5 \times 10^{-15} \text{ m} = \boxed{29.5 \text{ fm}}$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626 \times 10^{-34}}{\sqrt{2(6.64 \times 10^{-27})7.70(1.60 \times 10^{-13})}} = 5.18 \times 10^{-15} \text{ m} = \boxed{5.18 \text{ fm}}$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

44.5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_e qQ}{r_{\min}}$$

$$r_{\min} = \frac{k_e qQ}{K_i} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(2)(79)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{(0.500 \text{ MeV})\left(1.60 \times 10^{-13} \text{ J/MeV}\right)} = \boxed{4.55 \times 10^{-13} \text{ m}}$$

(b) Since $K_i = \frac{1}{2} m_\alpha v_i^2 = \frac{k_e qQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_e qQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(2)(79)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{(4.00 \text{ u})\left(1.66 \times 10^{-27} \text{ kg/u}\right)\left(3.00 \times 10^{-13} \text{ m}\right)}} = \boxed{6.04 \times 10^6 \text{ m/s}}$$

Goal Solution

(a) Use energy methods to calculate the distance of closest approach for a head-on collision between an alpha particle having an initial energy of 0.500 MeV and a gold nucleus (^{197}Au) at rest. (Assume the gold nucleus remains at rest during the collision.) (b) What minimum initial speed must the alpha particle have in order to get as close as 300 fm?

G: The positively charged alpha particle ($q = +2e$) will be repelled by the positive gold nucleus ($Q = +79e$), so that the particles probably will not touch each other in this electrostatic "collision." Therefore, the closest the alpha particle can get to the gold nucleus would be if the two nuclei did touch, in which case the distance between their centers would be about 6 fm (using $r = r_0 A^{1/3}$ for the radius of each nucleus). To get this close, or even within 300 fm, the alpha particle must be traveling very fast, probably close to the speed of light (but of course v must be less than c).

O: At the distance of closest approach, r_{min}, the initial kinetic energy will equal the electrostatic potential energy between the alpha particle and gold nucleus.

A: (a) $K_\alpha = U = k_e \dfrac{qQ}{r_{\text{min}}}$ and $r_{\text{min}} = k_e \dfrac{qQ}{K_\alpha} = \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{(0.500 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})} = 455 \text{ fm}$

(b) Since $K_\alpha = \frac{1}{2} m v^2 = k_e \dfrac{qQ}{r_{\text{min}}}$

$$v = \sqrt{\frac{2k_e qQ}{m r_{\text{min}}}} = \sqrt{\frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{4(1.66 \times 10^{-27} \text{ kg})(3.00 \times 10^{-13} \text{ m})}} = 6.04 \times 10^6 \text{ m/s}$$

L: The minimum distance in part (a) is about 100 times greater than the combined radii of the particles. For part (b), the alpha particle must have more than 0.5 MeV of energy since it gets closer to the nucleus than the 455 fm found in part (a). Even so, the speed of the alpha particle in part (b) is only about 2% of the speed of light, so we are justified in not using a relativistic approach. In solving this problem, we ignored the effect of the electrons around the gold nucleus that tend to "screen" the nucleus so that the alpha particle sees a reduced positive charge. If this screening effect were considered, the potential energy would be slightly reduced and the alpha particle could get closer to the gold nucleus for the same initial energy.

***44.6** It must start with kinetic energy equal to $K_i = U_f = k_e qQ / r_f$. Here r_f stands for the sum of the radii of the ^4_2He and $^{197}_{79}Au$ nuclei, computed as

$$r_f = r_0 A_1^{1/3} + r_0 A_2^{1/3} = \left(1.20 \times 10^{-15} \text{ m}\right)\left(4^{1/3} + 197^{1/3}\right) = 8.89 \times 10^{-15} \text{ m}$$

Thus, $$K_i = U_f = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(2)(79)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{8.89 \times 10^{-15} \text{ m}} = 4.09 \times 10^{-12} \text{ J} = \boxed{25.6 \text{ MeV}}$$

44.7 (a) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(4)^{1/3} = \boxed{1.90 \times 10^{-15}\text{ m}}$

(b) $r = r_0 A^{1/3} = (1.20 \times 10^{-15}\text{ m})(238)^{1/3} = \boxed{7.44 \times 10^{-15}\text{ m}}$

***44.8** From $r = r_0 A^{1/3}$, the radius of uranium is $r_U = r_0(238)^{1/3}$.

Thus, if $r = \frac{1}{2} r_U$ then $r_0 A^{1/3} = \frac{1}{2} r_0(238)^{1/3}$

from which $\boxed{A = 30}$

44.9 The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99 \times 10^{30}\text{ kg}\right)}{1.67 \times 10^{-27}\text{ kg/nucleon}} = 2.38 \times 10^{57}\text{ nucleons}$$

Therefore $r = r_0 A^{1/3} = \left(1.20 \times 10^{-15}\text{ m}\right)\left(2.38 \times 10^{57}\right)^{1/3} = \boxed{16.0\text{ km}}$

***44.10** $V = \frac{4}{3}\pi r^3 = 4.16 \times 10^{-5}\text{ m}^3$

$m = \rho V = (2.31 \times 10^{17}\text{ kg / m}^3)(4.16 \times 10^{-5}\text{ m}^3) = 9.61 \times 10^{12}\text{ kg}$ and

$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\text{ Nm}^2\text{ / kg}^2)\frac{(9.61 \times 10^{12}\text{ kg})^2}{(1.00\text{ m})^2} = \boxed{6.16 \times 10^{15}\text{ N}}$ toward the other ball.

44.11 The stable nuclei that correspond to magic numbers are:

Z magic: ^{2}He ^{8}O ^{20}Ca ^{28}Ni ^{50}Sn ^{82}Pb 126

N magic: T_1^3, He_2^4, N_7^{15}, O_8^{16}, Cl_{17}^{37}, K_{19}^{39}, Ca_{20}^{40}, V_{23}^{51}, Cr_{24}^{52}, Sr_{38}^{88}, Y_{39}^{89},

Zr_{40}^{90}, Xe_{54}^{136}, Ba_{56}^{138}, La_{57}^{139}, Ce_{58}^{140}, Pr_{59}^{141}, Nd_{60}^{142}, Pb_{82}^{208}, Bi_{83}^{209}, Po_{84}^{210}

44.12 Of the 102 stable nuclei listed in Table A.3,

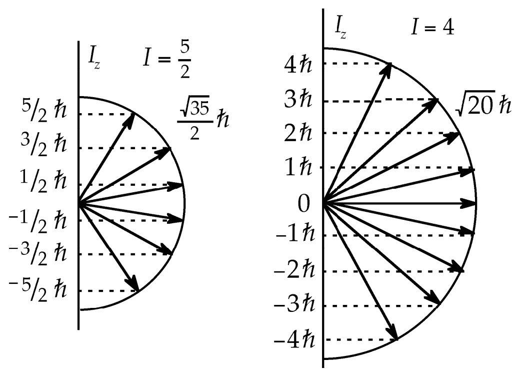
(a) Even Z, Even N $\boxed{48}$

(b) Even Z, Odd N $\boxed{6}$

(c) Odd Z, Even N $\boxed{44}$

(d) Odd Z, Odd N $\boxed{4}$

44.13



(a) (b)

44.14 (a) $f_n = \frac{|2\mu B|}{h} = \frac{2(1.9135)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{29.2\text{ MHz}}$

(b) $f_p = \frac{2(2.7928)(5.05\times10^{-27}\text{ J / T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$

(c) In the Earth's magnetic field, $f_p = \frac{2(2.7928)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\text{ kHz}}$

44.15 Using atomic masses as given in Table A.3,

(a) For $^2H_1$, $\frac{-2.014\,102 + 1(1.008\,665) + 1(1.007\,825)}{2}$

$$E_b = (0.00119\ 4\ \text{u})\left(\frac{931.5\ \text{MeV}}{\text{u}}\right) = \boxed{1.11\ \text{MeV/nucleon}}$$

(b) For ^{4}He, $\dfrac{2(1.008\ 665) + 2(1.007\ 825) - 4.002\ 602}{4}$

$E_b = 0.00759 \text{ u} = \boxed{7.07 \text{ MeV/nucleon}}$

(c) For $^{56}\text{Fe}_{26}$, $30(1.008\ 665) + 26(1.007\ 825) - 55.934\ 940 = 0.528 \text{ u}$

$E_b = \dfrac{0.528}{56} = 0.00944 \text{ u} = \boxed{8.79 \text{ MeV/nucleon}}$

(d) For $^{238}\text{U}_{92}$, $146(1.008\ 665) + 92(1.007\ 825) - 238.050\ 784 = 1.934\ 2 \text{ u}$

$E_b = \dfrac{1.934\ 2}{238} = 0.00813 \text{ u} = \boxed{7.57 \text{ MeV/nucleon}}$

44.16 $\Delta M = Zm_\text{H} + Nm_\text{n} - M$ $\quad \dfrac{\text{BE}}{A} = \dfrac{\Delta M(931.5)}{A}$

Nuclei	Z	N	M in u	ΔM in u	BE/A in MeV
^{55}Mn	25	30	54.938048	0.517527	8.765
^{56}Fe	26	30	55.934940	0.528460	8.786
^{59}Co	27	32	58.933198	0.555357	8.768

$\therefore$ ^{56}Fe has a greater BE/A than its neighbors. This tells us finer detail than is shown in Figure 44.8.

44.17 (a) The neutron-to-proton ratio, $(A - Z)/Z$ is greatest for $\boxed{^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ^{139}Cs with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.3, the neutron-proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

44.18 Use Equation 44.4.

The $^{23}_{11}$Na, $\dfrac{E_b}{A} = 8.11 \text{ MeV/nucleon}$

and for $^{23}_{12}$Mg, $\dfrac{E_b}{A} = 7.90 \text{ MeV/nucleon}$

The binding energy per nucleon is greater for $^{23}_{11}$Na by $\boxed{0.210 \text{ MeV}}$. (There is less proton repulsion in Na^{23}.)

44.19 The binding energy of a nucleus is $E_b(\text{MeV}) = \left[Z\,M(\text{H}) + Nm_n - M\left({}^{A}_{Z}\text{X}\right)\right](931.494\ \text{MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = \left[8(1.007\ 825\ \text{u}) + 7(1.008\ 665\ \text{u}) - 15.003\ 065\ \text{u}\right](931.494\ \text{MeV/u}) = 111.96\ \text{MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = \left[7(1.007\ 825\ \text{u}) + 8(1.008\ 665\ \text{u}) - 15.000\ 108\ \text{u}\right](931.494\ \text{MeV/u}) = 115.49\ \text{MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

44.20 (a) The radius of the ${}^{40}\text{Ca}$ nucleus is: $R = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)(40)^{1/3} = 4.10\times10^{-15}\ \text{m}$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[20\left(1.60\times10^{-19}\ \text{C}\right)\right]^2}{5\left(4.10\times10^{-15}\ \text{m}\right)} = 1.35\times10^{-11}\ \text{J} = \boxed{84.1\ \text{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = \left[20(1.007\ 825\ \text{u}) + 20(1.008\ 665\ \text{u}) - 39.962\ 591\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{342\ \text{MeV}}$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

44.21 Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is S_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass(n)}$$

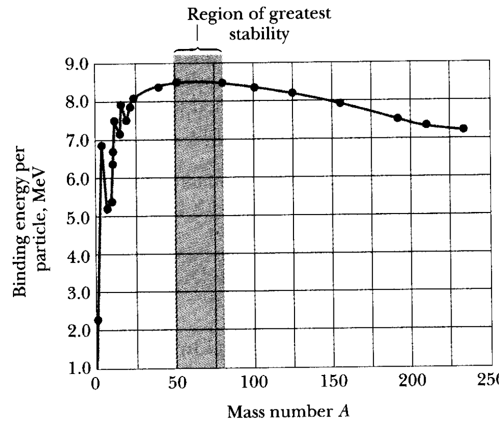
$$S_n = (41.958\ 618 + 1.008\ 665 - 42.958\ 767)\ \text{u} = (0.008\ 516\ \text{u})(931.5\ \text{MeV/u}) = \boxed{7.93\ \text{MeV}}$$

44.22 (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \boxed{\dfrac{R}{3}}$ For cubical volume $\dfrac{R^3}{6R^2} = \boxed{\dfrac{R}{6}}$

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

44.23 $\Delta E_b = E_{bf} - E_{bi}$



For $A = 200$, $\dfrac{E_b}{A} = 7.4 \text{ MeV}$

so $E_{bi} = 200(7.4 \text{ MeV}) = 1480 \text{ MeV}$

For $A \approx 100$, $E_b / A \approx 8.4 \text{ MeV}$

so $E_{bf} = 2(100)(8.4 \text{ MeV}) = 1680 \text{ MeV}$

$\Delta E_b = E_{bf} - E_{bi}$

$E_b = 1680 \text{ MeV} - 1480 \text{ MeV} = \boxed{200 \text{ MeV}}$

44.24 (a) "Volume" term: $E_1 = C_1 A = (15.7 \text{ MeV})(56) = 879 \text{ MeV}$

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8 \text{ MeV})(56)^{2/3} = -260 \text{ MeV}$

"Coulomb" term: $E_3 = -C_3 \dfrac{Z(Z-1)}{A^{1/3}} = -(0.71 \text{ MeV}) \dfrac{(26)(25)}{(56)^{1/3}} = -121 \text{ MeV}$

"Asymmetry" term: $E_4 = C_4 \dfrac{(A-2Z)^2}{A} = -(23.6 \text{ MeV}) \dfrac{(56-52)^2}{56} = -6.74 \text{ MeV}$

$\boxed{E_b = 491 \text{ MeV}}$

(b) $\dfrac{E_1}{E_b} = 179\%$; $\dfrac{E_2}{E_b} = -53.0\%$, $\dfrac{E_3}{E_b} = -24.6\%$; $\dfrac{E_4}{E_b} = -1.37\%$

44.25 $\dfrac{dN}{dt} = -\lambda N$ so $\lambda = \dfrac{1}{N}\left(-\dfrac{dN}{dt}\right) = (1.00 \times 10^{-15})(6.00 \times 10^{11}) = 6.00 \times 10^{-4} \text{ s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = \boxed{1.16 \times 10^3 \text{ s}}$ (= 19.3 min)

***44.26** $R = R_0 e^{-\lambda t} = (6.40 \text{ mCi}) e^{-\left(\frac{\ln 2}{8.04 \text{ d}}\right)(40.2 \text{ d})} = (6.40 \text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40 \text{ mCi})\left(\dfrac{1}{2^5}\right) = \boxed{0.200 \text{ mCi}}$

44.27 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times10^{-2}\text{ h}^{-1} = \boxed{1.55\times10^{-5}\text{ s}^{-1}} \qquad T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{10.0\times10^{-3}\text{ Ci}}{1.55\times10^{-5}/\text{s}}\left(\frac{3.70\times10^{10}/\text{s}}{1\text{ Ci}}\right) = \boxed{2.39\times10^{13}\text{ atoms}}$$

(c) $$R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp(-5.58\times10^{-2}\times30.0) = \boxed{1.87\text{ mCi}}$$

Goal Solution

A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. (a) Find the decay constant and half-life. (b) How many atoms of the isotope were contained in the freshly prepared sample? (c) What is the sample's activity 30.0 h after it is prepared?

G: Over the course of 4 hours, this isotope lost 20% of its activity, so its half-life appears to be around 10 hours, which means that its activity after 30 hours (~3 half-lives) will be about 1 mCi. The decay constant and number of atoms are not so easy to estimate.

O: From the rate equation, $R = R_0 e^{-\lambda t}$, we can find the decay constant λ, which can then be used to find the half life, the original number of atoms, and the activity at any other time, t.

A: (a) $$\lambda = \frac{1}{t}\ln\left(\frac{R_o}{R}\right) = \left(\frac{1}{(4.00\text{ h})(60.0\text{ s/h})}\right)\ln\left(\frac{10.0\text{ mCi}}{8.00\text{ mCi}}\right) = 1.55\times10^{-5}\text{ s}^{-1}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.0558\text{ h}^{-1}} = 12.4\text{ h}$$

(b) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second): $1\text{ Ci} \equiv 3.7\times10^{10}\text{ Bq}$

$$R_0 = 10.0\text{ mCi} = \left(10.0\times10^{-3}\text{ Ci}\right)\left(3.70\times10^{10}\text{ Bq/Ci}\right) = 3.70\times10^{8}\text{ Bq}$$

Since $R_0 = \lambda N_0$, $$N_0 = \frac{R_0}{\lambda} = \frac{3.70\times10^{8}\text{ decays/s}}{1.55\times10^{-5}\text{ s}} = 2.39\times10^{13}\text{ atoms}$$

(c) $$R = R_o e^{-\lambda t} = (10.0\text{ mCi})e^{-(5.58\times10^{-2}\text{ h}^{-1})(30.0\text{ h})} = 1.87\text{ mCi}$$

L: Our estimate of the half life was about 20% short because we did not account for the non-linearity of the decay rate. Consequently, our estimate of the final activity also fell short, but both of these calculated results are close enough to be reasonable.

The number of atoms is much less than one mole, so this appears to be a very small sample. To get a sense of how small, we can assume that the molar mass is about 100 g/mol, so the sample has a mass of only $m \approx \left(2.4\times10^{13}\text{ atoms}\right)\left(100\text{ g/mol}\right)\Big/\left(6.02\times10^{23}\text{ atoms/mol}\right) \approx 0.004\ \mu\text{g}$

This sample is so small it cannot be measured by a commercial mass balance! The problem states that this sample was "freshly prepared," from which we assumed that **all** the atoms within the sample are initially radioactive. Generally this is not true, so that N_0 only accounts for the formerly radioactive atoms, and does not include additional atoms in the sample that were not radioactive. Realistically then, the sample mass should be significantly greater than our above estimate.

44.28 $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{26.0 \text{ h}} = 0.0266/\text{h}$

$\dfrac{R}{R_0} = 0.100 = e^{-\lambda t}$ so $\ln(0.100) = -\lambda t$

$2.30 = \left(\dfrac{0.0266}{\text{h}}\right) t$ $\quad t = \boxed{86.4 \text{ h}}$

44.29 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8 \text{ h}} = 0.0107 \text{ h}^{-1} = 2.97 \times 10^{-6} \text{ s}^{-1}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})(3.70 \times 10^4 \text{ cps}/\mu\text{Ci})}{2.97 \times 10^{-6} \text{ s}^{-1}} = 4.98 \times 10^{11} \text{ nuclei}$

Substituting these values, $N_1 - N_2 = \left(4.98 \times 10^{11}\right)\left[e^{-(0.0107 \text{ h}^{-1})(10.0 \text{ h})} - e^{-(0.0107 \text{ h}^{-1})(12.0 \text{ h})}\right]$

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47 \times 10^9 \text{ nuclei}}$

44.30 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2(-t/T_{1/2})} = 2^{-t/T_{1/2}}$ and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

44.31 $R = \lambda N = \left(\dfrac{\ln 2}{5.27 \text{ yr}}\right)\left(\dfrac{1.00 \text{ g}}{59.93 \text{ g/mol}}\right)\left(6.02 \times 10^{23}\right)$

$R = \left(1.32 \times 10^{21} \dfrac{\text{decays}}{\text{yr}}\right)\left(\dfrac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) = 4.18 \times 10^{13} \text{ Bq}$

44.32 (a) $^{65}_{28}\text{Ni}^*$

(b) $^{211}_{82}\text{Pb}$

(c) $^{55}_{27}\text{Co}$

(d) $^{0}_{-1}\text{e}$

(e) $^{1}_{1}\text{H}$ (or p)

44.33

$$Q = \left(M_{^{238}\text{U}} - M_{^{234}\text{Th}} - M_{^{4}\text{He}}\right)(931.5\ \text{MeV/u})$$

$$Q = (238.050\,784 - 234.043\,593 - 4.002\,602)\,\text{u}\,(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$$

44.34

$$N_C = \left(\frac{0.0210\ \text{g}}{12.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)$$

$\left(N_C = 1.05\times10^{21}\ \text{carbon atoms}\right)$ of which 1 in 7.70×10^{11} is a ^{14}C atom

$$(N_0)_{^{14}\text{C}} = 1.37\times10^{9}, \quad \lambda_{^{14}\text{C}} = \frac{\ln 2}{5730\ \text{yr}} = 1.21\times10^{-4}\ \text{yr}^{-1} = 3.83\times10^{-12}\ \text{s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$, $$R_0 = \lambda N_0 = \left(3.83\times10^{-12}\ \text{s}^{-1}\right)\left(1.37\times10^{9}\right)\left[\frac{7(86400\ \text{s})}{1\ \text{week}}\right] = 3.17\times10^{3}\ \frac{\text{decays}}{\text{week}}$$

At time t, $$R = \frac{837}{0.88} = 951\ \text{decays/week}$$

Taking logarithms, $\ln\dfrac{R}{R_0} = -\lambda t$ so $t = \dfrac{-1}{\lambda}\ln\left(\dfrac{R}{R_0}\right)$

$$t = \frac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times10^{3}}\right) = \boxed{9.96\times10^{3}\ \text{yr}}$$

44.35 In the decay $^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$, the energy released is $E = (\Delta m)c^2 = \left[M_{^{3}_{1}\text{H}} - M_{^{3}_{2}\text{He}}\right]c^2$ since the antineutrino is massless and the mass of the electron is accounted for in the masses of $^{3}_{1}\text{H}$ and $^{3}_{2}\text{He}$.

Thus, $$E = [3.016\,049\,\text{u} - 3.016\,029\,\text{u}](931.5\ \text{MeV/u}) = 0.0186\ \text{MeV} = \boxed{18.6\ \text{keV}}$$

44.36 (a) For e^+ decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2 = \left[39.962\ 591\ \text{u} - 39.964\ 000\ \text{u} - 2\left(0.0000\ 549\ \text{u}\right)\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.34\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[97.905\ 287\ \text{u} - 4.002\ 602\ \text{u} - 93.905\ 085\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = -2.24\ \text{MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = \left(M_X - M_\alpha - M_Y\right)c^2 = \left[143.910\ 082\ \text{u} - 4.002\ 602\ \text{u} - 139.905\ 434\ \text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$Q = 1.91\ \text{MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

44.37 (a) $\boxed{e^- + p \to n + \nu}$

(b) For nuclei, $^{15}\text{O} + e^- \to {}^{15}\text{N} + \nu$.

Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}\text{O atom} \to {}^{15}_{7}\text{N atom} + \nu}$.

(c) From Table A.3, $m\left({}^{15}\text{O}\right) = m\left({}^{15}\text{N}\right) + \dfrac{Q}{c^2}$

Δm = 15.003 065 u – 15.000 108 u = 0.002 957 u

Q = (931.5 MeV / u)(0.002 957 u) = $\boxed{2.75\ \text{MeV}}$

44.38 (a) Let N be the number of ^{238}U nuclei and N' be ^{206}Pb nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = (\ln 2)/T_{1/2}$.

Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$

If $\frac{N}{N'} = 1.164$ for the $^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$

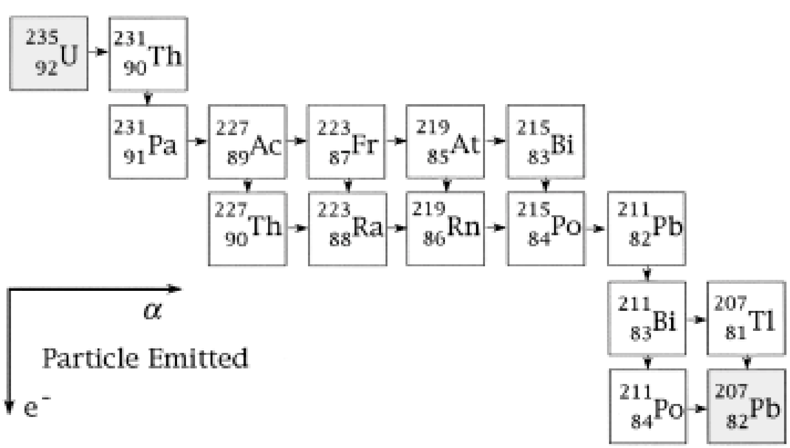
With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \text{ and } \boxed{\frac{N}{N'} = 0.0199}$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.1966 \text{ and } \boxed{\frac{N}{N'} = 4.60}$$

44.39



***44.40** (a) $4.00\ \text{pCi/L} = \left(\dfrac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\dfrac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\dfrac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$

(b) $N = \dfrac{R}{\lambda} = R\left(\dfrac{T_{1/2}}{\ln 2}\right) = \left(148\ \dfrac{\text{Bq}}{\text{m}^3}\right)\left(\dfrac{3.82\ \text{d}}{\ln 2}\right)\left(\dfrac{86\ 400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$

(c) $\text{mass} = \left(7.05\times10^{7}\ \dfrac{\text{atoms}}{\text{m}^3}\right)\left(\dfrac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\dfrac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times10^{-14}\ \dfrac{\text{g}}{\text{m}^3}$

Since air has a density of $1.20\ \text{kg/m}^3$, the fraction consisting of radon is

$\text{fraction} = \dfrac{2.60\times10^{-14}\ \text{g/m}^3}{1.20\ \text{kg/m}^3} = \boxed{2.17\times10^{-17}}$

***44.41** Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$

Fraction remaining: $\dfrac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$

(a) With $T_{1/2} = 3.82\ \text{d}$ and $t = 7.00\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$

(b) When $t = 1.00\ \text{yr} = 365.25\ \text{d}$, $\dfrac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$

(c) Radon is continuously created as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

44.42 $Q = \left[M_{^{27}\text{Al}} + M_{\alpha} - M_{^{30}\text{P}} - m_n\right]c^2$

$Q = [26.981\ 538 + 4.002\ 602 - 29.978\ 307 - 1.008\ 665]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{-2.64\ \text{MeV}}$

44.43 (a) ${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \to {}^{197}_{79}\text{Au}^{*} \to {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$

(b) Consider adding 79 electrons: ${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \to {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$

$Q = \left[M_{^{197}\text{Au}} + m_{\text{n}} - M_{^{198}\text{Hg}}\right]c^2$

$Q = [196.966\ 543 + 1.008\ 665 - 197.966\ 743]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{7.89\ \text{MeV}}$

***44.44** (a) For X, $A = 24 + 1 - 4 = 21$ and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}\text{Ne}}$

(b) $A = 235 + 1 - 90 - 2 = 144$ and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{{}^{144}_{54}\text{Xe}}$

(c) $A = 2 - 2 = 0$ and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{X = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{X' = {}^{0}_{0}\nu}$

***44.45** Neglect recoil of product nucleus, (i.e., do not require momentum conservation). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[\left(M_{\text{H}} + M_{\text{Al}}\right) - \left(M_{\text{Si}} + m_n\right)\right]c^2$$

$$Q = \left[(1.007\ 825 + 26.981\ 528) - (26.986\ 721 + 1.008\ 665)\right]\text{u}\ (931.5\ \text{MeV/u}) = -5.61\ \text{MeV}$$

Thus, $K_{\text{emerging}} = 6.61\ \text{MeV} - 5.61\ \text{MeV} = \boxed{1.00\ \text{MeV}}$

***44.46** (a) ${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$

The product nucleus is $\boxed{{}^{13}_{6}\text{C}}$

(b) ${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{10}_{5}\text{B} + {}^{4}_{2}\text{He}$

The product nucleus is $\boxed{{}^{10}_{5}\text{B}}$

44.47 ${}^{9}_{4}\text{Be} + 1.666\ \text{MeV} \rightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n}$, so $M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \dfrac{Q}{c^2} - m_n$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\ 174\ \text{u} - \frac{(-1.666\ \text{MeV})}{931.5\ \text{MeV/u}} - 1.008\ 665\ \text{u} = \boxed{8.005\ 3\ \text{u}}$$

${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \rightarrow {}^{10}_{4}\text{Be} + 6.810\ \text{MeV}$, so $M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \dfrac{Q}{c^2}$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\ 174\ \text{u} + 1.008\ 665\ \text{u} - \frac{6.810\ \text{MeV}}{931.5\ \text{MeV/u}} = \boxed{10.013\ 5\ \text{u}}$$

Goal Solution

Using the Q values of appropriate reactions and from Table 44.5, calculate the masses of ^{8}Be and ^{10}Be in atomic mass units to four decimal places.

G: The mass of each isotope in atomic mass units will be approximately the number of nucleons (8 or 10), also called the mass number. The electrons are much less massive and contribute only about 0.03% to the total mass.

O: In addition to summing the mass of the subatomic particles, the net mass of the isotopes must account for the binding energy that holds the atom together. Table 44.5 includes the energy released for each nuclear reaction. Precise atomic masses values are found in Table A.3.

A: The notation $^{9}\text{Be}\,(\gamma,\ \text{n})\,^{8}\text{Be}$ with $Q = -1.666\text{ MeV}$

means $^{9}\text{Be} + \gamma \to {}^{8}\text{Be} + \text{n} - 1.666\text{ MeV}$

Therefore $$m\left(^{8}\text{Be}\right) = m\left(^{9}\text{Be}\right) - m_n + \frac{1.666\text{ MeV}}{931.5\text{ MeV / u}}$$

$$m\left(^{8}\text{Be}\right) = 9.012174 - 1.008665 + 0.001789 = 8.0053\text{ u}$$

The notation $^{9}\text{Be}\,(\text{n},\ \gamma)\,^{10}\text{Be}$ with $Q = 6.810\text{ MeV}$

means $^{9}\text{Be} + \text{n} \to {}^{10}\text{Be} + \gamma + 6.810\text{ MeV}$

$$m\left(^{10}\text{Be}\right) = m\left(^{9}\text{Be}\right) + m_n + \frac{6.810\text{ MeV}}{931.5\text{ MeV / u}}$$

$$m\left(^{10}\text{Be}\right) = 9.012174 + 1.008665 - 0.001789 = 10.0135\text{ u}$$

L: As expected, both isotopes have masses slightly greater than their mass numbers. We were asked to calculate the masses to four decimal places, but with the available data, the results could be reported accurately to as many as six decimal places.

44.48 $$^{236}_{92}\text{U} \to {}^{90}_{37}\text{Rb} + {}^{143}_{55}\text{Cs} + 3\,^{1}_{0}\text{n},$$

so $$Q = \left[M_{^{236}_{92}\text{U}} - M_{^{90}_{37}\text{Rb}} - M_{^{143}_{55}\text{Cs}} - 3m_n\right]c^2$$

From Table A.3,

$$Q = \left[236.045\,562 - 89.914\,811 - 142.927\,220 - 3(1.008\,665)\right]\text{u}\,(931.5\ \text{MeV/u}) = \boxed{165\text{ MeV}}$$

44.49 $$\frac{N_1}{N_2} = \frac{N_0 - N_0 e^{-\lambda T_h/2}}{N_0 e^{-\lambda T_h/2} - N_0 e^{-\lambda T_h}} = \frac{1 - e^{-\ln 2/2}}{e^{-\ln 2/2} - e^{-\ln 2}} = \frac{1 - 2^{-1/2}}{2^{-1/2} - 2^{-1}} = \boxed{\sqrt{2}}$$

***44.50** $^1_1\text{H} + {}^7_3\text{Li} \to {}^7_4\text{Be} + {}^1_0\text{n}$

$$Q = \left[(M_{\text{H}} + M_{\text{Li}}) - (M_{\text{Be}} + M_{\text{n}})\right](931.5 \text{ MeV / u})$$

$$Q = \left[(1.007\ 825 \text{ u} + 7.016\ 003 \text{ u}) - (7.016\ 928 \text{ u} + 1.008\ 665 \text{ u})\right](931.5 \text{ MeV / u})$$

$$Q = (-1.765 \times 10^{-3} \text{ u})(931.5 \text{ MeV / u}) = -1.644 \text{ MeV}$$

Thus, $$KE_{\text{min}} = \left(1 + \frac{m_{\text{incident projectile}}}{m_{\text{target nucleus}}}\right)|Q| = \left(1 + \frac{1.007\ 825}{7.016\ 003}\right)(1.644 \text{ MeV}) = \boxed{1.88 \text{ MeV}}$$

44.51 (a) $$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00 \text{ kg}}{(239.05 \text{ u})\left(1.66 \times 10^{-27} \text{ kg/u}\right)} = \boxed{2.52 \times 10^{24}}$$

(b) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(2.412 \times 10^4 \text{ yr}\right)\left(3.156 \times 10^7 \text{ s/yr}\right)} = 9.106 \times 10^{-13} \text{ s}^{-1}$$

$$R_0 = \lambda N_0 = \left(9.106 \times 10^{-13} \text{ s}^{-1}\right)\left(2.52 \times 10^{24}\right) = \boxed{2.29 \times 10^{12} \text{ Bq}}$$

(c) $$R = R_0 e^{-\lambda t}, \quad \text{so} \quad t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$

$$t = \frac{1}{9.106 \times 10^{-13} \text{ s}^{-1}}\ln\left(\frac{2.29 \times 10^{12} \text{ Bq}}{0.100 \text{ Bq}}\right) = 3.38 \times 10^{13} \text{ s}\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{1.07 \times 10^6 \text{ yr}}$$

44.52 (a) $^{57}_{27}\text{Co} \to {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + {}^{0}_{0}\nu$

The Q – value for this positron emission is $Q = \left[M_{^{57}\text{Co}} - M_{^{57}\text{Fe}} - 2m_e\right]c^2$

$$Q = \left[56.936\ 294 - 56.935\ 396 - 2(0.000\ 549)\right]\text{u}\ (931.5 \text{ MeV/u}) = -0.186 \text{ MeV}$$

Since $Q < 0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) $^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$

The Q – value for this e^- decay is $Q = \left[M_{^{14}\text{C}} - M_{^{14}\text{N}}\right]c^2$.

$$Q = \left[14.003\ 242 - 14.003\ 074\right]\text{u}\ (931.5 \text{ MeV/u}) = 0.156 \text{ MeV} = 156 \text{ keV}$$

Since $Q > 0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e \text{ can range from zero to 156 keV}}$.

44.53 (a) $r = r_0 A^{1/3} = 1.20 \times 10^{-15} A^{1/3}$ m. When $A = 12$, $r = \boxed{2.75 \times 10^{-15}\text{ m}}$

(b) $$F = \frac{k_e (Z-1)e^2}{r^2} = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(Z-1)(1.60 \times 10^{-19}\text{ C})^2}{r^2}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $F = \boxed{152\text{ N}}$

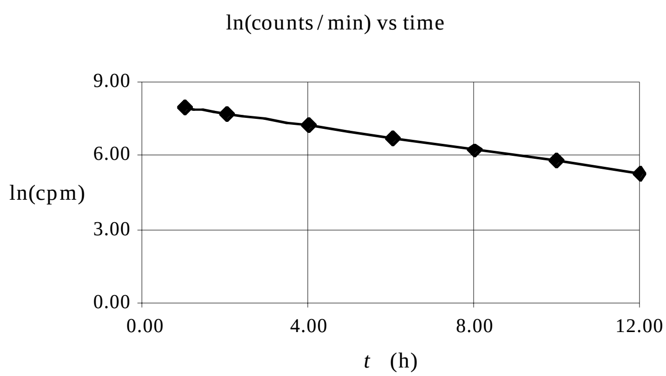
(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e (Z-1)e^2}{r} = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2 (Z-1)}{r}$$

When $Z = 6$ and $r = 2.75 \times 10^{-15}$ m, $U = 4.19 \times 10^{-13}\text{ J} = \boxed{2.62\text{ MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44 \times 10^{-15}\text{ m}}$ $F = \boxed{379\text{ N}}$

and $U = 2.82 \times 10^{-12}\text{ J} = \boxed{17.6\text{ MeV}}$

44.54 (a)



A least-square fit to the graph yields: $\lambda = -\text{slope} = -\left(-0.250\text{ h}^{-1}\right) = 0.250\text{ h}^{-1}$,

and $\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$

(b) $$\lambda = 0.250\text{ h}^{-1}\left(\frac{1\text{ h}}{60.0\text{ min}}\right) = \boxed{4.17 \times 10^{-3}\text{ min}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17 \times 10^{-3}\text{ min}^{-1}} = 166\text{ min} = \boxed{2.77\text{ h}}$$

(c) From (a), intercept = $\ln(\text{cpm})_0 = 8.30$.

Thus, $(\text{cpm})_0 = e^{8.30}\text{ counts/min} = \boxed{4.02 \times 10^3\text{ counts/min}}$

(d) $$N_0 = \frac{R_0}{\lambda} = \frac{1}{\lambda}\frac{(\text{cpm})_0}{\text{Eff}} = \frac{4.02 \times 10^3\text{ counts/min}}{\left(4.17 \times 10^{-3}\text{ min}^{-1}\right)(0.100)} = \boxed{9.65 \times 10^6\text{ atoms}}$$

44.55 (a) Because the reaction $p \to n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$,

$m_p = 1.007\ 276$ u $\quad m_n = 1.008\ 665$ u $\quad m_{e^+} = 5.49 \times 10^{-4}$ u $\qquad$ Note that $\quad m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei $\quad {}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + e^+ + \nu$

to obtain the reaction for neutral atoms $\quad {}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5\text{ MeV / u})\left[13.005\ 738 - 13.003\ 355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5\text{ MeV/u})(1.285 \times 10^{-3}\text{ u}) = \boxed{1.20\text{ MeV}}$$

44.56 (a) If we assume all the ^{87}Sr came from ^{87}Rb, then $N = N_0 e^{-\lambda t}$ yields

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right), \quad \text{where} \quad N = N_{^{87}\text{Rr}} \text{ and } N_0 = N_{^{87}\text{Sr}} + N_{^{87}\text{Rb}}.$$

$$t = \frac{\left(4.75 \times 10^{10}\text{ yr}\right)}{\ln 2}\ln\left(\frac{1.82 \times 10^{10} + 1.07 \times 10^{9}}{1.82 \times 10^{10}}\right) = \boxed{3.91 \times 10^{9}\text{ yr}}$$

(b) It could be $\boxed{\text{no longer}}$. The rock could be younger if some ^{87}Sr were originally present.

44.57 (a) Let us assume that the parent nucleus (mass M_p) is initially at rest, and let us denote the masses of the daughter nucleus and alpha particle by M_d and M_α, respectively. Applying the equations of conservation of momentum and energy for the alpha decay process gives

$$M_d v_d = M_\alpha v_\alpha \tag{1}$$

$$M_p c^2 = M_d c^2 + M_\alpha c^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \tag{2}$$

The disintegration energy Q is given by $\quad Q = (M_p - M_d - M_\alpha)c^2 = \tfrac{1}{2} M_\alpha v_\alpha^2 + \tfrac{1}{2} M_d v_d^2 \quad (3)$

Eliminating v_d from Equations (1) and (3) gives

$$Q = \tfrac{1}{2} M_d\left(\frac{M_\alpha}{M_d} v_\alpha\right)^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2}\frac{M_\alpha^2}{M_d} v_\alpha^2 + \tfrac{1}{2} M_\alpha v_\alpha^2 = \tfrac{1}{2} M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_d}\right) = \boxed{K_\alpha\left(1 + \frac{M_\alpha}{M_d}\right)}$$

(b) $$K_\alpha = \frac{Q}{1 + \left(M_\alpha / M_d\right)} = \frac{4.87\text{ MeV}}{1 + (4 / 222)} = \boxed{4.78\text{ MeV}}$$

44.58 (a) The reaction is ${}^{145}_{61}\mathrm{Pm} \to {}^{141}_{59}\mathrm{Pr} + \alpha$

(b) $Q = (M_{\mathrm{Pm}} - M_\alpha - M_{\mathrm{Pr}})931.5 = (144.912\ 745 - 4.002\ 602 - 140.907\ 647)931.5 = \boxed{2.32\ \mathrm{MeV}}$

(c) The alpha and daughter have equal and opposite momenta $p_\alpha = p_d$

$$E_\alpha = \frac{p_\alpha^2}{2m_\alpha} \qquad E_d = \frac{p_d^2}{2m_d}$$

$$\frac{E_\alpha}{E_{\mathrm{tot}}} = \frac{E_\alpha}{E_\alpha + E_d} = \frac{\frac{p_\alpha^2}{2m_\alpha}}{\frac{p_\alpha^2}{2m_a} + \frac{p_\alpha^2}{2m_d}} = \frac{\frac{1}{2m_\alpha}}{\frac{1}{2m_\alpha} + \frac{1}{2m_d}} = \frac{m_d}{m_d + m_\alpha} = \frac{141}{141+4} = \boxed{97.2\%} \text{ or } 2.26\ \mathrm{MeV}$$

This is carried away by the alpha

44.59 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy gives

$$\Delta E = hf + E_r \qquad (1)$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad (2)$$

Since momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad (3)$$

Hence, E_r can be expressed as $E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf << Mc^2$, we can make the approximation that $hf \approx \Delta E$, so $E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.0144\ \mathrm{MeV}$ and $Mc^2 = (57\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 5.31 \times 10^4\ \mathrm{MeV}$

Therefore, $E_r = \dfrac{(1.44 \times 10^{-2}\ \mathrm{MeV})^2}{(2)(5.31 \times 10^4\ \mathrm{MeV})} = \boxed{1.94 \times 10^{-3}\ \mathrm{eV}}$

***44.60** (a) One liter of milk contains this many ^{40}K nuclei:

$$N = (2.00\text{ g})\left(\frac{6.02\times 10^{23}\text{ nuclei/mol}}{39.1\text{ g/mol}}\right)\left(\frac{0.0117}{100}\right) = 3.60\times 10^{18}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times 10^{9}\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times 10^{7}\text{ s}}\right) = 1.72\times 10^{-17}\text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times 10^{-17}\text{ s}^{-1}\right)\left(3.60\times 10^{18}\right) = \boxed{61.8\text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \frac{\ln 2}{8.04\text{ d}}$.

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04\text{ d}}{\ln 2}\ln\left(\frac{2000}{61.8}\right) = \boxed{40.3\text{ d}}$$

***44.61** (a) For cobalt-56, $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1\text{ d}}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right) = 3.28\text{ yr}^{-1}$.

The elapsed time from July 1054 to July 2000 is 946 yr.

$R = R_0 e^{-\lambda t}$ implies $\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-3106} = e^{-(\ln 10)1349} = \boxed{\sim 10^{-1349}}$

(b) For carbon-14, $\lambda = \frac{\ln 2}{5730\text{ yr}} = 1.21\times 10^{-4}\text{ yr}^{-1}$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times 10^{-4}\text{ yr}^{-1}\right)(946\text{ yr})} = e^{-0.114} = \boxed{0.892}$$

***44.62** We have $N_{235} = N_{0,235}e^{-\lambda_{235}t}$ and $N_{238} = N_{0,238}e^{-\lambda_{238}t}$

$$\frac{N_{235}}{N_{238}} = 0.00725 = e^{\left(-(\ln 2)t/T_{h,235} + (\ln 2)t/T_{h,238}\right)}$$

Taking logarithms, $-4.93 = \left(-\frac{\ln 2}{0.704\times 10^{9}\text{ yr}} + \frac{\ln 2}{4.47\times 10^{9}\text{ yr}}\right)t$

or $-4.93 = \left(-\frac{1}{0.704\times 10^{9}\text{ yr}} + \frac{1}{4.47\times 10^{9}\text{ yr}}\right)(\ln 2)t$

$$t = \frac{-4.93}{\left(-1.20\times 10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times 10^{9}\text{ yr}}$$

44.63 (a) Add two electrons to both sides of the reaction to have it in energy terms: $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q$

$$Q = \Delta mc^2 = \left[4\,M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2$$

$$Q = \left[4(1.007\ 825\ \text{u}) - 4.002\ 602\ \text{u}\right](931.5\ \text{MeV/u})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{4.28\times10^{-12}\ \text{J}}$$

(b) $$N = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg/atom}} = \boxed{1.19\times10^{57}\ \text{atoms}} = 1.19\times10^{57}\ \text{protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\ \text{protons}\right)\left(\frac{4.28\times10^{-12}\ \text{J}}{4\ \text{protons}}\right) = 1.27\times10^{45}\ \text{J}$$

$$\mathcal{P} = \frac{E}{t} \qquad \text{so} \qquad t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\ \text{J}}{3.77\times10^{26}\ \text{W}} = 3.38\times10^{18}\ \text{s} = \boxed{107\ \text{billion years}}$$

44.64 (a) $$Q = \left[M_{{}^9\text{Be}} + M_{{}^4\text{He}} - M_{{}^{12}\text{C}} - m_n\right]c^2$$

$$Q = \left[9.012\ 174\ \text{u} + 4.002\ 602\ \text{u} - 12.000\ 000\ \text{u} - 1.008\ 665\ \text{u}\right](931.5\ \text{MeV/u}) = \boxed{5.69\ \text{MeV}}$$

(b) $$Q = \left[2\,M_{{}^2\text{H}} - M_{{}^3\text{He}} - m_n\right]$$

$$Q = \left[2(2.014\ 102) - 3.016\ 029 - 1.008\ 665\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{3.27\ \text{MeV (exothermic)}}$$

44.65 $E = -\boldsymbol{\mu}\cdot\mathbf{B}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.7928\mu_\text{n}$ and $\mu_\text{n} = 5.05\times10^{-27}\ \text{J/T}$

$$\Delta E = 2\mu B = 2(2.7928)(5.05\times10^{-27}\ \text{J/T})(12.5\ \text{T}) = 3.53\times10^{-25}\ \text{J} = \boxed{2.20\times10^{-6}\ \text{eV}}$$

44.66 (a) $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27 \text{ yr}}\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = 4.17 \times 10^{-9} \text{ s}^{-1}$

$$t = 30.0 \text{ months} = (2.50 \text{ yr})\left(\frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}}\right) = 7.89 \times 10^7 \text{ s}$$

$$R = R_0 e^{-\lambda t} = (\lambda N_0) e^{-\lambda t}$$

so $N_0 = \left(\frac{R}{\lambda}\right) e^{\lambda t} = \left[\frac{(10.0 \text{ Ci})(3.70 \times 10^{10} \text{ Bq/Ci})}{4.17 \times 10^{-9} \text{ s}^{-1}}\right] e^{(4.17 \times 10^{-9} \text{ s}^{-1})(7.89 \times 10^7 \text{ s})}$

$$N_0 = 1.23 \times 10^{20} \text{ nuclei}$$

$$\text{Mass} = (1.23 \times 10^{20} \text{ atoms})\left(\frac{59.93 \text{ g/mol}}{6.02 \times 10^{23} \text{ atoms/mol}}\right) = 1.23 \times 10^{-2} \text{ g} = \boxed{12.3 \text{ mg}}$$

(b) We suppose that each decaying nucleus promptly puts out both a beta particle and two gamma rays, for

$$Q = (0.310 + 1.17 + 1.33) \text{ MeV} = 2.81 \text{ Mev}$$

$$\mathcal{P} = QR = (2.81 \text{ Mev})(1.6 \times 10^{-13} \text{ J/MeV})(3.70 \times 10^{11} \text{ s}^{-1}) = \boxed{0.166 \text{ W}}$$

44.67 For an electric charge density $\rho = \frac{Ze}{\frac{4}{3}\pi R^3}$

Using Gauss's Law inside the sphere,

$$E \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3}{\epsilon_0} \frac{Ze}{\frac{4}{3}\pi R^3}: \qquad E = \frac{1}{4\pi\epsilon_0}\frac{Zer}{R^3} \quad (r \le R)$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r^2} \quad (r \ge R)$$

We now find the electrostatic energy: $U = \int_{r=0}^{\infty} \frac{1}{2}\epsilon_0 E^2 4\pi r^2 \, dr$

$$U = \frac{1}{2}\epsilon_0 \int_0^R \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{Z^2 e^2 r^2}{R^6} 4\pi r^2 \, dr + \frac{1}{2}\epsilon_0 \int_R^\infty \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{Z^2 e^2}{r^4} 4\pi r^2 \, dr = \frac{Z^2 e^2}{8\pi\epsilon_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] = \boxed{\frac{3}{20}\frac{Z^2 e^2}{\pi\epsilon_0 R}}$$

44.68 (a) For the electron capture, $^{93}_{43}\text{Tc} + {}^{0}_{-1}\text{e} \to {}^{93}_{42}\text{Mo} + \gamma$

The disintegration energy is $Q = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}}\right]c^2$.

$$Q = [92.910\ 2 - 92.906\ 8]\text{u}\ (931.5\ \text{MeV/u}) = 3.17\ \text{MeV} > 2.44\ \text{MeV}$$

Electron capture is allowed $\boxed{\text{to all specified excited states}}$ in $^{93}_{42}\text{Mo}$.

For positron emission, $^{93}_{43}\text{Tc} \to {}^{93}_{42}\text{Mo} + {}^{0}_{+1}\text{e} + \gamma$

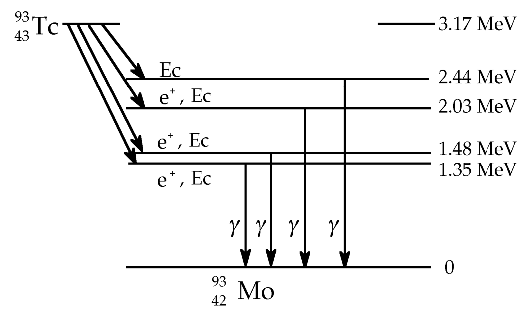
The disintegration energy is $Q' = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} - 2m_e\right]c^2$.

$$Q' = [92.910\ 2 - 92.906\ 8 - 2(0.000\ 549)]\text{u}\ (931.5\ \text{MeV/u}) = 2.14\ \text{MeV}$$

Positron emission can reach

$\boxed{\text{the 1.35, 1.48, and 2.03 MeV states}}$

but there is insufficient energy to reach the 2.44 MeV state.



(b) The daughter nucleus in both forms of decay is $\boxed{^{93}_{42}\text{Mo}}$.

44.69 $K = \frac{1}{2}mv^2$,

so $$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.0400\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.67\times10^{-27}\ \text{kg}}} = 2.77\times10^{3}\ \text{m s}$$

The time for the trip is $t = \frac{x}{v} = \frac{1.00\times10^{4}\ \text{m}}{2.77\times10^{3}\ \text{m/s}} = 3.61\ \text{s}$

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $1 - \frac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61\ \text{s}/624\ \text{s})} = 0.004\ 00 = \boxed{0.400\%}$

44.70 (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles which have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation: $M_a v_a = (M_a + M_X)v_c$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is: $E_f = \frac{1}{2}(M_a + M_X)v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{(M_a + M_X)}\right]E_i$

From this, we see that E_f is always less than E_i and the *loss* in energy, $E_i - E_f$, is given by

$$E_i - E_f = \left[1 - \frac{M_a}{M_a + M_X}\right]E_i = \left[\frac{M_X}{M_a + M_X}\right]E_i$$

In this problem, the energy loss is the disintegration energy $-Q$ and the initial energy is the threshold energy E_{th}. Therefore,

$$-Q = \left[\frac{M_X}{M_a + M_X}\right]E_{th} \quad \text{or} \quad E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the $Q-$value for the reaction: $Q = \left[M_{^{14}\text{N}} + M_{^{4}\text{He}} - M_{^{17}\text{O}} - M_{^{1}\text{H}}\right]c^2$

$$Q = [14.003\ 074 + 4.002\ 602 - 16.999\ 132 - 1.007\ 825]\text{u}\ (931.5\ \text{MeV/u}) = -1.19\ \text{MeV}$$

Then, $E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\ 602}{14.003\ 074}\right] = \boxed{1.53\ \text{MeV}}$

44.71 $R = R_0 \exp(-\lambda t)$

$\ln R = \ln R_0 - \lambda t$ (the equation of a straight line)

$|\text{slope}| = \lambda$

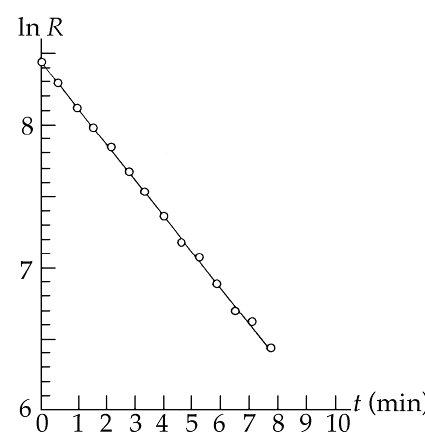
The logarithmic plot shown in Figure P44.71 is fitted by

$\ln R = 8.44 - 0.262t$.

If t is measured in minutes, then the decay constant λ is 0.262 per minute. The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262\ /\ \text{min}} = \boxed{2.64\ \text{min}}$$

The reported half-life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.



Chapter 45 Solutions

***45.1** $\Delta m = (m_n + M_{\text{U}}) - (M_{\text{Zr}} + M_{\text{Te}} + 3m_n)$

$\Delta m = (1.008\,665 \text{ u} + 235.043\,924 \text{ u}) - (97.912\,0 \text{ u} + 134.908\,7 \text{ u} + 3(1.008\,665 \text{ u}))$

$\Delta m = 0.205\,89 \text{ u} = 3.418 \times 10^{-28} \text{ kg}$ so $Q = \Delta mc^2 = 3.076 \times 10^{-11} \text{ J} = \boxed{192 \text{ MeV}}$

45.2 Three different fission reactions are possible: ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{144}_{54}\text{Xe} + 2\,{}^{1}_{0}\text{n}$ $\boxed{{}^{144}_{54}\text{Xe}}$

${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{143}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n}$ $\boxed{{}^{143}_{54}\text{Xe}}$ ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \to {}^{90}_{38}\text{Sr} + {}^{142}_{54}\text{Xe} + 4\,{}^{1}_{0}\text{n}$ $\boxed{{}^{142}_{54}\text{Xe}}$

45.3 ${}^{1}_{0}\text{n} + {}^{232}_{90}\text{Th} \to {}^{233}_{90}\text{Th} \to {}^{233}_{91}\text{Pa} + e^- + \bar{\nu}$ ${}^{233}_{91}\text{Pa} \to {}^{233}_{92}\text{U} + e^- + \bar{\nu}$

45.4 ${}^{1}_{0}\text{n} + {}^{238}_{92}\text{U} \to {}^{239}_{92}\text{U} \to {}^{239}_{93}\text{Np} + e^- + \bar{\nu}$ ${}^{239}_{93}\text{Np} \to {}^{239}_{94}\text{Pu} + e^- + \bar{\nu}$

45.5 (a) $Q = (\Delta m)c^2 = \left[m_n + M_{\text{U235}} - M_{\text{Ba141}} - M_{\text{Kr92}} - 3m_n\right]c^2$

$\Delta m = \left[(1.008\,665 + 235.043\,924) - (140.913\,9 + 91.897\,3 + 3 \times 1.008\,665)\right]\text{u} = 0.215\,39 \text{ u}$

$Q = (0.215\,39 \text{ u})(931.5 \text{ MeV/u}) = \boxed{201 \text{ MeV}}$

(b) $f = \dfrac{\Delta m}{m_i} = \dfrac{0.215\,39 \text{ u}}{236.052\,59 \text{ u}} = 9.13 \times 10^{-4} = \boxed{0.0913\%}$

45.6 If the electrical power output of 1000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1000 \text{ MW}}{0.400} = \left(2.50 \times 10^9 \ \frac{\text{J}}{\text{s}}\right)\left(\frac{8.64 \times 10^4}{\text{d}}\right) = 2.16 \times 10^{14} \text{ J/d}$$

The number of fissions per day is $\left(2.16 \times 10^{14} \ \dfrac{\text{J}}{\text{d}}\right)\left(\dfrac{1 \text{ fission}}{200 \times 10^6 \text{ eV}}\right)\left(\dfrac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 6.74 \times 10^{24} \text{ d}^{-1}$

This also is the number of ${}^{235}\text{U}$ nuclei used, so the mass of ${}^{235}\text{U}$ used per day is

$$\left(6.74 \times 10^{24} \ \frac{\text{nuclei}}{\text{d}}\right)\left(\frac{235 \text{ g/mol}}{6.02 \times 10^{23} \text{ nuclei/mol}}\right) = 2.63 \times 10^3 \text{ g/d} = \boxed{2.63 \text{ kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6×10^6 kg/d of coal.

45.7 The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\text{ kg fuel})\left(\frac{0.0340\ ^{235}\text{U}}{\text{fuel}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ mol}}{235\text{ g}}\right)\left(\frac{6.02\times10^{23}}{\text{mol}}\right)\left(\frac{(208)(1.60\times10^{-13}\text{ J})}{\text{fission}}\right)$$

$$\left(2.90\times10^{12}\text{ J}\right)(0.200) = 5.80\times10^{11}\text{ J} = \left(1.00\times10^{5}\text{ N}\right)\cdot d$$

$d = 5.80\times10^{6}$ m = $\boxed{5.80\text{ Mm}}$

Goal Solution

Suppose enriched uranium containing 3.40% of the fissionable isotope $^{235}_{92}\text{U}$ is used as fuel for a ship. The water exerts an average frictional drag of 1.00×10^{5} N on the ship. How far can the ship travel per kilogram of fuel? Assume that the energy released per fission event is 208 MeV and that the ship's engine has an efficiency of 20.0%.

G: Nuclear fission is much more efficient for converting mass to energy than burning fossil fuels. However, without knowing the rate of diesel fuel consumption for a comparable ship, it is difficult to estimate the nuclear fuel rate. It seems plausible that a ship could cross the Atlantic ocean with only a few kilograms of nuclear fuel, so a reasonable range of uranium fuel consumption might be 10 km / kg to 10 000 km / kg.

O: The fuel consumption rate can be found from the energy released by the nuclear fuel and the work required to push the ship through the water.

A: One kg of enriched uranium contains 3.40% $^{235}_{92}\text{U}$ so $m_{235} = (1000\text{ g})(0.0340) = 34.0\text{ g}$

In terms of number of nuclei, this is equivalent to

$$N_{235} = (34.0\text{ g})\left(\frac{1}{235\text{ g / mol}}\right)\left(6.02\times10^{23}\text{ atoms / mol}\right) = 8.71\times10^{22}\text{ nuclei}$$

If all these nuclei fission, the thermal energy released is equal to

$$\left(8.71\times10^{22}\text{ nuclei}\right)\left(208\ \frac{\text{MeV}}{\text{nucleus}}\right)\left(1.602\times10^{-19}\text{ J/eV}\right) = 2.90\times10^{12}\text{ J}$$

Now, for the engine, $\text{efficiency} = \dfrac{\text{work output}}{\text{heat input}}$ or $e = \dfrac{fd\cos\theta}{Q_h}$

So the distance the ship can travel per kilogram of uranium fuel is

$$d = \frac{eQ_h}{f\cos(0)} = \frac{0.200\left(2.90\times10^{12}\text{ J}\right)}{1.00\times10^{5}\text{ N}} = 5.80\times10^{6}\text{ m}$$

L: The ship can travel 5 800 km / kg of uranium fuel, which is on the high end of our prediction range. The distance between New York and Paris is 5 851 km, so this ship could cross the Atlantic ocean on just one kilogram of uranium fuel.

45.8 (a) For a sphere: $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3V}{4\pi}\right)^{1/3}$ so $\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \boxed{4.84V^{-1/3}}$

(b) For a cube: $V = l^3$ and $l = V^{1/3}$ so $\frac{A}{V} = \frac{6l^2}{l^3} = \boxed{6V^{-1/3}}$

(c) For a parallelepiped: $V = 2a^3$ and $a = \left(\frac{V}{2}\right)^{1/3}$ so $\frac{A}{V} = \frac{\left(2a^2 + 8a^2\right)}{2a^3} = \boxed{6.30V^{-1/3}}$

(d) Therefore, the $\boxed{\text{sphere has the least leakage}}$ and the $\boxed{\text{parallelepiped has the greatest leakage}}$ for a given volume.

45.9 mass of ^{235}U available $\approx (0.007)\left(10^9 \text{ metric tons}\right)\left(\frac{10^6 \text{ g}}{1 \text{ metric ton}}\right) = 7 \times 10^{12}$ g

number of nuclei $\sim \left(\frac{7 \times 10^{12} \text{ g}}{235 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{nuclei}}{\text{mol}}\right) = 1.8 \times 10^{34}$ nuclei

The energy available from fission (at 208 MeV/event) is

$$E \sim \left(1.8 \times 10^{34} \text{ events}\right)(208 \text{ MeV / event})\left(1.60 \times 10^{-13} \text{ J / MeV}\right) = 6.0 \times 10^{23} \text{ J}$$

This would last for a time of $t = \frac{E}{\mathcal{P}} \sim \frac{6.0 \times 10^{23} \text{ J}}{7.0 \times 10^{12} \text{ J/s}} = \left(8.6 \times 10^{10} \text{ s}\right)\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) \sim \boxed{3000 \text{ yr}}$

45.10 In one minute there are $\frac{60.0 \text{ s}}{1.20 \text{ ms}} = 5.00 \times 10^4$ fissions.

So the rate increases by a factor of $(1.000\ 25)^{50000} = \boxed{2.68 \times 10^5}$

45.11 $\mathcal{P} = 10.0 \text{ MW} = 1.00 \times 10^7$ J/s

If each decay delivers 1.00 MeV = 1.60×10^{-13} J, then the number of decays/s = $\boxed{6.25 \times 10^{19} \text{ Bq}}$

45.12 (a) The Q value for the D-T reaction is 17.59 MeV.

Heat content in fuel for D-T reaction: $\dfrac{(17.59\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(5\text{ u})(1.66\times10^{-27}\text{ kg/u})}=3.39\times10^{14}\text{ J/kg}$

$$r_{\text{DT}}=\frac{(3.00\times10^{9}\text{ J / s})(3600\text{ s / hr})}{(3.39\times10^{14}\text{ J / kg})(10^{-3}\text{ kg / g})}=\boxed{31.9\text{ g/h burning of D and T}}$$

(b) Heat content in fuel for D-D reaction: $Q=\frac{1}{2}(3.27+4.03)=3.65$ MeV average of two Q values

$$\frac{(3.65\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(4\text{ u})(1.66\times10^{-27}\text{ kg/u})}=8.80\times10^{13}\text{ J/kg}$$

$$r_{\text{DD}}=\frac{(3.00\times10^{9}\text{ J / s})(3600\text{ s / hr})}{(8.80\times10^{12}\text{ J / kg})(10^{-3}\text{ kg / g})}=\boxed{122\text{ g/h burning of D}}$$

45.13 (a) At closest approach, the electrostatic potential energy equals the total energy E.

$$U_f=\frac{k_e(Z_1e)(Z_2e)}{r_{\min}}=E:\qquad E=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\text{ C}\right)^2Z_1Z_2}{1.00\times10^{-14}\text{ m}}=\boxed{\left(2.30\times10^{-14}\text{ J}\right)Z_1Z_2}$$

(b) For both the D-D and the D-T reactions, $Z_1=Z_2=1$. Thus, the minimum energy required in both cases is

$$E=\left(2.30\times10^{-14}\text{ J}\right)\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)=\boxed{0.144\text{ MeV}}$$

45.14 (a) $r_f=r_{\text{D}}+r_{\text{T}}=\left(1.20\times10^{-15}\text{ m}\right)\left[(2)^{1/3}+(3)^{1/3}\right]=\boxed{3.24\times10^{-15}\text{ m}}$

(b) $$U_f=\frac{k_ee^2}{r_f}=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{3.24\times10^{-15}\text{ m}}=7.10\times10^{-14}\text{ J}=\boxed{444\text{ keV}}$$

(c) Conserving momentum, $m_{\text{D}}v_i=(m_{\text{D}}+m_{\text{T}})v_f$, or $v_f=\left(\dfrac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)v_i=\boxed{\tfrac{2}{5}v_i}$

(d) $K_i+U_i=K_f+U_f$:

$$K_i+0=\tfrac{1}{2}(m_{\text{D}}+m_{\text{T}})v_f^2+U_f=\tfrac{1}{2}(m_{\text{D}}+m_{\text{T}})\left(\frac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)^2v_i^2+U_f$$

$$K_i+0=\left(\frac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)\left(\tfrac{1}{2}m_{\text{D}}v_i^2\right)+U_f=\left(\frac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)K_i+U_f$$

$\left(1-\dfrac{m_{\text{D}}}{m_{\text{D}}+m_{\text{T}}}\right)K_i=U_f$: $\qquad K_i=U_f\left(\dfrac{m_{\text{D}}+m_{\text{T}}}{m_{\text{T}}}\right)=\tfrac{5}{3}(444\text{ keV})=\boxed{740\text{ keV}}$

(e) **Possibly by tunneling.**

45.15 (a) Average KE per particle is $\frac{3}{2}k_BT = \frac{1}{2}mv^2$.

Therefore, $v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(4.00\times10^8\text{ K})}{2(1.67\times10^{-27}\text{ kg})}} = \boxed{2.23\times10^6\text{ m/s}}$

(b) $t = \frac{x}{v} \sim \frac{0.1\text{ m}}{10^6\text{ m/s}}$ $\boxed{\sim 10^{-7}\text{ s}}$

45.16 (a) $V = (317\times10^6\text{ mi}^3)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)^3 = 1.32\times10^{18}\text{ m}^3$

$m_{water} = \rho V = (10^3\text{ kg/m}^3)(1.32\times10^{18}\text{ m}^3) = 1.32\times10^{21}\text{ kg}$

$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right)m_{H_2O} = \left(\frac{2.016}{18.015}\right)(1.32\times10^{21}\text{ kg}) = 1.48\times10^{20}\text{ kg}$

$m_{Deuterium} = (0.0300\%)m_{H_2} = (0.0300\times10^{-2})(1.48\times10^{20}\text{ kg}) = 4.43\times10^{16}\text{ kg}$

The number of deuterium nuclei in this mass is

$N = \frac{m_{Deuterium}}{m_{Deuteron}} = \frac{4.43\times10^{16}\text{ kg}}{(2.014\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 1.33\times10^{43}$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He} + Q$, the number of events is $N/2 = 6.63\times10^{42}$.

The energy released per event is

$Q = \left[M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right]c^2 = \left[2(2.014\,102) - 4.002\,602\right]\text{u}\ (931.5\text{ MeV/u}) = 23.8\text{ MeV}$

The total energy available is then

$E = \left(\frac{N}{2}\right)Q = (6.63\times10^{42})(23.8\text{ MeV})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{2.52\times10^{31}\text{ J}}$

(b) The time this energy could possibly meet world requirements is

$t = \frac{E}{\mathcal{P}} = \frac{2.52\times10^{31}\text{ J}}{100(7.00\times10^{12}\text{ J/s})} = (3.61\times10^{16}\text{ s})\left(\frac{1\text{ yr}}{3.16\times10^7\text{ s}}\right) = \boxed{1.14\times10^9\text{ yr}}$ ~ 1 billion years.

45.17 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$ where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_BT = 3nVk_BT = 3\left(2.0\times10^{13}\text{ cm}^{-3}\right)\left[\left(50\text{ m}^3\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)\right]\left(1.38\times10^{-23}\text{ J/K}\right)\left(4.0\times10^8\text{ K}\right)$$

$$E = \boxed{1.7\times10^7\text{ J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6$ J/kg. The mass of water that could be boiled away is

$$m = \frac{E}{L_v} = \frac{1.7\times10^7\text{ J}}{2.26\times10^6\text{ J/kg}} = \boxed{7.3\text{ kg}}$$

45.18 (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\text{ s/cm}^3$. For a confinement time of $\tau = 1.00$ s, this requires a minimum ion density of $n = \boxed{10^{14}\text{ cm}^{-3}}$

(b) At the ignition temperature of $T = 4.5\times10^7$ K and the ion density found above, the plasma pressure is

$$P = 2nk_BT = 2\left[\left(10^{14}\text{ cm}^{-3}\right)\left(\frac{10^6\text{cm}^3}{1\text{ m}^3}\right)\right]\left(1.38\times10^{-23}\text{ J/K}\right)\left(4.5\times10^7\text{ K}\right) = \boxed{1.24\times10^5\text{ J/m}^3}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.24\times10^5\text{ J/m}^3\right) = 1.24\times10^6\text{ J/m}^3,$$

$$B \geq \sqrt{2\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(1.24\times10^6\text{ J/m}^3\right)} = \boxed{1.77\text{ T}}$$

45.19 Let the number of ^{6}Li atoms, each having mass 6.015 u, be N_6 while the number of ^{7}Li atoms, each with mass 7.016 u, is N_7.

Then, $N_6 = 7.50\%$ of $N_{total} = 0.0750(N_6 + N_7)$, or $N_7 = \left(\frac{0.925}{0.0750}\right)N_6$

Also, total mass $= \left[N_6(6.015\text{ u}) + N_7(7.016\text{ u})\right]\left(1.66\times10^{-27}\text{ kg/u}\right) = 2.00$ kg,

or $$N_6\left[(6.015\text{ u}) + \left(\frac{0.925}{0.0750}\right)(7.016\text{ u})\right]\left(1.66\times10^{-27}\text{ kg/u}\right) = 2.00\text{ kg}.$$

This yields $N_6 = \boxed{1.30\times10^{25}}$ as the number of ^{6}Li atoms and

$$N_7 = \left(\frac{0.925}{0.0750}\right)\left(1.30\times10^{25}\right) = \boxed{1.61\times10^{26}}$$ as the number of ^{7}Li atoms.

45.20 The number of nuclei in 1.00 metric ton of trash is

$$N = 1000\text{ kg}(1000\text{ g / kg})\left(6.02\times10^{23}\text{ nuclei / mol}\right)\Big/(56.0\text{ g / mol}) = 1.08\times10^{28}\text{ nuclei}$$

At an average charge of 26.0 e / nucleus, $q = \left(1.08\times10^{28}\right)(26.0)\left(1.60\times10^{-19}\right) = 4.47\times10^{10}\text{ C}$

Therefore $t = \dfrac{q}{I} = \dfrac{4.47\times10^{10}}{1.00\times10^{6}} = 4.47\times10^{4}\text{ s} = \boxed{12.4\text{ h}}$

45.21 $$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00\text{ kg}}{(89.9077\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)} = 3.35\times10^{25}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1\text{ yr}} = 2.38\times10^{-2}\text{ yr}^{-1} = 4.52\times10^{-8}\text{ min}^{-1}$$

$$R_0 = \lambda N_0 = \left(4.52\times10^{-8}\text{ min}^{-1}\right)\left(3.35\times10^{25}\right) = 1.52\times10^{18}\text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0\text{ counts/min}}{1.52\times10^{18}\text{ counts/min}} = 6.60\times10^{-18}$$ and $$\lambda t = -\ln\left(6.60\times10^{-18}\right) = 39.6$$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times10^{-2}\text{ yr}^{-1}} = \boxed{1.66\times10^{3}\text{ yr}}$

45.22 Source: 100 mrad of 2-MeV γ-rays / h at a 1.00-m distance.

(a) For γ-rays, dose in rem = dose in rad.

Thus a person would have to stand $\boxed{10.0\text{ hours}}$ to receive 1.00 rem from a 100-mrad / h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $1/r^2$.

Thus a dosage 10.0 mrad / h would be received at a distance $r = \sqrt{10.0}\text{ m} = \boxed{3.16\text{ m}}$.

45.23 (a) The number of x-rays taken per year is

$$n = (8\text{ x-ray/d})(5\text{ d/wk})(50\text{ wk/yr}) = 2.0\times10^{3}\text{ x-ray/yr}$$

The average dose per photograph is $\dfrac{5.0\text{ rem/yr}}{2.0\times10^{3}\text{ x-ray/yr}} = \boxed{2.5\times10^{-3}\text{ rem/x-ray}}$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The dose of 5.0 rem/yr received as a result of the job is

$$\frac{5.0 \text{ rem/yr}}{0.13 \text{ rem/yr}} = \boxed{\text{38 times background levels}}$$

45.24 (a) $I = I_0 e^{-\mu x}$, so $x = \frac{1}{\mu}\ln\left(\frac{I_0}{I}\right)$

With $\mu = 1.59\ \text{cm}^{-1}$, the thickness when $I = I_0/2$ is $x = \frac{1}{1.59\ \text{cm}^{-1}}\ln(2) = \boxed{0.436\ \text{cm}}$

(b) When $\frac{I_0}{I} = 1.00\times10^4$, $x = \frac{1}{1.59\ \text{cm}^{-1}}\ln\left(1.00\times10^4\right) = \boxed{5.79\ \text{cm}}$

45.25 $1\ \text{rad} = 10^{-2}\ \text{J / kg}$ $\quad Q = mc\Delta T \quad \mathcal{P}t = mc\Delta T$

$$t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4186\ \text{J / kg}\cdot{}^\circ\text{C})(50.0\ {}^\circ\text{C})}{(10)\left(10^{-2}\ \text{J / kg}\cdot\text{s}\right)(m)} = \boxed{2.09\times10^6\ \text{s}} \approx 24\ \text{days!}$$

Note that power is the product of dose rate and mass.

45.26 $$\frac{Q}{m} = \frac{\text{absorbed energy}}{\text{unit mass}} = (1000\ \text{rad})\frac{10^{-2}\ \text{J / kg}}{1\ \text{rad}} = 10.0\ \text{J / kg}$$

The rise in body temperature is calculated from $Q = mc\,\Delta T$ where $c = 4186\ \text{J/kg}\cdot{}^\circ\text{C}$ for water and the human body

$$\Delta T = \frac{Q}{mc} = (10.0\ \text{J / kg})\frac{1}{4186\ \text{J / kg}\cdot{}^\circ\text{C}} = \boxed{2.39\times10^{-3}\ {}^\circ\text{C}} \quad \text{(Negligible)}$$

45.27 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right] = 4.26\times10^{12}\ \text{MeV}$$

$$E = \left(4.26\times10^{12}\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 0.682\ \text{J}$$

Thus, the dose received is $\text{Dose} = \frac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\frac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$

45.28 The nuclei initially absorbed are $N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$

The number of decays in time t is $\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$

At the end of 1 year, $\dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.0344$

and $\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1-e^{-0.0238}\right) = 1.58\times10^{11}$

The energy deposited is $E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.0277\text{ J}$

Thus, the dose received is $\text{Dose} = \left(\dfrac{0.0277\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.0396\text{ rad}$

45.29 (a) $\dfrac{E}{E_\beta} = \dfrac{\frac{1}{2}C(\Delta V)^2}{0.500\text{ MeV}} = \dfrac{\frac{1}{2}\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J / MeV}\right)} = \boxed{3.12\times10^{7}}$

(b) $N = \dfrac{Q}{e} = \dfrac{C(\Delta V)}{e} = \dfrac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}} = \boxed{3.12\times10^{10}\text{ electrons}}$

45.30 (a) $\text{amplification} = \dfrac{\text{energy discharged}}{E} = \dfrac{\frac{1}{2}C(\Delta V)^2}{E} = \boxed{\dfrac{C(\Delta V)^2}{2E}}$

(b) $N = \dfrac{\text{charge released}}{\text{charge of electron}} = \boxed{\dfrac{C(\Delta V)}{e}}$

45.31 (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e(\Delta V)/E_I$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^{1}\text{ electrons}}$

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \frac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$.

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \frac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$.

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6\, e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

***45.32** (a) The average time between slams is $60\text{ min}/38 = 1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2\times 1.6\text{ min}$. Perhaps about half as often, it is $4\times 1.6\text{ min}$. Somewhere around $5\times 1.6\text{ min} = \boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long wait might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R = R_0 e^{-\lambda t}$. Subtracting the background counts,

$$337 - 5(15) = [372 - 5(15)]e^{-(\ln 2/T_{1/2})(5.00\text{ min})}$$

or $\ln\left(\frac{262}{297}\right) = \ln(0.882) = -3.47\text{ min}/T_{1/2}$ which yields $T_{1/2} = \boxed{27.6\text{ min}}$.

(c) As in the random events in part (a), we imagine a ± 5 count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -3.47\text{ min}/T_{1/2},\ \text{or}\ \left(T_{1/2}\right)_{\min} = 21.1\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right) = -3.47\text{ min}/T_{1/2},\ \text{yielding}\ \left(T_{1/2}\right)_{\max} = 38.8\text{ min}$$

Thus, $T_{1/2} = \left(\frac{38.8+21.1}{2}\right) \pm \left(\frac{38.8-21.1}{2}\right)\text{ min} = (30\pm 9)\text{ min} = \boxed{30\text{ min} \pm 30\%}$

45.33 The initial specific activity of ^{59}Fe in the steel,

$$(R/m)_0 = \frac{20.0\ \mu\text{Ci}}{0.200\ \text{kg}} = \frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times10^4\ \text{Bq}}{1\ \mu\text{Ci}}\right) = 3.70\times10^6\ \text{Bq/kg}$$

After 1000 h, $$\frac{R}{m} = (R/m)_0 e^{-\lambda t} = (3.70\times10^6\ \text{Bq/kg})e^{-(6.40\times10^{-4}\ \text{h}^{-1})(1000\ \text{h})} = 1.95\times10^6\ \text{Bq/kg}$$

The activity of the oil, $$R_{\text{oil}} = \left(\frac{800}{60.0}\ \text{Bq/liter}\right)(6.50\ \text{liters}) = 86.7\ \text{Bq}$$

Therefore, $$m_{\text{in oil}} = \frac{R_{\text{oil}}}{(R/m)} = \frac{86.7\ \text{Bq}}{1.95\times10^6\ \text{Bq/kg}} = 4.45\times10^{-5}\ \text{kg}$$

So that wear rate is $$\frac{4.45\times10^{-5}\ \text{kg}}{1000\ \text{h}} = \boxed{4.45\times10^{-8}\ \text{kg/h}}$$

***45.34** The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{70.6\ \text{s}} = 0.009\,82\ \text{s}^{-1}$

The ^{14}O nuclei remaining after five min is $N = N_0 e^{-\lambda t} = \left(10^{10}\right)e^{-\left(0.009\,82\ \text{s}^{-1}\right)(300\ \text{s})} = 5.26\times10^8$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00\ \text{cm}^3}{\text{total vol. of blood}}\right) = \left(5.26\times10^8\right)\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right) = 2.63\times10^5$$

and their activity is $R = \lambda N' = \left(0.009\,82\ \text{s}^{-1}\right)\left(2.63\times10^5\right) = 2.58\times10^3\ \text{Bq}$ $\boxed{\sim10^3\ \text{Bq}}$

***45.35** (a) The number of photons is $10^4\ \text{MeV}/1.04\ \text{MeV} = 9.62\times10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92\times10^4$ and $N_0 = 2.56\times10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is
$N_{63} = 0.6917\,N_{\text{Cu}}$ and $N_{65} = 0.3083\,N_{\text{Cu}}$.

Therefore, $$\frac{N_{63}}{N_{65}} = \frac{0.6917}{0.3083} \quad\text{or}\quad N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56\times10^6\right) = 5.75\times10^6$$

The total mass of copper present is then $m_{\text{Cu}} = (62.93\ \text{u})N_{63} + (64.93\ \text{u})N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75\times10^6\right) + (64.93)\left(2.56\times10^6\right)\right]\text{u}\left(1.66\times10^{-24}\ \text{g/u}\right) = 8.77\times10^{-16}\ \text{g}\ \boxed{\sim10^{-15}\ \text{g}}$$

45.36 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)dt$$

The variables are separable.

$$\int_{N=0}^{N} \frac{dN}{R - \lambda N} = \int_{t=0}^{t} dt$$

$$-\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t \qquad \text{so} \qquad \ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t$$

$$\left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t} \qquad \text{and} \qquad 1 - \frac{\lambda}{R}N = e^{-\lambda t}$$

Therefore,

$$N = \boxed{\frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)}$$

(b) The maximum number of radioactive nuclei would be $\boxed{R / \lambda}$

45.37 (a) At 6×10^8 K, each carbon nucleus has thermal energy of

$$\tfrac{3}{2}k_B T = (1.5)(8.62 \times 10^{-5} \text{ eV / K})(6 \times 10^8 \text{ K}) = \boxed{8 \times 10^4 \text{ eV}}$$

(b) The energy released is

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Ne}^{20}\right) - m\left(\text{He}^4\right)\right]c^2$$

$$E = (24.000\,000 - 19.992\,435 - 4.002\,602)(931.5) \text{ MeV} = \boxed{4.62 \text{ MeV}}$$

In the second reaction,

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Mg}^{24}\right)\right](931.5)\text{MeV / u}$$

$$E = (24.000\,000 - 23.985\,042)(931.5)\text{MeV} = \boxed{13.9 \text{ MeV}}$$

(c) The energy released is the energy of reaction of the # of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00 \times 10^3 \text{ g}\right)\left(\frac{6.02 \times 10^{23} \text{ atoms / mol}}{12.0 \text{ g / mol}}\right)\left(\frac{4.62 \text{ MeV / fusion event}}{2 \text{ nuclei / fusion event}}\right)\left(\frac{1 \text{ kWh}}{2.25 \times 10^{19} \text{ MeV}}\right)$$

$$\Delta E = \frac{\left(1.00 \times 10^{26}\right)(4.62)}{2\left(2.25 \times 10^{19}\right)} \text{ kWh} = \boxed{1.03 \times 10^7 \text{ kWh}}$$

45.38 (a) Suppose each ^{235}U fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5\times10^{13}\text{ J}}{(208\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)} = \boxed{1.5\times10^{24}\text{ nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5\times10^{24}\text{ nuclei}}{6.02\times10^{23}\text{ nuclei/mol}}\right)(235\text{ g/mol}) \approx \boxed{0.6\text{ kg}}$$

45.39 For a typical ^{235}U, $Q = 208$ MeV; and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208\text{ MeV}}{(235\text{ u})(931.5\text{ MeV/u})} = 9.50\times10^{-4} = \boxed{0.0950\%}$$

For the D-T fusion reaction, $Q = 17.6$ MeV

The initial mass is $m = (2.014\text{ u}) + (3.016\text{ u}) = 5.03\text{ u}$

The fractional loss in this reaction is $$\frac{Q}{mc^2} = \frac{17.6\text{ MeV}}{(5.03\text{ u})(931.5\text{ MeV/u})} = 3.75\times10^{-3} = \boxed{0.375\%}$$

$$\frac{0.375\%}{0.0950\%} = 3.95 \quad \text{or} \quad \boxed{\text{the fractional loss in D} - \text{T fusion is about 4 times that in }^{235}\text{U fission}}$$

.

45.40 To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{m_1}{m_2}\right)v_1$$

The kinetic energies after the break-up are then

$$K_1 = \tfrac{1}{2}m_1 v_1^2 \quad \text{and} \quad K_2 = \tfrac{1}{2}m_2 v_2^2 = \tfrac{1}{2}m_2\left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right)K_1$$

The fraction of the total kinetic energy carried off by m_1 is $$\frac{K_1}{K_1+K_2} = \frac{K_1}{K_1 + (m_1/m_2)K_1} = \boxed{\frac{m_2}{m_1+m_2}}$$

and the fraction carried off by m_2 is $$1 - \frac{m_2}{m_1+m_2} = \boxed{\frac{m_1}{m_1+m_2}}$$

45.41 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12.3 \text{ yr})(3.16 \times 10^7 \text{ s/yr})} = 1.78 \times 10^{-9} \text{ s}^{-1}$

The tritium in the plasma decays at a rate of

$$R = \lambda N = (1.78 \times 10^{-9} \text{ s}^{-1})\left[\left(\frac{2.00 \times 10^{14}}{\text{cm}^3}\right)\left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right)(50.0 \text{ m}^3)\right]$$

$$R = 1.78 \times 10^{13} \text{ Bq} = (1.78 \times 10^{13} \text{ Bq})\left(\frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ Bq}}\right) = \boxed{482 \text{ Ci}}$$

$\boxed{\text{The fission inventory is } \frac{4 \times 10^{10} \text{ Ci}}{482 \text{ Ci}} \sim 10^8 \text{ times greater}}$ than this amount.

Goal Solution

The half-life of tritium is 12.3 yr. If the TFTR fusion reactor contained 50.0 m^3 of tritium at a density equal to $2.00 \times 10^{14} \text{ ions/cm}^3$, how many curies of tritium were in the plasma? Compare this value with a fission inventory (the estimated supply of fissionable material) of 4×10^{10} Ci.

G: It is difficult to estimate the activity of the tritium in the fusion reactor without actually calculating it; however, we might expect it to be a small fraction of the fission (not fusion) inventory.

O: The decay rate (activity) can be found by multiplying the decay constant λ by the number of ${}^3_1\text{H}$ particles. The decay constant can be found from the half-life of tritium, and the number of particles from the density and volume of the plasma.

A: The number of Hydrogen-3 nuclei is

$$N = (50.0 \text{ m}^3)\left(2.00 \times 10^{14} \frac{\text{particles}}{\text{m}^3}\right)\left(100 \frac{\text{cm}}{\text{m}}\right)^3 = 1.00 \times 10^{22} \text{ particles}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{12.3 \text{ yr}}\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) = 1.78 \times 10^{-9} \text{ s}^{-1}$$

The activity is then

$$R = \lambda N = (1.78 \times 10^{-9} \text{ s}^{-1})(1.00 \times 10^{22} \text{ nuclei}) = 1.78 \times 10^{13} \text{ Bq} = (1.78 \times 10^{13} \text{ Bq})\left(\frac{1 \text{ Ci}}{3.70 \times 10^{10} \text{ Bq}}\right) = 482 \text{ Ci}$$

L: Even though 482 Ci is a large amount of radioactivity, it is smaller than 4.00×10^{10} Ci by about a hundred million. Therefore, loss of containment is a smaller hazard for a fusion power reactor than for a fission reactor.

45.42 Momentum conservation: $0 = m_{\text{Li}}\mathbf{v}_{\text{Li}} + m_\alpha \mathbf{v}_\alpha$, or, $m_{\text{Li}} v_{\text{Li}} = m_\alpha v_\alpha$

Thus,

$$K_{\text{Li}} = \frac{1}{2} m_{\text{Li}} v_{\text{Li}}^2 = \frac{1}{2}\frac{(m_{\text{Li}} v_{\text{Li}})^2}{m_{\text{Li}}} = \frac{(m_\alpha v_\alpha)^2}{2m_{\text{Li}}} = \left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right) v_\alpha^2$$

$$K_{\text{Li}} = \left(\frac{(4.002\,6 \text{ u})^2}{2(7.016\,9 \text{ u})}\right)(9.30 \times 10^6 \text{ m/s})^2 = (1.14 \text{ u})(9.30 \times 10^6 \text{ m/s})^2$$

$$K_{\text{Li}} = 1.14(1.66 \times 10^{-27} \text{ kg})(9.30 \times 10^6 \text{ m/s})^2 = 1.64 \times 10^{-13} \text{ J} = \boxed{1.02 \text{ MeV}}$$

45.43 The complete fissioning of 1.00 gram of U^{235} releases

$$\Delta Q = \frac{(1.00 \text{ g})}{235 \text{ grams / mol}}\left(6.02 \times 10^{23} \ \frac{\text{atoms}}{\text{mol}}\right)\left(200 \ \frac{\text{MeV}}{\text{fission}}\right)\left(1.60 \times 10^{-13} \ \frac{\text{J}}{\text{MeV}}\right) = 8.20 \times 10^{10} \text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values), then

$$\Delta Q = mc_w \Delta T + mL_v + mc_s \Delta T$$

$$\Delta Q = m\left[(4186 \text{ J / kg}\cdot{}^\circ\text{C})(80.0\ {}^\circ\text{C}) + 2.26 \times 10^6 \text{ J / kg} + (2010 \text{ J / kg}\cdot{}^\circ\text{C})(300\ {}^\circ\text{C})\right]$$

Therefore $$m = \frac{8.20 \times 10^{10} \text{ J}}{3.20 \times 10^6 \text{ J / kg}} = \boxed{2.56 \times 10^4 \text{ kg}}$$

45.44 When mass m of ^{235}U undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q = \left(\frac{m}{235 \text{ g/mol}}\right) N_{\text{A}}(200 \text{ MeV})$$ where N_{A} is Avogadro's number.

If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then,

$$Q = m_w\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]} = \boxed{\frac{m N_{\text{A}}(200 \text{ MeV})}{(235 \text{ g/mol})\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]}}$$

45.45 (a) The number of molecules in 1.00 liter of water (mass = 1000 g) is

$$N = \left(\frac{1.00 \times 10^3 \text{ g}}{18.0 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ molecules/mol}\right) = 3.34 \times 10^{25} \text{ molecules}$$

The number of deuterium nuclei contained in these molecules is

$$N' = \left(3.34 \times 10^{25} \text{ molecules}\right)\left(\frac{1 \text{ deuteron}}{3300 \text{ molecules}}\right) = 1.01 \times 10^{22} \text{ deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $N'/2 = 5.07 \times 10^{21}$ reactions, and the energy released is

$$E_{\text{fusion}} = \left(5.07 \times 10^{21} \text{ reactions}\right)\left(3.27 \text{ MeV/reaction}\right) = 1.66 \times 10^{22} \text{ MeV}$$

$$E_{\text{fusion}} = \left(1.66 \times 10^{22} \text{ MeV}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right) = \boxed{2.65 \times 10^9 \text{ J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}} = \frac{2.65 \times 10^9 \text{ J}}{3.40 \times 10^7 \text{ J}} = \boxed{78.0 \text{ times larger}}\ .$$

45.46 The number of nuclei in 0.155 kg of ^{210}Po is

$$N_0 = \left(\frac{155 \text{ g}}{209.98 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/g}\right) = 4.44 \times 10^{23} \text{ nuclei}$$

The half-life of ^{210}Po is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38 \text{ d})\left(8.64 \times 10^4 \text{ s/d}\right)} = 5.80 \times 10^{-8} \text{ s}^{-1}$$

The initial activity is $R_0 = \lambda N_0 = \left(5.80 \times 10^{-8} \text{ s}^{-1}\right)\left(4.44 \times 10^{23} \text{ nuclei}\right) = 2.58 \times 10^{16} \text{ Bq}$

The energy released in each ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is $Q = \left[M_{{}^{210}_{84}\text{Po}} - M_{{}^{206}_{82}\text{Pb}} - M_{{}^{4}_{2}\text{He}}\right]c^2$:

$$Q = \left[209.982\ 848 - 205.974\ 440 - 4.002\ 602\right]\text{u}\left(931.5 \ \frac{\text{MeV}}{\text{u}}\right) = 5.41 \text{ MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.0100)R_0 Q = (0.0100)\left(2.58 \times 10^{16} \ \frac{\text{decays}}{\text{s}}\right)\left(5.41 \ \frac{\text{MeV}}{\text{decay}}\right)\left(1.60 \times 10^{-13} \ \frac{\text{J}}{\text{MeV}}\right) = \boxed{223 \text{ W}}$$

45.47 (a) The thermal power transferred to the water is $\mathcal{P}_w = 0.970(\text{waste heat})$

$$\mathcal{P}_w = 0.970(3065 - 1000)\,\text{MW} = 2.00 \times 10^9\ \text{J/s}$$

r_w is the mass of heated per hour: $r_w = \dfrac{\mathcal{P}_w}{c(\Delta T)} = \dfrac{(2.00 \times 10^9\ \text{J/s})(3600\ \text{s/h})}{(4186\ \text{J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})} = \boxed{4.91 \times 10^8\ \text{kg/h}}$

The volume used per hour is $\dfrac{4.91 \times 10^8\ \text{kg/h}}{1.00 \times 10^3\ \text{kg/m}^3} = \boxed{4.91 \times 10^5\ \text{m}^3/\text{h}}$

(b) The ^{235}U fuel is consumed at a rate $r_f = \left(\dfrac{3065 \times 10^6\ \text{J/s}}{7.80 \times 10^{10}\ \text{J/g}}\right)\left(\dfrac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right) = \boxed{0.141\ \text{kg/h}}$

***45.48** (a) $\Delta V = 4\pi r^2(\Delta r) = 4\pi(14.0 \times 10^3\ \text{m})^2(0.05\ \text{m}) = 1.23 \times 10^8\ \text{m}^3$ $\boxed{\sim 10^8\ \text{m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W = P(\Delta V) = \left(1.013 \times 10^5\ \frac{\text{N}}{\text{m}^2}\right)(1.23 \times 10^8\ \text{m}^3) = 1.25 \times 10^{13}\ \text{J} \quad \boxed{\sim 10^{13}\ \text{J}}$$

(c) $1.25 \times 10^{13}\ \text{J} = \frac{1}{10}(\text{yield})$, so yield $= 1.25 \times 10^{14}\ \text{J}$ $\boxed{\sim 10^{14}\ \text{J}}$

(d) $\dfrac{1.25 \times 10^{14}\ \text{J}}{4.2 \times 10^9\ \text{J/ton TNT}} = 2.97 \times 10^4\ \text{ton TNT}\ \sim 10^4\ \text{ton TNT}$ or $\boxed{\sim 10\ \text{kilotons}}$

***45.49** (a) $V = \ell^3 = \dfrac{m}{\rho}$, so $\ell = \left(\dfrac{m}{\rho}\right)^{1/3} = \left(\dfrac{70.0\ \text{kg}}{18.7 \times 10^3\ \text{kg/m}^3}\right)^{1/3} = \boxed{0.155\ \text{m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes $^{238}_{92}\text{U atom} \to 8\,^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}$.

$$Q_{\text{net}} = \left[M_{^{238}_{92}\text{U}} - 8M_{^{4}_{2}\text{He}} - M_{^{206}_{82}\text{Pb}}\right]c^2 = [238.050\,784 - 8(4.002\,602) - 205.974\,440]\,\text{u}\ (931.5\ \text{MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7\ \text{MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00\times10^{4}\ \text{g}}{238\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{nuclei/mol}\right) = 1.77\times10^{26}\ \text{nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^{9}\ \text{yr}} = 1.55\times10^{-10}\ \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55\times10^{-10}\ \frac{1}{\text{yr}}\right)\left(1.77\times10^{26}\ \text{nuclei}\right) = 2.75\times10^{16}\ \text{decays/yr},$$

so $$\mathcal{P} = QR = (51.7\ \text{MeV})\left(2.75\times10^{16}\ \frac{1}{\text{yr}}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = \boxed{2.27\times10^{5}\ \text{J/yr}}$$

(e) dose in rem = dose in rad x RBE

$$5.00\ \frac{\text{rem}}{\text{yr}} = \left(\text{dose in}\ \frac{\text{rad}}{\text{yr}}\right)1.10,\ \text{giving}\ \left(\text{dose in}\ \frac{\text{rad}}{\text{yr}}\right) = 4.55\ \frac{\text{rad}}{\text{yr}}$$

The allowed whole-body dose is then $(70.0\ \text{kg})\left(4.55\ \frac{\text{rad}}{\text{yr}}\right)\left(\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}}\right) = \boxed{3.18\ \text{J/yr}}$

45.50 $E_T \equiv E(\text{thermal}) = \frac{3}{2}k_BT = 0.039\ \text{eV}$

$E_T = \left(\frac{1}{2}\right)^{n} E$ where $n \equiv$ number of collisions, and $0.039 = \left(\frac{1}{2}\right)^{n}\left(2.0\times10^{6}\right)$

Therefore, $n = 25.6 = \boxed{26\ \text{collisions}}$

45.51 From conservation of energy: $K_\alpha + K_n = Q$ or $\frac{1}{2}m_\alpha {v_\alpha}^2 + \frac{1}{2}m_n {v_n}^2 = 17.6\ \text{MeV}$

Conservation of momentum: $m_\alpha v_\alpha = m_n v_n$ or $v_\alpha = \left(\frac{m_n}{m_\alpha}\right)v_n.$

The energy equation becomes: $\frac{1}{2}m_\alpha\left(\frac{m_n}{m_\alpha}\right)^2 v_n^2 + \frac{1}{2}m_n v_n^2 = \left(\frac{m_n + m_\alpha}{m_\alpha}\right)\left(\frac{1}{2}m_n v_n^2\right) = 17.6\ \text{MeV}$

Thus, $$K_n = \left(\frac{m_\alpha}{m_n + m_\alpha}\right)(17.6\ \text{MeV}) = \left(\frac{4.002\ 602}{1.008\ 665 + 4.002\ 602}\right) = \boxed{14.1\ \text{MeV}}$$

Goal Solution

Assuming that a deuteron and a triton are at rest when they fuse according to

$$^{2}\text{H} + {}^{3}\text{H} \to {}^{4}\text{He} + \text{n} + 17.6\ \text{MeV},$$

determine the kinetic energy acquired by the neutron.

G: The products of this nuclear reaction are an alpha particle and a neutron, with total kinetic energy of 17.6 MeV. In order to conserve momentum, the lighter neutron will have a larger velocity than the more massive alpha particle (which consists of two protons and two neutrons). Since the kinetic energy of the particles is proportional to the square of their velocities but only linearly proportional to their mass, the neutron should have the larger kinetic energy, somewhere between 8.8 and 17.6 MeV.

O: Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.

A: The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\mathbf{v}_n + m_\alpha\mathbf{v}_\alpha = 0 \qquad \text{or} \qquad (1.0087\ \text{u})v_n = (4.0026\ \text{u})v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E = \tfrac{1}{2}m_n v_n^2 + \tfrac{1}{2}m_\alpha v_\alpha^2 = \tfrac{1}{2}(1.0087\ \text{u})v_n^2 + \tfrac{1}{2}(4.0026\ \text{u})v_\alpha^2 = 17.6\ \text{MeV}$$

Substitute $v_\alpha = 0.2520v_n$: $\quad E = (0.50435\ \text{u})v_n^2 + (0.12710\ \text{u})v_n^2 = 17.6\ \text{MeV}\left(\dfrac{1\ \text{u}}{931.494\ \text{MeV}/c^2}\right)$

$$v_n = \sqrt{\frac{0.0189c^2}{0.63145}} = 0.173c = 5.19\times10^7\ \text{m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.0087\ \text{u})(0.173c)\left(\frac{931.494\ \text{MeV}/c^2}{\text{u}}\right) = 14.1\ \text{MeV}$$

L: The kinetic energy of the neutron is within the range we predicted. For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n\mathbf{v}_n + \gamma_\alpha m_\alpha\mathbf{v}_\alpha = 0 \qquad 1.0087\frac{v_n}{\sqrt{1-v_n^2/c^2}} = 4.0026\frac{v_\alpha}{\sqrt{1-v_\alpha^2/c^2}}$$

yielding $\quad \dfrac{v_\alpha^2}{c^2} = \dfrac{v_n^2}{15.746c^2 - 14.746v_n^2}$

Then $\quad (\gamma_n - 1)m_nc^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6\ \text{MeV}$

and $v_n = 0.171c$, implying that $(\gamma_n - 1)m_nc^2 = 14.0\ \text{MeV}$

45.52 From Table A.3, the half-life of ${}^{32}P$ is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26 \text{ d}} = 0.0486 \text{ d}^{-1} = 5.63 \times 10^{-7} \text{ s}^{-1}.$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22 \times 10^6 \text{ decay/s}}{5.63 \times 10^{-7} \text{ s}^{-1}} = 9.28 \times 10^{12} \text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28 \times 10^{12} \text{ nuclei}\right) e^{-\left(0.0486 \text{ d}^{-1}\right)(10.0 \text{ d})} = 5.71 \times 10^{12} \text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E = \left(3.57 \times 10^{12}\right)(700 \text{ keV})\left(1.60 \times 10^{-16} \frac{\text{J}}{\text{keV}}\right) = 0.400 \text{ J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400 \text{ J}}{0.100 \text{ kg}}\right)\left(\frac{1 \text{ rad}}{10^{-2} \text{ J/kg}}\right) = \boxed{400 \text{ rad}}$$

45.53 (a) The number of Pu nuclei in 1.00 kg = $\dfrac{6.02 \times 10^{23} \text{ nuclei/mol}}{239.05 \text{ g/mol}}(1000 \text{ g})$

The total energy = $(25.2 \times 10^{23} \text{ nuclei})(200 \text{ MeV}) = 5.04 \times 10^{26}$ MeV

$E = (5.04 \times 10^{26} \text{ MeV})(4.44 \times 10^{-20} \text{ kWh/MeV}) = \boxed{2.24 \times 10^{7} \text{ kWh}}$ or 22 million kWh

(b) $E = \Delta mc^2 = (3.016\,049 \text{ u} + 2.014\,102 \text{ u} - 4.002\,602 \text{ u} - 1.008\,665 \text{ u})(931.5 \text{ MeV/u})$

$E = \boxed{17.6 \text{ MeV for each D-T fusion}}$

(c) E_n = (Total number of D nuclei)(17.6)(4.44×10^{-20})

$E_n = (6.02 \times 10^{23})(1000/2.014)(17.6)(4.44 \times 10^{-20}) = \boxed{2.34 \times 10^{8} \text{ kWh}}$

(d) E_n = the number of C atoms in 1.00 kg × 4.20 eV

$E_n = (6.02 \times 10^{26}/12.0)(4.20 \times 10^{-6} \text{ MeV})(4.44 \times 10^{-20}) = \boxed{9.36 \text{ kWh}}$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels.

***45.54** Add two electrons to both sides of the given reaction. Then $4\,{}^{1}_{1}\text{H atom} \rightarrow {}^{4}_{2}\text{He atom} + Q$

where $Q = (\Delta m)c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})\left(1.60 \times 10^{-13}\ \text{J/MeV}\right) = 4.28 \times 10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.77 \times 10^{26}\ \text{J/s}}{\left(4.28 \times 10^{-12}\ \text{J}\right)/(4\ \text{protons})} = \boxed{3.53 \times 10^{38}\ \text{protons/s}}$$

***45.55** (a) $Q_{\text{I}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}\right]c^2$, and $Q_{\text{II}} = \left[M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives ${}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{\ 0}_{-1}\text{e} + {}^{1}_{1}\text{H} + {}^{1}_{1}\text{H} + {}^{\ 0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + 2\nu + Q_{\text{net}}$

or $4\,{}^{1}_{1}\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \rightarrow {}^{4}_{2}\text{He} + 2\nu + Q_{\text{net}}$

Adding two electrons to each side $4\,{}^{1}_{1}\text{H atom} \rightarrow {}^{4}_{2}\text{He atom} + Q_{\text{net}}$

Thus, $Q_{\text{net}} = \left[4M_{{}^{1}_{1}\text{H}} - M_{{}^{4}_{2}\text{He}}\right]c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{26.7\ \text{MeV}}$

45.56 (a) The mass of the pellet is $m = \rho V = \left(0.200\ \dfrac{\text{g}}{\text{cm}^3}\right)\left[\dfrac{4\pi}{3}\left(\dfrac{1.50 \times 10^{-2}\ \text{cm}}{2}\right)^3\right] = 3.53 \times 10^{-7}\ \text{g}$

The pellet consists of equal numbers of ${}^{2}\text{H}$ and ${}^{3}\text{H}$ atoms, so the average atomic weight is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53 \times 10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02 \times 10^{23}\ \text{atoms/mol}\right) = 8.51 \times 10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T = \frac{E}{3Nk_{\text{B}}} = \frac{2.00 \times 10^3\ \text{J}}{3\left(8.51 \times 10^{16}\right)\left(1.38 \times 10^{-23}\ \text{J/K}\right)} = \boxed{5.68 \times 10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so $N/2$ events will occur. The energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51 \times 10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}}\right) = 1.20 \times 10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

***45.57** (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^1_1\text{H} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + \text{e}^+ + \nu$, estimated as $k_e(e)(2e)/r$. The Coulomb barrier to Bethe's fifth and eight reactions is like $k_e(e)(7e)/r$, larger by $\frac{7}{2}$ times, so the temperature should be like $\frac{7}{2}\left(15\times 10^6\text{ K}\right) \approx \boxed{5\times 10^7\text{ K}}$.

(b) For ${}^{12}\text{C} + {}^1\text{H} \rightarrow {}^{13}\text{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,738)(931.5\text{ MeV}) = \boxed{1.94\text{ MeV}}$$

For the second step, add seven electrons to both sides to have: ${}^{13}\text{N atom} \rightarrow {}^{13}\text{C atom} + \text{e}^- + \text{e}^+ + Q$.

$$Q_2 = [13.005\,738 - 13.003\,355 - 2(0.000\,549)](931.5\text{ MeV}) = \boxed{1.20\text{ MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\text{ MeV}) = \boxed{1.02\text{ MeV}}$$

$$Q_4 = [13.003\,355 + 1.007\,825 - 14.003\,074](931.5\text{ MeV}) = \boxed{7.55\text{ MeV}}$$

$$Q_5 = [14.003\,074 + 1.007\,825 - 15.003\,065](931.5\text{ MeV}) = \boxed{7.30\text{ MeV}}$$

$$Q_6 = [15.003\,065 - 15.000\,108 - 2(0.000\,549)](931.5\text{ MeV}) = \boxed{1.73\text{ MeV}}$$

$$Q_8 = [15.000\,108 + 1.007\,825 - 12 - 4.002\,602](931.5\text{ MeV}) = \boxed{4.97\text{ MeV}}$$

The sum is $\boxed{26.7\text{ MeV}}$, the same as for the proton-proton cycle.

(c) Not all of the energy released heats the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

45.58 (a) $\dfrac{I_2}{I_1} = \dfrac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2 - \mu_1)x}}$

(b) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(0.100)} = e^{3.56} = \boxed{35.2}$

(c) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(1.00)} = e^{35.6} = \boxed{2.89\times 10^{15}}$

Thus, a 1.00-cm aluminum plate has essentially removed the long-wavelength x-rays from the beam.

Chapter 46 Solutions

46.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy of the photon E, must be

$$E = 2E_0 = 2(938.3 \text{ MeV}) = 1876.6 \text{ MeV} = 3.00 \times 10^{-10} \text{ J}$$

Thus, $E = hf = 3.00 \times 10^{-10}$ J

$$f = \frac{3.00 \times 10^{-10} \text{ J}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{4.53 \times 10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{4.53 \times 10^{23} \text{ Hz}} = \boxed{6.62 \times 10^{-16} \text{ m}}$$

46.2 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy. Thus,

$$E_{\min} = hf_{\min} = \frac{(2E_0)}{2} = E_0 = 938.3 \text{ MeV}$$

Thus, $f_{\min} = \dfrac{(938.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{2.27 \times 10^{23} \text{ Hz}}$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.27 \times 10^{23} \text{ Hz}} = \boxed{1.32 \times 10^{-15} \text{ m}}$$

***46.3** In $\gamma \rightarrow p^+ + p^-$, we start with energy 2.09 GeV

we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + K_2

where K_2 is the kinetic energy of the second proton.

Conservation of energy gives $\boxed{K_2 = 118 \text{ MeV}}$

Goal Solution

A photon with an energy $E_\gamma = 2.09$ GeV creates a proton-antiproton pair in which the proton has a kinetic energy of 95.0 MeV. What is the kinetic energy of the antiproton? ($m_p c^2 = 938.3$ MeV).

G: An antiproton has the same mass as a proton, so it seems reasonable to expect that both particles will have similar kinetic energies.

O: The total energy of each particle is the sum of its rest energy and its kinetic energy. Conservation of energy requires that the total energy before this pair production event equal the total energy after.

A: $E_\gamma = \left(E_{Rp} + K_p\right) + \left(E_{R\bar{p}} + K_{\bar{p}}\right)$

The energy of the photon is given as $E_\gamma = 2.09\text{ GeV} = 2.09 \times 10^3\text{ MeV}$. From Table 46.2, we see that the rest energy of both the proton and the antiproton is

$$E_{Rp} = E_{R\bar{p}} = m_p c^2 = 938.3\text{ MeV}$$

If the kinetic energy of the proton is observed to be 95.0 MeV, the kinetic energy of the antiproton is

$$K_{\bar{p}} = E_\gamma - E_{R\bar{p}} - E_{Rp} - K_p = 2.09 \times 10^3\text{ MeV} - 2(938.5\text{ MeV}) - 95.0\text{ MeV} = 118\text{ MeV}$$

L: The kinetic energy of the antiproton is slightly (~20%) greater than the proton. The two particles most likely have different shares in momentum of the gamma ray, and therefore will not have equal energies, either.

***46.4** The reaction is $\mu^+ + e^- \to \nu + \nu$

muon-lepton number before reaction: (–1) + (0) = –1

electron-lepon number before reaction: (0) + (1) = 1

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$$\boxed{\bar{\nu}_\mu \quad \text{and} \quad \nu_e}$$

Then $\mu^+ + e^- \to \bar{\nu}_\mu + \nu_e$

46.5 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 93 \times 10^9$ eV. It can last no longer than $\Delta t = \hbar/2\Delta E$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\,\Delta E} = \frac{\left(6.626 \times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00 \times 10^8\text{ m/s}\right)}{4\pi\left(93 \times 10^9\text{ eV}\right)}\left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) = 1.06 \times 10^{-18}\text{ m} = \boxed{\sim 10^{-18}\text{ m}}$$

46.6 (a) $$\Delta E = (m_n - m_p - m_e)c^2$$

From Table A-3, $\Delta E = (1.008\,665 - 1.007\,825)931.5 = \boxed{0.782 \text{ MeV}}$

(b) Assuming the neutron at rest, momentum is conserved, $p_p = p_e$

relativistic energy is conserved, $\sqrt{(m_pc^2)^2 + p_p^2c^2} + \sqrt{(m_ec^2)^2 + p_e^2c^2} = m_nc^2$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6 \text{ MeV}$

Solving the algebra $pc = 1.19 \text{ MeV}$

If $p_ec = \gamma m_e v_e c = 1.19$ MeV, then $\dfrac{\gamma v_e}{c} = \dfrac{1.19 \text{ MeV}}{0.511 \text{ MeV}} = \dfrac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \dfrac{v_e}{c}$

Solving, $x^2 = (1 - x^2)5.43$ and $x = v_e/c = 0.919$

$$\boxed{v_e = 0.919c}$$

Then $m_pv_p = \gamma_e m_e v_e$:

$$v_p = \frac{\gamma_e m_e v_e c}{m_p c} = \frac{(1.19 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{(1.67 \times 10^{-27})(3.00 \times 10^8)} = 3.80 \times 10^5 \text{ m/s} = \boxed{380 \text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic, the proton is not.}}$

***46.7** The time for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3 \times 10^{-15} \text{ m}}{3 \times 10^8 \text{ m/s}} = \boxed{\sim 10^{-23} \text{ s}}$$

***46.8** With energy 938.3 MeV, the time that a virtual proton could last is at most Δt in $\Delta E \Delta t \sim \hbar$.

The distance it could move is at most

$$c\,\Delta t \sim \frac{\hbar c}{\Delta E} = \frac{(1.055 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{(938.3)(1.6 \times 10^{-13} \text{ J})} = \boxed{\sim 10^{-16} \text{ m}}$$

46.9 By Table 46.2, $M_{\pi^0} = 135\text{ MeV}/c^2$

Therefore, $E_\gamma = \boxed{67.5\text{ MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \frac{\text{MeV}}{c}} \quad \text{and} \quad f = \frac{E_\gamma}{h} = \boxed{1.63 \times 10^{22}\text{ Hz}}$$

***46.10** In $? + p^+ \to n + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\bar{\nu}_\mu}$.

46.11 $\Omega^+ \to \overline{\Lambda}^0 + K^+$

$\overline{K}_S^0 \to \pi^+ + \pi^-$ (or $\pi^0 + \pi^0$)

$\overline{\Lambda}^0 \to \overline{p} + \pi^+$

$\overline{n} \to \overline{p} + e^+ + \nu_e$

46.12
(a) $p + \overline{p} \to \mu^+ + e^-$ $\boxed{L_e}$ $0 + 0 \to 0 + 1$ and $\boxed{L_\mu}$ $0 + 0 \to -1 + 0$

(b) $\pi^- + p \to p + \pi^+$ $\boxed{\text{charge}}$ $-1 + 1 \to +1 + 1$

(c) $p + p \to p + \pi^+$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 0$

(d) $p + p \to p + p + n$ $\boxed{\text{baryon number}}$ $1 + 1 \to 1 + 1 + 1$

(e) $\gamma + p \to n + \pi^0$ $\boxed{\text{charge}}$ $0 + 1 \to 0 + 0$

***46.13** (a) Baryon number and charge are conserved, with values of 0 + 1 = 0 + 1 and 1 + 1 = 1 + 1 in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

46.14 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

46.15 (a) $\pi^- \to \mu^- + \boxed{\bar{\nu}_\mu}$ L_μ: $0 \to 1 - 1$

(b) $K^+ \to \mu^+ + \boxed{\nu_\mu}$ L_μ: $0 \to -1 + 1$

(c) $\boxed{\bar{\nu}_e} + p^+ \to n + e^+$ L_e: $-1 + 0 \to 0 - 1$

(d) $\boxed{\nu_e} + n \to p^+ + e^-$ L_e: $1 + 0 \to 0 + 1$

(e) $\boxed{\nu_\mu} + n \to p^+ + \mu^-$ L_μ: $1 + 0 \to 0 + 1$

(f) $\mu^- \to e^- + \boxed{\bar{\nu}_e} + \boxed{\nu_\mu}$ L_μ: $1 \to 0 + 0 + 1$ and L_e: $0 \to 1 - 1 + 0$

***46.16** Momentum conservation requires the pions to have equal speeds.

The total energy of each is 497.7 MeV/2

so $E^2 = p^2c^2 + (mc^2)^2$ gives $(248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2$

Solving, $pc = 206\text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$(v/c) = 1.48\sqrt{1-(v/c)^2}$ and $(v/c)^2 = 2.18\left[1-(v/c)^2\right] = 2.18 - 2.18(v/c)^2$

$3.18(v/c)^2 = 2.18$ so $\dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$ and $\boxed{v = 0.828c}$

46.17 (a) $p^+ \to \pi^+ + \pi^0$ $\boxed{\text{Baryon number}}$ is violated: $1 \to 0 + 0$

(b) $p^+ + p^+ \to p^+ + p^+ + \pi^0$ This reaction $\boxed{\text{can occur}}$.

(c) $p^+ + p^+ \to p^+ + \pi^+$ $\boxed{\text{Baryon number}}$ is violated: $1 + 1 \to 1 + 0$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^0 \to p^+ + e^- + \bar{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^+ \to \mu^+ + n$ Violates $\boxed{\text{baryon number}}$: $0 \to 0 + 1$

Violates muon-lepton number : $0 \to -1 + 0$

46.18 (a) $p \to e^+ + \gamma$ Baryon number: $+1 \to 0 + 0$ $\Delta B \neq 0$, so baryon number is violated.

(b) From conservation of momentum: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = (p_e c)^2 + E_{0,e}^2$ becomes $E_e^2 = (p_\gamma c)^2 + E_{0,e}^2 = E_\gamma^2 + E_{0,e}^2$

From conservation of energy: $E_{0,p} = E_e + E_\gamma$ or $E_e = E_{0,p} - E_\gamma$

so $E_e^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$.

Equating this to the result from above gives $E_\gamma^2 + E_{0,e}^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$,

or $$E_\gamma = \frac{E_{0,p}^2 - E_{0,e}^2}{2E_{0,p}} = \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = \boxed{469\text{ MeV}}$$

Thus, $E_e = E_{0,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}$

Also, $p_\gamma = \frac{E_\gamma}{c} = \boxed{469\text{ MeV/c}}$ and $p_e = p_\gamma = \boxed{469\text{ MeV}/c}$

(c) The total energy of the positron is $E_e = 469$ MeV.

But, $E_e = \gamma E_{0,e} = \frac{E_{0,e}}{\sqrt{1-(v/c)^2}}$ so $\sqrt{1-(v/c)^2} = \frac{E_{0,e}}{E_e} = \frac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09 \times 10^{-3}$

which yields: $\boxed{v = 0.999\,999\,4c}$

***46.19** The relevant conservation laws are: $\Delta L_e = 0$, $\Delta L_\mu = 0$, and $\Delta L_\tau = 0$.

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $L_e:\ 0 \to 0 - 1 + L_e \Rightarrow L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $L_\mu:\ L_\mu + 0 \to +1 + 0 + 0 \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ $L_\mu:\ 0 \to 0 + 1 + L_\mu \Rightarrow L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $L_\mu:\ 0 \to -1 + L_\mu \Rightarrow L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

$L_\tau:\ +1 \to 0 + L_\tau \Rightarrow L_\tau = 1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = 1$ for the other particle.

We have $\boxed{\nu_\mu}$ and $\boxed{\bar{\nu}_\tau}$.

46.20 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.

The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

46.21 (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is **not conserved**)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ (0 = 0 and strangeness is **conserved**)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ (0 = 0 and strangeness is **conserved**)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is **not conserved**)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is **not conserved**)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is **not conserved**)

46.22 (a) $\mu^- \to e^- + \gamma$ L_e: $0 \to 1 + 0$, and L_μ: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ L_e: $0 \to 0 + 1 + 1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0 + 0$, and charge: $0 \to +1 + 0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0 + 0$

***46.23** (a) $\pi^- + p \to 2\eta$ violates conservation of baryon number as $0 + 1 \to 0$. **not allowed**

(b) $K^- + n \to \Lambda^0 + \pi^-$

Baryon number = $0 + 1 \to 1 + 0$ Charge = $-1 + 0 \to 0 - 1$

Strangeness, $-1 + 0 \to -1 + 0$ Lepton number, $0 \to 0$

The interaction may occur via the **strong interaction** since all are conserved.

(c) $K^- \to \pi^- + \pi^0$

Strangeness, $-1 \to 0 + 0$ Baryon number, $0 \to 0$

Lepton number, $0 \to 0$ Charge, $-1 \to -1 + 0$

Strangeness is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the **weak interaction**, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$

Baryon number, $1 \to 1 + 0$ Lepton number, $0 \to 0$

Charge, $-1 \to -1 + 0$ Strangeness, $-3 \to -2 + 0$

May occur by **weak interaction**, but not by strong or electromagnetic.

(e) $\eta \to 2\gamma$

Baryon number, $0 \to 0$ Lepton number, $0 \to 0$

Charge, $0 \to 0$ Strangeness, $0 \to 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

***46.24** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number: + 1 → + 1 + 0 + 0 Charge: –1 → 0 – 1 + 0

L_e: 0 → 0 + 0 + 0 L_μ: 0 → 0 + 1 + 1

L_τ: 0 → 0 + 0 + 0 Strangeness: –2 → –1 + 0 + 0

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, and } L_\tau}$

(b) $K_S^0 \to 2\pi^0$

Baryon number: 0 → 0 Charge: 0 → 0

L_e: 0 → 0 L_μ: 0 → 0

L_τ: 0 → 0 Strangeness: +1 → 0

Conserved quantities are: $\boxed{B\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

Baryon number: 0 + 1 → 1 + 1 Charge: –1 + 1 → 0 + 0

L_e: 0 + 0 → 0 + 0 L_μ: 0 + 0 → 0 + 0

L_τ: 0 + 0 → 0 + 0 Strangeness: –1 + 0 → –1 + 0

Conserved quantities are: $\boxed{S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(d) $\Sigma^0 \to \Lambda^0 + \gamma$

Baryon number: +1 → 1 + 0 Charge: 0 → 0

L_e: 0 → 0 + 0 L_μ: 0 → 0 + 0

L_τ: 0 → 0 + 0 Strangeness: –1 → –1 + 0

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number: 0 + 0 → 0 + 0 Charge: +1 – 1 → +1 – 1

L_e: –1 + 1 → 0 + 0 L_μ: 0 + 0 → + 1 – 1

L_τ: 0 + 0 → 0 + 0 Strangeness: 0 + 0 → 0 + 0

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

(f) $\overline{p} + n \to \overline{\Lambda}^0 + \Sigma^-$

Baryon number: –1 + 1 → –1 + 1 Charge: –1 + 0 → 0 – 1

L_e: 0 + 0 → 0 + 0 L_μ: 0 + 0 → 0 + 0

L_τ: 0 + 0 → 0 + 0 Strangeness: 0 + 0 → +1 – 1

Conserved quantities are: $\boxed{B\text{, } S\text{, charge, } L_e\text{, } L_\mu\text{, and } L_\tau}$

***46.25** (a) $K^+ + p \to \underline{?} + p$

The strong interaction conserves everything.

Baryon number,	$0+1 \to B+1$	so	$B=0$
Charge,	$+1+1 \to Q+1$	so	$Q=+1$
Lepton numbers,	$0+0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1+0 \to S+0$	so	$S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to \underline{?} + \pi^-$

Baryon number,	$+1 \to B+0$	so	$B=1$
Charge,	$-1 \to Q-1$	so	$Q=0$
Lepton numbers,	$0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \to S+0$	so $\Delta S = 1$:	$S=-2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \to \underline{?} + \mu^+ + \nu_\mu$:

Baryon number,	$0 \to B+0+0$	so	$B=0$
Charge,	$+1 \to Q+1+0$	so	$Q=0$
Lepton Numbers	L_e, $0 \to L_e + 0 + 0$	so	$L_e = 0$
	L_μ, $0 \to L_\mu - 1 + 1$	so	$L_\mu = 0$
	L_τ, $0 \to L_\tau + 0 + 0$	so	$L_\tau = 0$
Strangeness:	$1 \to S+0+0$	so	$\Delta S = \pm 1$ (for weak interaction): $S=0$

The particle must be a neutral meson with strangeness = 0 $\Rightarrow$ $\boxed{\pi^0}$.

***46.26** (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

***46.27** (a) The number of protons $N_p = 1000\text{ g}\left(\dfrac{6.02 \times 10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\dfrac{10\text{ protons}}{\text{molecule}}\right) = 3.34 \times 10^{26}\text{ protons}$

and there are $N_n = (1000\text{ g})\left(\dfrac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\dfrac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times10^{26}\text{ neutrons}$

So there are for electric neutrality $\boxed{3.34\times10^{26}\text{ electrons}}$

The up quarks have number $2\times3.34\times10^{26} + 2.68\times10^{26} = \boxed{9.36\times10^{26}\text{ up quarks}}$

and there are $2\times2.68\times10^{26} + 3.34\times10^{26} = \boxed{8.70\times10^{26}\text{ down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain $65\times3.34\times10^{26}$ $\boxed{\sim10^{28}\text{ electrons}}$

$65\times9.36\times10^{26}$ $\boxed{\sim10^{29}\text{ up quarks}}$

$65\times8.70\times10^{26}$ $\boxed{\sim10^{29}\text{ down quarks}}$

Only these fundamental particles form your body. You have no strangeness, charm, topness or bottomness.

46.28 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

46.29 Quark composition of proton = uud and of neutron = udd.

Thus, if we neglect binding energies, we may write $m_p = 2m_u + m_d$ (1)

and $m_n = m_u + 2m_d$ (2)

Solving simultaneously, we find

$$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938.3\text{ MeV}/c^2) - 939.6\text{ MeV}/c^2\right] = \boxed{312\text{ MeV}/c^2}$$

and from either (1) or (2), $m_d = \boxed{314\text{ MeV}/c^2}$

***46.30** In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + p \to K^0 + n$, the quarks in the particles are: $\bar{u}d + uud \to d\bar{s} + udd$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

46.31 (a) $\pi^- + p \to K^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{u}d + uud \to d\bar{s} + uds}$.

up quarks:	$-1 + 2 \to 0 + 1$, or $1 \to 1$
down quarks:	$1 + 1 \to 1 + 1$, or $2 \to 2$
strange quarks:	$0 + 0 \to -1 + 1$, or $0 \to 0$

(b) $\pi^+ + p \to K^+ + \Sigma^+ \Rightarrow$ $\boxed{u\bar{d} + uud \to u\bar{s} + uus}$

up quarks:	$1 + 2 \to 1 + 2$, or $3 \to 3$
down quarks:	$-1 + 1 \to 0 + 0$, or $0 \to 0$
strange quarks:	$0 + 0 \to -1 + 1$, or $0 \to 0$

(c) $K^- + p \to K^+ + K^0 + \Omega^- \Rightarrow$ $\boxed{\bar{u}s + uud \to u\bar{s} + d\bar{s} + sss}$

up quarks:	$-1 + 2 \to 1 + 0 + 0$, or $1 \to 1$
down quarks:	$0 + 1 \to 0 + 1 + 0$, or $1 \to 1$
strange quarks:	$1 + 0 \to -1 - 1 + 3$, or $1 \to 1$

(d) $p + p \to K^0 + p + \pi^+ + \underline{?} \Rightarrow$ $uud + uud \to d\bar{s} + uud + u\bar{d} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks:	$2 + 2 = 0 + 2 + 1 + ?$	(has 1 u quark)
down quarks:	$1 + 1 = 1 + 1 - 1 + ?$	(has 1 d quark)
strange quarks:	$0 + 0 = -1 + 0 + 0 + ?$	(has 1 s quark)

quark composite = uds = $\boxed{\Lambda^0 \text{ or } \Sigma^0}$

46.32 $\Sigma^0 + p \to \Sigma^+ + \gamma + X$ dds + uud → uds + 0 + ?

The left side has a net 3d, 2u and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

***46.33** Compare the given quark states to the entries in Tables 46.4 and 46.5.

(a) $\mathrm{suu} = \boxed{\Sigma^+}$

(b) $\bar{\mathrm{u}}\mathrm{d} = \boxed{\pi^-}$

(c) $\bar{\mathrm{s}}\mathrm{d} = \boxed{\mathrm{K}^0}$

(d) $\mathrm{ssd} = \boxed{\Xi^-}$

***46.34** (a) $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right)+\left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

***46.35** Section 39.4 says $$f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1+v_a/c}{1-v_a/c}}$$

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-v/c}{1+v/c}} \quad \text{and} \quad \boxed{\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}}$$

46.36 $v = HR$ (Equation 46.7) $$H = \frac{(1.7\times 10^{-2}\ \mathrm{m/s})}{\mathrm{ly}}$$

(a) $v\,(2.00\times 10^6\ \mathrm{ly}) = 3.4\times 10^4\ \mathrm{m/s}$ $$\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}} = 590(1.0001133) = \boxed{590.07\ \mathrm{nm}}$$

(b) $v\,(2.00\times 10^8\ \mathrm{ly}) = 3.4\times 10^6\ \mathrm{m/s}$ $$\lambda' = 590\sqrt{\frac{1+0.01133}{1-0.01133}} = \boxed{597\ \mathrm{nm}}$$

(c) $v\,(2.00\times 10^9\ \mathrm{ly}) = 3.4\times 10^7\ \mathrm{m/s}$ $$\lambda' = 590\sqrt{\frac{1+0.1133}{1-0.1133}} = \boxed{661\ \mathrm{nm}}$$

46.37 (a) $$\frac{\lambda'}{\lambda} = \frac{650\ \mathrm{nm}}{434\ \mathrm{nm}} = 1.50 = \sqrt{\frac{1+v/c}{1-v/c}} \qquad \frac{1+v/c}{1-v/c} = 2.24$$

$v = 0.383c$, $\boxed{38.3\%\ \text{the speed of light}}$

(b) Equation 46.7, $v = HR$ $$R = \frac{v}{H} = \frac{(0.383)(3.00\times 10^8)}{(1.7\times 10^{-2})} = \boxed{6.76\times 10^9\ \text{light years}}$$

Goal Solution

A distant quasar is moving away from Earth at such high speed that the blue 434-nm hydrogen line is observed at 650 nm, in the red portion of the spectrum. (a) How fast is the quasar receding? You may use the result of Problem 35. (b) Using Hubble's law, determine the distance from Earth to this quasar.

G: The problem states that the quasar is moving very fast, and since there is a significant red shift of the light, the quasar must be moving away from Earth at a relativistic speed ($v > 0.1c$). Quasars are very distant astronomical objects, and since our universe is estimated to be about 15 billion years old, we should expect this quasar to be $\sim 10^9$ light-years away.

O: As suggested, we can use the equation in Problem 35 to find the speed of the quasar from the Doppler red shift, and this speed can then be used to find the distance using Hubble's law.

A: (a) $\dfrac{\lambda'}{\lambda} = \dfrac{650\text{ nm}}{434\text{ nm}} = 1.498 = \sqrt{\dfrac{1+v/c}{1-v/c}}$ or squared, $\dfrac{1+v/c}{1-v/c} = 2.243$

Therefore, $v = 0.383c$ or 38.3% the speed of light

(b) Hubble's law asserts that the universe is expanding at a constant rate so that the speeds of galaxies are proportional to their distance R from Earth, $v = HR$

so, $$R = \frac{v}{H} = \frac{(0.383)\left(3.00\times10^{8}\text{ m / s}\right)}{\left(1.70\times10^{-2}\text{ m / s}\cdot\text{ly}\right)} = 6.76\times10^{9}\text{ ly}$$

L: The speed and distance of this quasar are consistent with our predictions. It appears that this quasar is quite far from Earth but not the most distant object in the visible universe.

***46.38** (a) $\lambda'_n = \lambda_n\sqrt{\dfrac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\qquad \dfrac{1+v/c}{1-v/c} = (Z+1)^2$

$$1 + \frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2$$

$$\left(\frac{v}{c}\right)(Z^2 + 2Z + 2) = Z^2 + 2Z$$

$$v = \boxed{c\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

(b) $$R = \frac{v}{H} = \boxed{\frac{c}{H}\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)}$$

***46.39** The density of the Universe is $\rho = 1.20\rho_c = 1.20\left(3H^2/8\pi G\right)$.

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4\pi r^3}{3}\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4\pi r^3}{3}\right) = \frac{0.600H^2r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy is constant as it moves to apogee distance R:

$$\tfrac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \quad \text{so} \quad \tfrac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \quad \text{so} \quad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

***46.40** (a) $k_BT \approx 2m_pc^2$ so $T \approx \frac{2m_pc^2}{k_B} = \frac{2(938.3\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{13}\text{ K}}$

(b) $k_BT \approx 2m_ec^2$ $T \approx \frac{2m_ec^2}{k_B} = \frac{2(0.511\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$ $\boxed{\sim 10^{10}\text{ K}}$

***46.41** (a) Wien's law: $\lambda_{\text{max}}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$

Thus, $\lambda_{\text{max}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2.73\text{ K}} = 1.06\times10^{-3}\text{ m} = \boxed{1.06\text{ mm}}$

(b) This is a $\boxed{\text{microwave}}$.

***46.42** (a) $L = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times10^8\text{ m/s}\right)^3}} = \boxed{1.61\times10^{-35}\text{ m}}$

(b) This time is given as $T = \frac{L}{c} = \frac{1.61\times10^{-35}\text{ m}}{3.00\times10^8\text{ m/s}} = \boxed{5.38\times10^{-44}\text{ s}}$,

which is approximately equal to the ultra-hot epoch.

46.43 (a) $\Delta E \Delta t \approx \hbar$, and $\Delta t \approx \dfrac{r}{c} = \dfrac{1.4 \times 10^{-15}\ \text{m}}{3.0 \times 10^{8}\ \text{m/s}} = 4.7 \times 10^{-24}\ \text{s}$

$$\Delta E \approx \frac{\hbar}{\Delta t} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{4.7 \times 10^{-24}\ \text{s}} = 2.3 \times 10^{-11}\ \text{J}\left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = 1.4 \times 10^{2}\ \text{MeV}$$

$$m = \frac{\Delta E}{c^2} \approx 1.4 \times 10^{2}\ \text{MeV}/c^2 \quad \boxed{\sim 10^{2}\ \text{MeV}/c^2}$$

(b) From Table 46.2, $m_\pi c^2 = 139.6\ \text{MeV}$, $\boxed{\text{a pi-meson}}$

***46.44** (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

46.45 The total energy in neutrinos emitted per second by the Sun is:

$(0.4)(4\pi)(1.5 \times 10^{11})^2 = 1.1 \times 10^{23}\ \text{W}$

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$mc^2 = 3.6 \times 10^{39}\ \text{J}$

$m = 4.0 \times 10^{22}\ \text{kg}$

About $\boxed{\text{1 part in 50,000,000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

Goal Solution

The energy flux carried by neutrinos from the Sun is estimated to be on the order of 0.4 W/m^2 at Earth's surface. Estimate the fractional mass loss of the Sun over 10^9 years due to the radiation of neutrinos. (The mass of the Sun is 2×10^{30} kg. The Earth-Sun distance is 1.5×10^{11} m.)

G: Our Sun is estimated to have a life span of about 10 billion years, so in this problem, we are examining the radiation of neutrinos over a considerable fraction of the Sun's life. However, the mass carried away by the neutrinos is a very small fraction of the total mass involved in the Sun's nuclear fusion process, so even over this long time, the mass of the Sun may not change significantly (probably less than 1%).

O: The change in mass of the Sun can be found from the energy flux received by the Earth and Einstein's famous equation, $E = mc^2$.

A: Since the neutrino flux from the Sun reaching the Earth is 0.4 W/m^2, the total energy emitted per second by the Sun in neutrinos in all directions is

$$\left(0.4 \text{ W / m}^2\right)\left(4\pi r^2\right) = \left(0.4 \text{ W / m}^2\right)(4\pi)\left(1.5\times10^{11} \text{ m}\right)^2 = 1.13\times10^{23} \text{ W}$$

In a period of 10^9 yr, the Sun emits a total energy of

$$\left(1.13\times10^{23} \text{ J / s}\right)\left(10^9 \text{ yr}\right)\left(3.156\times10^7 \text{ s / yr}\right) = 3.57\times10^{39} \text{ J}$$

in the form of neutrinos. This energy corresponds to an annihilated mass of

$$E = m_\nu c^2 = 3.57\times10^{39} \text{ J} \qquad \text{so} \qquad m_\nu = 3.97\times10^{22} \text{ kg}$$

Since the Sun has a mass of about 2×10^3 kg, this corresponds to a loss of only about 1 part in 50 000 000 of the Sun's mass over 10^9 yr in the form of neutrinos.

L: It appears that the neutrino flux changes the mass of the Sun by so little that it would be difficult to measure the difference in mass, even over its lifetime!

46.46 $\text{p} + \text{p} \to \text{p} + \pi^+ + X$

We suppose the protons each have 70.4 MeV of kinetic energy. From conservation of momentum, particle X has zero momentum and thus zero kinetic energy. Conservation of energy then requires

$$M_\text{p}c^2 + M_\pi c^2 + M_X c^2 = \left(M_\text{p}c^2 + K_\text{p}\right) + \left(M_\text{p}c^2 + K_\text{p}\right)$$

$$M_X c^2 = M_\text{p}c^2 + 2K_\text{p} - M_\pi c^2 = 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} = 939.5 \text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

***46.47** We find the number N of neutrinos: $10^{46}\text{ J} = N(6\text{ MeV}) = N(6 \times 1.6 \times 10^{-13}\text{ J})$

$$N = 1.0 \times 10^{58}\text{ neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi(1.7\times 10^5\text{ ly})^2}\left(\frac{1\text{ ly}}{(3.0\times 10^8\text{ m/s})(3.16\times 10^7\text{ s})}\right)^2 = 3.1\times 10^{14}\text{ / m}^2$$

The number passing through a body presenting $5000\text{ cm}^2 = 0.50\text{ m}^2$ is then

$$\left(3.1\times 10^{14}\,\frac{1}{\text{m}^2}\right)\left(0.50\text{ m}^2\right) = 1.5\times 10^{14} \quad \text{or} \quad \boxed{\sim 10^{14}}$$

***46.48** By relativistic energy conservation, $E_\gamma + m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1-v^2/c^2}}$ (1)

By relativistic momentum conservation, $\dfrac{E_\gamma}{c} = \dfrac{3m_e v}{\sqrt{1-v^2/c^2}}$ (2)

Dividing (2) by (1), $X = \dfrac{E_\gamma}{E_\gamma + m_e c^2} = \dfrac{v}{c}$

Subtracting (2) from (1), $m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1-X^2}} - \dfrac{3m_e c^2 X}{\sqrt{1-X^2}}$

Solving, $1 = \dfrac{3-3X}{\sqrt{1-X^2}}$ and $X = \dfrac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04\text{ MeV}}$

46.49 $m_\Lambda c^2 = 1115.6\text{ MeV}$ $\Lambda^0 \to \text{p} + \pi^-$

$m_p c^2 = 938.3\text{ MeV}$ $m_\pi c^2 = 139.6\text{ MeV}$

The difference between starting mass-energy and final mass-energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7\text{ MeV} \quad \text{and} \quad p_p = p_\pi = p$$

Applying conservation of relativistic energy,

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7\text{ MeV}$$

Solving the algebra yields $p_\pi c = p_p c = 100.4\text{ MeV}$

Then, $K_p = \sqrt{(m_p c^2)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35\text{ MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3\text{ MeV}}$$

46.50 Momentum of proton is $qBr = \left(1.60\times10^{-19}\text{ C}\right)\left(0.250\text{ kg / C}\cdot\text{s}\right)\left(1.33\text{ m}\right)$

$p_p = 5.32\times10^{-20}\ \dfrac{\text{kg}\cdot\text{m}}{\text{s}}$ $cp_p = 1.60\times10^{-11}\ \dfrac{\text{kg m}^2}{\text{s}^2} = 1.60\times10^{-11}\text{ J} = 99.8\text{ MeV}$

Therefore, $p_p = 99.8\ \dfrac{\text{MeV}}{c}$

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(938.3)^2 + (99.8)^2} = 944\text{ MeV}$

For pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \dfrac{\text{MeV}}{c}$ $E_{0\pi} = 139.6\text{ MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\text{ MeV}$$

Thus, $E_{\text{Total after}} = E_{\text{Total before}} = \text{Rest Energy}$

Rest Energy of unknown particle = 944 MeV + 172 MeV = 1116 MeV (This is a Λ^0 particle!)

Mass = $\boxed{1116\text{ MeV}/c^2}$

46.51 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1192.5\text{ MeV}/c^2$ and $m_\Lambda = 1115.6\text{ MeV}/c^2$

Conservation of energy requires $E_{0,\Sigma} = \left(E_{0,\Lambda} + K_\Lambda\right) + E_\gamma$,

or $$1192.5\text{ MeV} = \left(1115.6\text{ MeV} + \frac{p_\Lambda^2}{2m_\Lambda}\right) + E_\gamma$$

Momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1192.5\text{ MeV} = \left(1115.6\text{ MeV} + \frac{p_\gamma^2}{2m_\Lambda}\right) + E_\gamma$$

or $$1192.5\text{ MeV} = \left(1115.6\text{ MeV} + \frac{p_\gamma^2 c^2}{2m_\Lambda c^2}\right) + E_\gamma$$

Recognizing that $m_\Lambda c^2 = 1115.6\text{ MeV}$ and $p_\gamma c = E_\gamma$,

we now have $$1192.5\text{ MeV} = 1115.6\text{ MeV} + \frac{E_\gamma^2}{2(1115.6\text{ MeV})} + E_\gamma$$

Solving this quadratic equation, $E_\gamma \approx \boxed{74.4\text{ MeV}}$

46.52 $p + p \to p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} = \boxed{70.4\text{ MeV}}$$

46.53 Time-dilated lifetime: $T = \gamma T_0 = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - v^2/c^2}} = \dfrac{0.900 \times 10^{-10}\text{ s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10}\text{ s}$

$$\text{distance} = (0.960)(3.00 \times 10^8\text{ m/s})(3.214 \times 10^{-10}\text{ s}) = \boxed{9.26\text{ cm}}$$

46.54 $\pi^- \to \mu^- + \bar{\nu}_\mu$

From the conservation laws, $m_\pi c^2 = 139.5\text{ MeV} = E_\mu + E_\nu$ [1]

and $p_\mu = p_\nu, \quad E_\nu = p_\nu c$

Thus, $E_\mu^2 = \left(p_\mu c\right)^2 + (105.7\text{ MeV})^2 = \left(p_\nu c\right)^2 + (105.7\text{ MeV})^2$

or $E_\mu^2 - E_\nu^2 = (105.7\text{ MeV})^2$ [2]

Since $E_\mu + E_\nu = 139.5\text{ MeV}$ [1]

and $\left(E_\mu + E_\nu\right)\left(E_\mu - E_\nu\right) = (105.7\text{ MeV})^2$ [2]

then $E_\mu - E_\nu = \dfrac{(105.7\text{ MeV})^2}{139.5\text{ MeV}} = 80.1$ [3]

Subtracting [3] from [1], $2E_\nu = 59.4\text{ MeV}$ and $\boxed{E_\nu = 29.7\text{ MeV}}$

***46.55** The expression $e^{-E/k_BT}\,dE$ gives the fraction of the photons that have energy between E and $E+dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}\,dE}{\int_0^\infty e^{-E/k_BT}\,dE}=\frac{\int_E^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}{\int_0^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}=\frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty}=e^{-E/k_BT}$$

We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

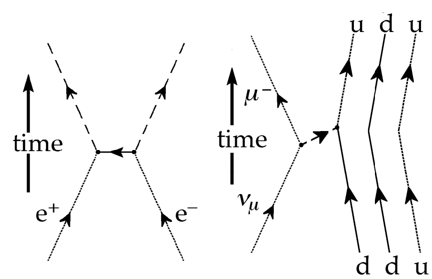
and $E=1.00\text{ eV}=1.60\times10^{-19}\text{ J}$

Thus, $0.0100=e^{-\left(1.60\times10^{-19}\text{ J}\right)/\left(1.38\times10^{-23}\text{ J/K}\right)T}$

or $\ln(0.0100)=-\dfrac{1.60\times10^{-19}\text{ J}}{\left(1.38\times10^{-23}\text{ J/K}\right)T}=-\dfrac{1.16\times10^4\text{ K}}{T}$ giving $T=\boxed{2.52\times10^3\text{ K}}$

46.56 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

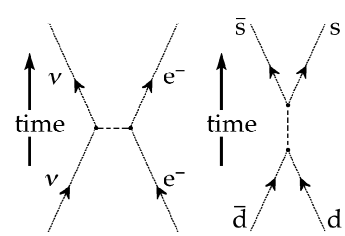
(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge +1*e* and is a $\boxed{W^+}$.



(a) (b)

46.57 (a) The mediator of this weak interaction is a $\boxed{Z^0\text{ boson}}$.

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$. For conservation of both energy and momentum, we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.57. Depending on the color charges of the d and $\bar{d}$ quarks,



(a) **(b)**

the ephemeral particle could also be a gluon , as suggested in the discussion of Figure 46.13(b).